# Experiments Regarding Transition in a Subcritical plane Poiseuille Flow

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**Abstract.** The purpose of this work is to demonstrate experimentally the scaling law for threshold amplitude of perturbations to trigger nonlinearity in subcritical plane Poiseuille flow as function of the Reynolds number. The disturbances are introduced through an almost streamwise independent slot drilled at the bottom wall of a horizontal air channel flow. Beyond a critical injection rate of the disturbance, nonlinear phenomenon is observed by the initiation of hairpin vortices. The vortices are located close to the central part of the channel and the separating distance between them is of the order of the half channel height. The normalized critical injection rate ($v_0$) scales with the Reynolds number ($Re$) as $v_0 \sim Re^{-3/2}$, as predicted by Chapman [3].

## 1. Introduction

Plane Couette flow and pipe Poiseuille flow are known to be linearly stable to infinitesimal disturbances at all Reynolds numbers ($Re$) [1]. Nevertheless, at a sufficiently large Reynolds number, these flows undergo transition to turbulence. Other flows, such as plane Poiseuille flow often become turbulent at relatively low Reynolds number (e.g. $Re \approx 1000$) well before their critical Reynolds number for normal mode instability ($Re_c \approx 5772$, [2]). As pointed out by Chapman (2000) [3], one school of thought (which goes back to Thomson, 1887 [4] and Orr, 1907 [5]) is that the domain of attraction of the laminar flows shrinks as $Re \rightarrow \infty$, so the flow becomes nonlinearly unstable to small but finite amplitude perturbations.

Trefethen *et al.* (1993)[6] suggested to determine the smallest value of $\gamma$ for which the threshold amplitude for transition decreases as $Re^{-\gamma}$ for positive values of $\gamma$ and for large Reynolds numbers. They in fact conjectured that $\gamma < -1$. Subsequent numerical experiments [7] indicated that $\gamma = -7/4$ for plane Poiseuille flow using subcritical Reynolds numbers. More recently, however, Chapman (2002)[3], through a formal asymptotic analysis of the Navier-Stokes equations found that in the case of plane Poiseuille flow, when the unstable modes are factored out, $\gamma = -1.5$ for streamwise initial perturbations whereas it equals to -5/4 for initial oblique disturbances.

In this work we focus on the route to transition which begins with the formation of streamwise vortices (approximately aligned with the basic laminar flow), the subsequent formation of streamwise streaks of relatively low and high velocity and the final evolution of secondary instability of oblique modes (i.e., any perturbation which is streamwise dependent). By examining the linearized equations asymptotically ($Re \rightarrow \infty$), it is evident that the transient growth of streamwise modes (perturbations with infinitely long streamwise wavelength $\sim Re$)

is proportional to $Re$ over a timescale of order $Re$. In other words, an initial vertical velocity perturbation ($v_0$) will have a transient growth in its vertical vorticity ($\eta$) to order $v_0 Re$ (e.g. Chapman, 2002 [3]). It should be noted that the initial vertical velocity is assumed to vary along the spanwise direction.

In the next step Chapman (2002)[3] determined how large a streamwise streak has to be in order to make an eigenvalue of a secondary instability become unstable. He found that the center modes (localized in the region around the center of the channel) are the most efficient modes to undergo secondary transition. Moreover, among these modes the one that requires minimum perturbation is an odd mode (i.e., $\partial v/\partial y = \eta = 0$) having a wavelength of order one, for which the streamwise streak ($\eta$) has to be of order ($Re^{-1/2}$) to generate instability. In summary, the required initial threshold amplitude ($v_0$) for transition in plane Poiseuille flow involving the growth of streamwise streaks (from streamwise independent counter-rotating vortices) and their breakdown through secondary instability of oblique modes should vary as $Re^{-3/2}$. In this transition we expect to see secondary odd mode instabilities located around the center of the channel and their wavelength should scale with the half channel height.

