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Numerical Investigation of Unsteady Flow and Heat Transfer in Wavy Channels

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Abstract

Two-dimensional Navier-Stokes and energy equations have been solved numerically for unsteady laminar flow in periodic wavy (sinusoidal and triangular) channels. The flow in the channels has been observed to be steady up to a critical Reynolds number. Beyond the critical Reynolds number the flow becomes self-sustained quasiperiodic oscillatory. This transition of flow occurs at lower Reynolds number for triangular channel relative to sinusoidal channel. The frequencies of oscillations, the friction factors and Nusselt numbers are reported.

Introduction

A simple geometry of the flow passage that is relatively easy to fabricate and may be used to enhance the heat transfer rate is wavy, periodic channel. Wavy channel can provide significant heat transfer augmentation if operated in an appropriate (transitional) Reynolds number (Re) range. Therefore, wavy passages have been considered in several earlier studies as a means to enhance heat / mass transfer in compact exchange devices. Both corrugated and converging-diverging cross sections have been studied experimentally and numerically. An important observation made is that wavy passages do not provide any significant heat transfer enhancement when the flow is steady. However, if the flow is made unsteady (either through external forcing or through natural transitioning to an unsteady state) significant increases in heat exchange are observed. This is a result of complex interactions between the core fluid and boundary layer fluid through shear layer destabilization and self-sustained oscillations. It is in this regime that such passages can be very effective and our objective, therefore, has been to quantify such gains as well as penalties (increased pressure drop) through accurate and well resolved numerical computations of the unsteady flow and heat exchange processes.

Several literatures are available on steady state solutions of wavy channels [8,11,7,9,5,6]. Saidi et al. [8] studied laminar flow past a sinusoidal cavity. They presented how increase of flow velocity gave birth vortex inside a cavity and affected the hydrodynamic and the thermal performance. Wang and Vanka [11] reported higher values of friction factor for wavy channel compared to the parallel plate channel of same inter-wall spacing. Nishimura et al. [7] investigated flow characteristics such as flow pattern, pressure drop and wall shear stress in a channel with symmetric sinusoidal wavy wall. This study reported that at Reynolds number greater than 700, turbulent flow occurred owing to the onset of unsteady vortex motion. Sparrow et al. [9] presented the effect of inlet condition, inter wall spacing and protruding edge on fluid flow and heat transfer.

Studies on fully developed flow in periodic converging-diverging passages with uniform in-flow report a Hopf bifurcation at Re=130, followed by a series of bifurcations leading to chaos [2]. The flow was observed to be quasi-periodic with up to three fundamental frequencies and multiple sub- and super-harmonics in the Reynolds number range of 130-800. At 850, the flow became aperiodic with broad band frequency spectra of the velocity signals. Stone and Vanka [10] have presented numerical results on developing flow and heat transfer characteristics in a furrowed wavy channel. They found that at low Reynolds numbers, the flow in the wavy passage is steady, characterized by steady separation bubbles in the troughs of the waves. However, as the Reynolds number is increased beyond a modest value, the flow becomes unsteady, with the rolling up of the shear layers on the channel walls. When the flow becomes unsteady, there is increased mixing between the core and near-wall fluids, resulting in enhanced heat transfer rates and pressure drops.

The present paper deals with the flow structure and heat transfer of wavy channels at unsteady state with periodic boundary conditions. Two different types of surface waviness one sinusoidal channel and another triangular channel have been considered for the present investigation. For both of the geometry, individual minimum height has been varied to understand the flow and heat transfer behavior properly.

Conservation Equations

In the present study, the flow is considered to be two-dimensional with no variation in the span wise direction. The governing equations for flow and energy transport can be written as:

$$\nabla.\boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{uu}) = -\frac{1}{r}\nabla p + n\nabla^2 \boldsymbol{u} \tag{2}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (\boldsymbol{u}q) = \frac{1}{a}\nabla^2 q \tag{3}$$

where $\boldsymbol{u}$ is the velocity vector, $\theta = (T-T_w)/(T_{b,in}-T_w)$, and $T_{b,in}$ is the bulk mean temperature of the flowing fluid at inlet, ν and α are kinematic viscosity and thermal diffusivity respectively.

