Solution of stability problems by a low-order finite volume method

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The present study is devoted to analysis of stability of steady buoyancy convection flows by a low-order finite volume method. We consider several benchmark problems, part of which are widely known, and another part is added here to complete the study. The motivation for this work is the necessity to perform the stability analysis for many applied problems, which cannot be treated by spectral of pseudospectral methods.

It is well-known that spectral and pseudospectral methods yield the most accurate solutions for benchmark problems considering flows in rectangular cavities, and especially instabilities of these flows. However, these methods are restricted to simple geometries and because of this cannot be applied to many practically important problems. As an example one can mention problems of melt instabilities in bulk crystal growth processes, which was the motivation for one of the most well-known convective benchmarks [1].

Here we apply the second-order finite volume method to several problems of buoyancy and thermocapillary convection in rectangular cavities. It is emphasized that the numerical technique briefly described below is not restricted to a certain class of problems and already was applied for stability studies in Czochralski [2] and floating zone [3] crystal growth configurations. The studies of this kind usually have two main bottlenecks. The first one is connected with the calculation of steady state flows, whose stability is to be studied. The Jacobian-free or other inexact Newton methods combined with a Krylov-subspace-based iterative linear solver usually are applied for this purpose. These solvers are very effective when relatively simple benchmark problems are considered, however fail to converge in more complicated cases. There were also some reports about possible loss of accuracy when Jacobian-free approach is applied. Therefore in the present study we calculate the Jacobian matrix using the corresponding analytical evaluations, which follow from the discretized equations. We also argue that when very fine grids are used and due to the high level of the sparseness of the Jacobian matrix it can be more effective to replace iterative solvers by direct ones.

The second bottleneck is connected with the eigenvalue problem of very large dimension, which must be solved for the study of linear stability of a steady flow. The usual approach here is the Arnoldi iteration method, which allows one to calculate only necessary part of the whole spectrum. The Arnoldi iteration also needs computation of the Krylov-subspace basis. An additional difficulty here is connected with the incompressible continuity equation, which does not contain the time derivative. The latter requires considering the eigenvalue problem in the shift-and-invert mode. Consequently, the Krylov basis vectors are to be computed as solutions of a system of linear algebraic equations. Again, we argue here that instead of iterative solvers, which can diverge and be CPU-time consuming, it is more effective to built an **LU**-decomposition of the matrix, so that the necessary amount of the Krylov basis vectors will be computed by the back substitution.

The effectiveness of the application of the direct sparse matrix solvers described here is the consequence of the matrix sparseness, which follows from the low-order discretization method applied. As reported below, we are able to perform calculations of steady states and stability analysis on the grids consisting of 450^2 nodes. Apparently, time-stepping algorithms can handle much finer grids. To the best of our knowledge, however, the direct stability analysis of numerically calculated flows was never performed on the grids of this size. The only restriction for the further grid refinement is the computer memory consumed by a direct sparse matrix solver. The results reported here are obtained on an Itanium-2 workstation with 16 Gbytes memory.

The convergence studies reported below show that correct critical parameters can be calculated only on rather fine grids having more than 100 nodes in the shortest direction. We show that use of uniform grids combined, where possible, with the Richardson extrapolation can significantly improve results. We show also that the mesh stretching can significantly speed up the convergence, but there can be also a certain loss of accuracy.

References

1. Roux B (ed.). Proc. GAMM Workshop on Numerical Solution of Oscillatory Convection in Low Prandtl Number Fluids. Marseille, 1988," Notes on Numerical Fluid Mechanics, Vieweg Braunschweig, 1989; **27**.
2. Gelfgat AY, Rubinov A, Bar-Yoseph PZ, Solan A. numerical study of three-dimensional instabilities in a hydrodynamic model of Czochralski growth, *J. Cryst. Growth*. 2005; **275**: e7-e13.
3. Gelfgat AY, Rubinov A, Bar-Yoseph PZ, Solan A. On the three-dimensional instability of thermocapillary convection in arbitrary heating floating zones in microgravity environment. *Fluid Dynamics & Material Processing*. 2005; **1**: 21-31.