For experimental investigation of transition, the above scenario was interpreted by Chapman (2002) to require a sufficiently long channel of $O(Re)$. In fact, for the case of a long pipe (785 diameters) Poiseuille flow, Hof, Juel and Mullin (2003)[8] demonstrated experimentally the scaling law for transition. Accordingly, the amplitude of perturbation required to cause transition scales as $O(Re^{-1})$. Here, the objective of the present article is to show experimentally that the scaling of threshold amplitude ($v_0 \sim Re^{-3/2}$) is valid for the case in which artificial disturbances having small but finite amplitude $v_0$ are introduced into plane Poiseuille flow. In this case the length required for the transient growth of the streaks is shortened significantly and nonlinearity is indicated by the first appearance of hairpin vortices.

## 2. Experimental Setup

The Poiseuille flow facility is described in Svizher and Cohen (2006) [9]. It consists of an open circuit air facility, a centrifugal blower, a noise reduction chamber, a diffuser and a settling chamber followed by a contraction nozzle and a channel. A 3 $m$ long channel is placed horizontally downstream of the nozzle outlet. The channel consists of two 10 $mm$ thick glass plates separated by 50 $mm$ thick bars of aluminium which are positioned to give a channel width of 570 $mm$ (see Fig. 1). Two 300 $mm$ diameter aluminium plugs are flush-mounted at the lower and upper plates in the middle of the channel, 2 $m$ downstream of the channel inlet. An optical window, 70 $mm$ in diameter, is installed within each plug, allowing one to observe or/and illuminate the region of interest from the vertical direction. Two rectangular ($80 \times 50$ $mm$) windows, mounted on the side-bars, provide observation of the region of interest from the horizontal (spanwise) direction.

Coherent structures are artificially generated in a laminar Poiseuille flow by using continuous air injection through a streamwise slot ($57 \times 1$ $mm$), drilled in the lower plug. This provides an almost streamwise independent initial vertical velocity with a spanwise variation. The fact that the injection is only introduced from the lower wall corresponds approximately to an odd mode. All velocities and lengths are normalized by the laminar centerline velocity $U_{cl}$ and the half-channel height, $h = 25$ $mm$, respectively. The coordinate system has the $X$-, $Y$- and $Z$-axes aligned in the streamwise, wall-normal and spanwise directions, respectively. The origin is at the downstream edge of the slot.

For given channel height and injection slot geometry, there are two parameters that govern the flow. The first one is the base flow Reynolds number, defined as $Re = U_{cl}h/\nu$, where $\nu = 1.5 \cdot 10^{-5}$ $m^2/sec$ is the air kinematic viscosity at the experimental conditions. The second governing parameter is the normalized disturbance amplitude, defined as the average injection velocity (over the outlet cross-section area of the slot) normalized by the laminar centerline