Computational Details

The base geometry (minimum height, H_{min} = 6mm, maximum height, H_{max} = 20mm, amplitude, a = 3.5mm, and wavelength, λ= 28mm) considered in the present investigation are shown in figure 1. These geometrical configurations conform to the channel studied experimentally by Nishimura et al. [7] and numerically by Wang and Vanka [11]. Here minimum heights are varied keeping the others constant. As boundary conditions, no-slip conditions with a constant wall temperature are prescribed along the wall. Thus

u_w =0, v_w =0, and θ_w = 1.

A uniform velocity is prescribed at the inlet. At the stream wise direction, the following periodic boundary conditions (equation 4) are used to attain fully-developed flow.

$$u(0,y)=u(\lambda,y),\ v(0,y)=v(\lambda,y),\ \theta(0,y) = \theta(\lambda,y) \qquad (4)$$

In this study, the integral forms of governing equations are discretized using control volume based Finite Volume method with collocated arrangement. The final discretized form of governing equations are solved iteratively using TDMA solver. Time integration is done using three time level method [1]. All the calculations are performed using 64×64 grid size with a time step of 0.001sec. A systematic grid refinement test has been done using 32×32, 48×48 and 64×64 grid sizes [3].

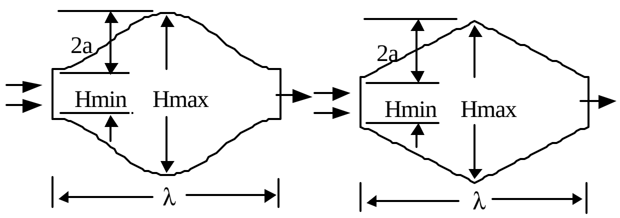

Figure 1. Detail geometry configurations of sinusoidal (left) and triangular (right) channels.

Results and Discussion

Calculations were performed for several Reynolds numbers ($Re=u_{avg,in}\ H_{min}/\nu$) from a low value to 500. It has been observed that at low Reynolds numbers, the flow in the wavy passages (both sinusoidal and triangular) is steady, characterized by steady separation bubbles in the troughs of the waves. With the increase of Reynolds number beyond a certain critical value the flow becomes unstable and bifurcates, with the rolling up of shear layers on the channel walls. The critical Reynolds numbers for both geometry are reported in table 1. It has been observed that the value of critical Reynolds number increased with the increase of H_{min} for sinusoidal channel, but it decreased in case of triangular channel. The transition to unsteady flow for triangular channel occurs at a lower Reynolds number than for the sinusoidal channel. This is the consequences of the fact that the edges of the triangular channel are relatively sharper than the edges of the sinusoidal channel, and thus contribute to the formation, at lower Reynolds number, of an unstable jet-pattern, which easily becomes unsteady.

H_{min} (mm)	Critical Reynolds Numbers			Frequency (Hz)	
	Sinusoidal channel		Triangular channel	Sinusoidal channel	Triangular channel
	Stone & Vanka [10]	Present Prediction			
3	130	150	150	31	25
6	190	205	130	20	60
9	240	240	100	27	27

Table 1. Critical Reynolds number and Frequency of oscillation

Instantaneous streamline plot at Re =300 for both the channels are shown in figure 2. It has been shown [4] that steady flow yields recirculating vortices in each of the cavities, accompanied by straight cross flow. A single trapped vortex fills each of the cavities. But here at unsteady state separation vortices are formed in the wavy cavity at an earlier instant that is slowly engulf by the shear layer. This interaction of the core fluid with the fluid in the cavities replenishes the thermal boundary layer and results in enhanced heat transfer. Corresponding temperature field at this Reynolds number is shown in figure 3.

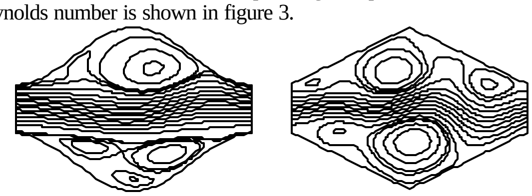

Figure 2. Instantaneous streamline plot for sinusoidal (left) and triangular (right) channel at Re = 300.

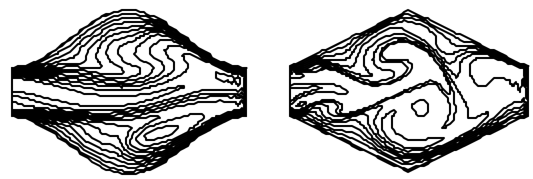

Figure 3. Temperature filed for sinusoidal (left) and triangular (right) channel at Re = 300.

A velocity probe was arbitrarily placed at height of $0.75H_{max}$ in the tallest part of the each of the channel. The time signal of the u-velocity at Reynolds number 300 at the probe height for sinusoidal and triangular channels and the corresponding FFT analyses are shown in figure 4 and 5 respectively. At this Reynolds number, self-sustained quasiperiodic oscillatory flow was observed. For sinusoidal channel the fundamental frequency was 20 with some of its harmonics as it is shown by the FFT analysis of the u-velocity. In the case of triangular channel, the fundamental frequency of oscillation was 60 and the FFT analysis is chaotic and shows multiple secondary harmonics. To find out cause for the higher value of the frequency of oscillation, the frequencies for the H_{min} of the closer values have been

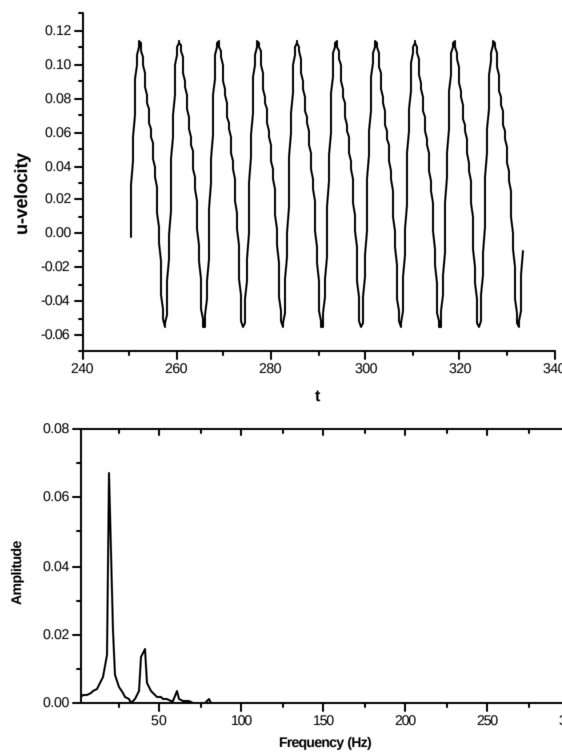

Figure 4. Time signal of u-velocity at Re =300 for sinusoidal wavy channel (top) and corresponding FFT (bottom) : H_{min} = 6mm.

obtained and frequencies for H_{min}= 4, 5, 7 and 8mm are found to be 40, 45, 24 and 26 respectively. Here, we got the frequencies of the flow disturbances are to be independent of Re, but function of the geometry, which is shown in table 1.