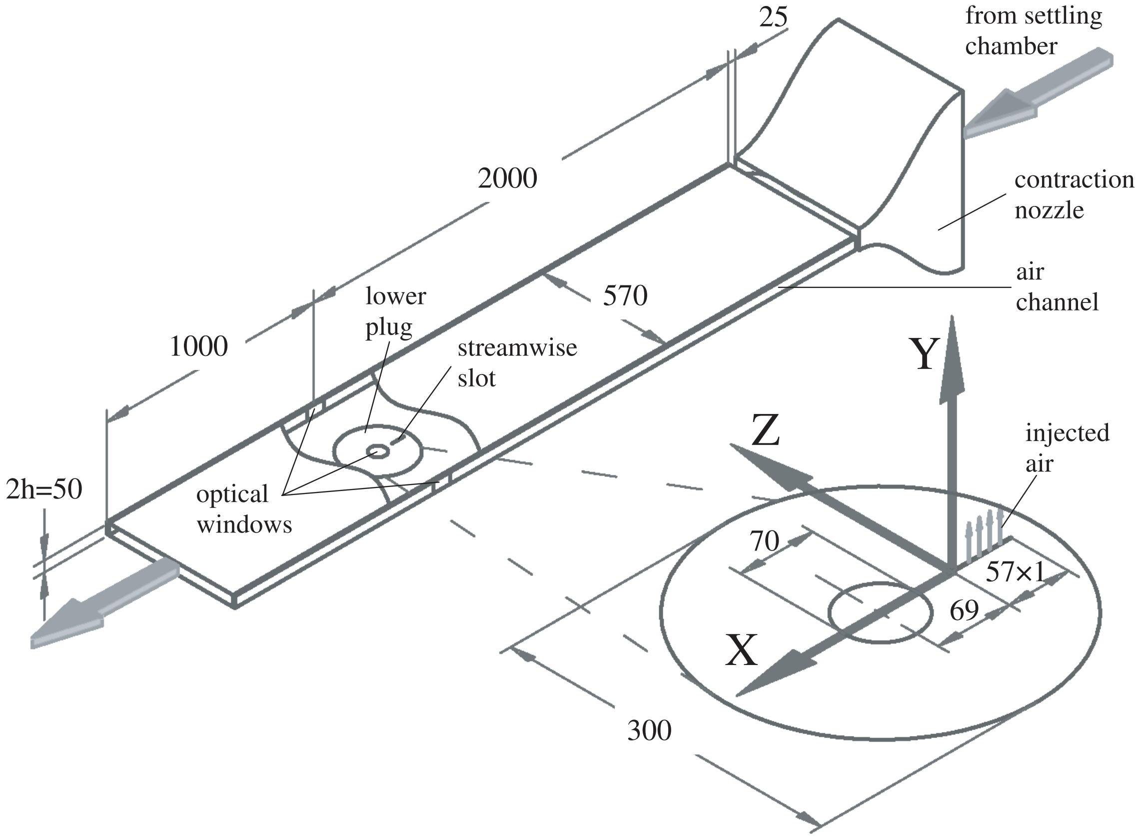

**Figure 1.** The air channel facility. All dimensions are given in millimeters.

velocity: $V_{inj} = Q_{inj}/(S_{inj}U_{cl})$, where $Q_{inj}$ is the injection flow rate and $S_{inj} = 57 \times 1\ mm^2$ is the slot cross-section area. It should be mentioned that a similar normalization was employed by Hof *et. al* (2003) [8]. The disturbance was visualized by adding tracer particles to the secondary flow, produced by a fog machine via the condensation of oil vapour, and illuminated by a laser light sheet.

## 3. Results

Flow visualization, in conjunction with hot wire measurements were performed to obtain the following results. The base flow profile (without any injection of secondary flow) was obtained by hot wire measurements. A parabolic laminar profile corresponding to plane Poiseuille flow was obtained except for a slight deviation, caused by the fact that the flow was not fully developed. Nevertheless, the effect of this deviation was found to be secondary [9]. The maximum level of turbulence of the undisturbed flow was observed to be smaller than 0.2% for all Reynolds numbers considered.

Flow visualization was used to monitor the initiation of hairpin generation in the Poiseuille flow. Initially, a steady base flow was attained for a fixed Reynolds number. Keeping this base flow constant, a continuous injection of the secondary flow was introduced through the streamwise slot, starting with very small flow rate. With very low injection rates a counter-rotating pair of vortex was observed as long as the injection rate was below a critical value. At this critical value the head of hairpin vortices was first observed. This injection rate was monitored and is called here the 'critical injection rate'. Fig. 2(a) shows the cross-section of the