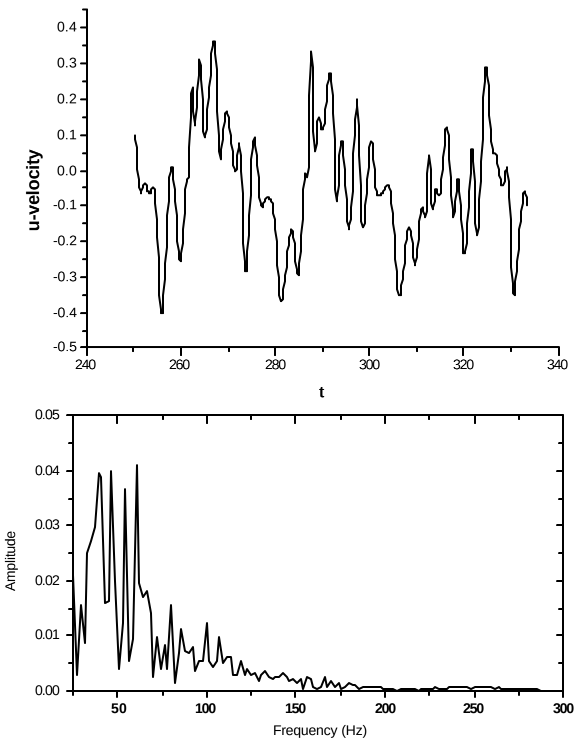

Figure 5. Time signal of u-velocity at Re =300 for triangular wavy channel (top) and corresponding FFT (bottom) : H_{min} = 6mm.

A comparison of the time mean friction factor averaged over the wave length is shown in figure 6 for sinusoidal wavy channel of H_{min} = 6 mm. The present prediction slightly under predicts the experimental data of Nishimura et al.[7] but shows a good agreement with the predicted values of Wang & Vanka [11]. In the steady regime the friction factor is approximately twice that of the planer channel. In the unsteady regime the friction factor is even more.

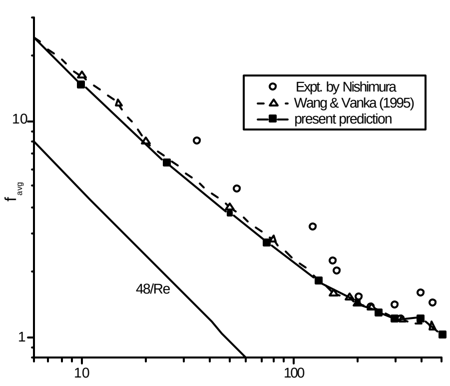

Figure 6. Comparison of present predicted average friction factor with Nishimura et. al. [7] and Wang & Vanka[11] for H_{min} = 6 mm of sinusoidal wavy channel.

The effect of the aspect ratio (by changing the minimum height) on average friction factor($f_{avg} = 2\Delta p.D_h/\rho\lambda u^2_{avg,in}$) is shown in figure 7 for both the channels. Friction factor is higher in all cases than the straight channel (48/Re) for all Reynolds number, and with the decrease of H_{min} friction factor increases. For the same H_{min}, friction factor of triangular channel is lower than the sine-shaped (wavy) channel, this is because of less effective area of the triangular channel.

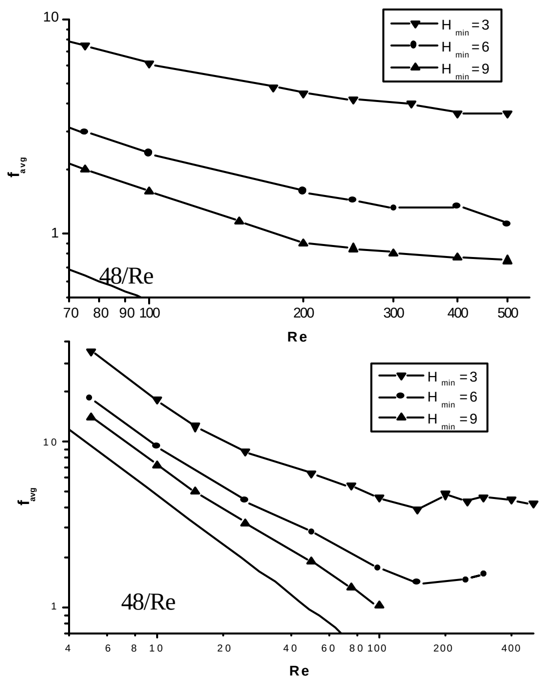

Figure 7. Effect of the minimum height on time averaged friction factor for sinusoidal channel (top) and triangular channel (bottom).

Figure 8 shows the time averaged friction factor and Nusselt number ($Nu = hD_h/k$, $D_h = H_{min} + H_{max}$) at various Re for both steady and unsteady flow in the sinusoidal channel. After slight increase in friction factor when the flow first becomes unsteady,

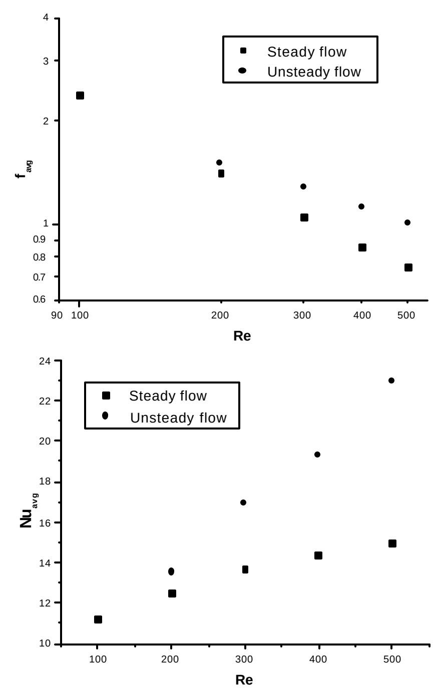

Figure 8. Time averaged friction factor (top) and Nusselt number (bottom) for the sinusoidal channel (base case).

time averaged friction factor continues to decrease with Re. However, the rate of this decrease slows down as Re is increased. On the other hand, Nusselt number increases with increase in Re. Steady flow gives modest increase in Nusselt number but unsteady flow gives rapid increase due to better mixing of core and near wall fluids, but this rate of increase again slows down as Re is increased more. Hence, there comes a point where the increasing Re renders diminishing benefits in heat transfer performance. The optimal value of Re depends on the specific criteria for evaluating performance, and on the dimensions of the passage. But it is known that wavy passages generally offer the best enhancement in the transitional regime.

Time mean Nusselt number averaged over the wave length for the sinusoidal and triangular channels of H_{min} = 6mm are shown in figure 9. The Nusselt number for both the cases is higher than that for a straight channel and increases with Reynolds number. The rate of increase of Nusselt number for triangular channel is higher than that of sinusoidal channel. At low Reynolds number (less than Re=275) sinusoidal channel has higher heat transfer capability than a triangular channel but above Re=275 the triangular channel channel gives more heat transfer.

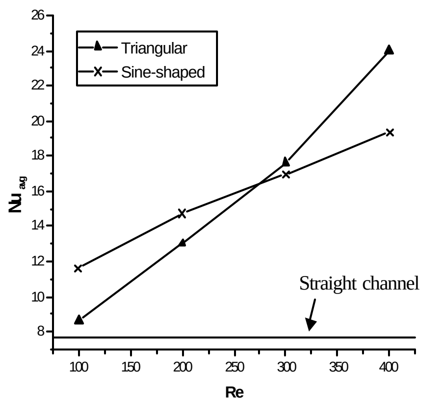

Figure 9. Average Nusselt number at different Reynolds number for H_{min} = 6mm.

Conclusion

Fluid flow and heat transfer in periodic, corrugated channels have been numerically investigated at unsteady flow conditions using finite volume method for a fluid with Prandtl number 0.7, representative value for air. Periodic boundary conditions are used to attain the fully developed flow condition. Two different types of wavy geometry, sinusoidal and triangular, are considered. Effect of aspect ratio has been studied by changing the H_{min} only. It has been observed that the flow becomes unstable with a self-sustained oscillation beyond a certain critical Reynolds number and thereby increase heat transfer rate. For sinusoidal channel the critical Reynolds number increases with the increase of H_{min}, but decreases in case of triangular channel. FFT analysis of the u-velocities shows that one fundamental frequency of oscillation prevails at all the Reynolds number for a particular geometry. The transition of flow occurs earlier, at lower Reynolds number, for triangular channel relative to sinusoidal channel.

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