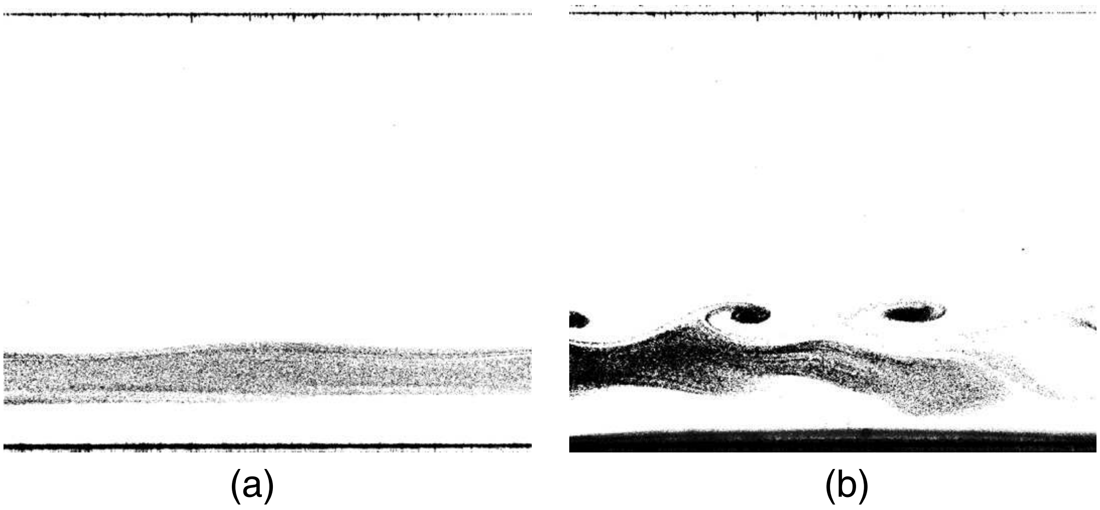

**Figure 2.** Smoke flow visulaization of secondary flow for $Re = 1660$ ($a$) Below critical $V_{inj} = 0.29$ and ($b$) At critical $V_{inj} = 0.29$.

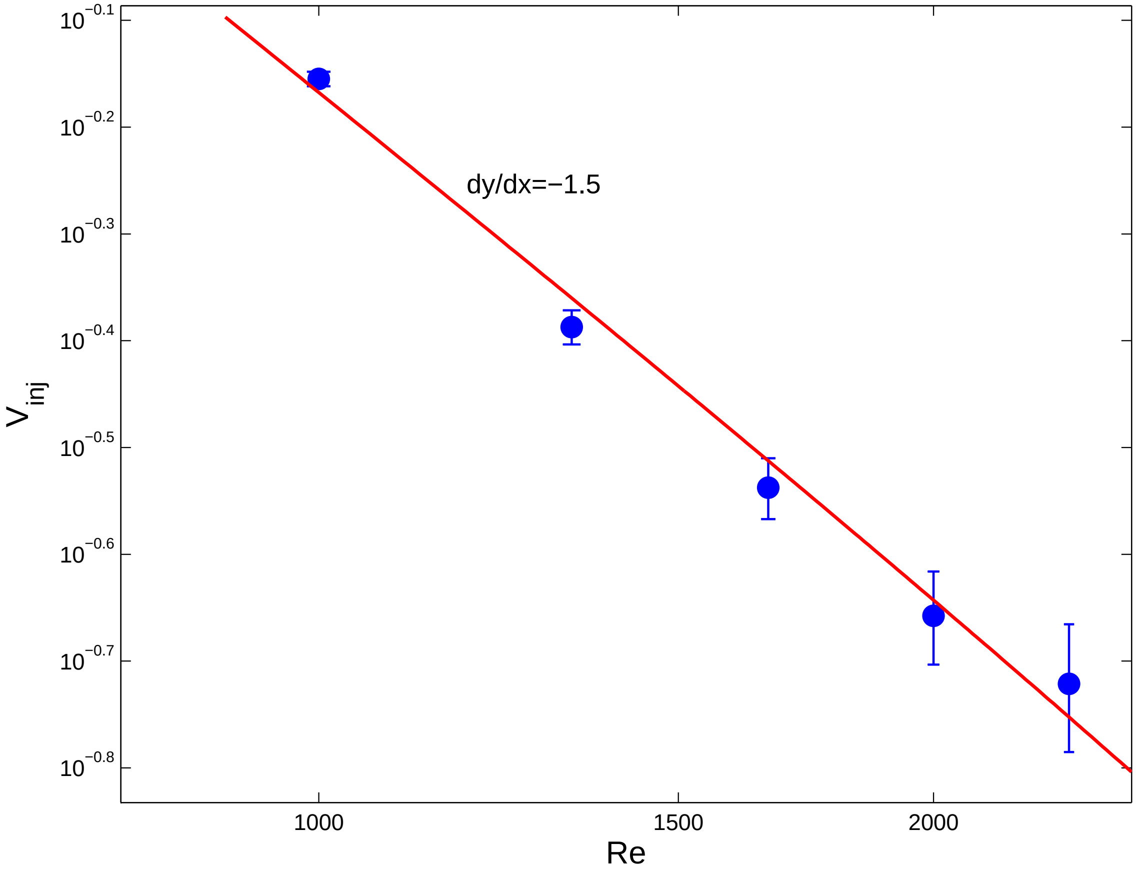

**Figure 3.** A log-log plot indicating the first appearance of hairpin vortices in subcritical conditions. The symbols represent the experimental data whereas the solid line represents the −1.5 slope predicted by Chapman [3].

streamwise vortex when the injection rate is just below its critical value at $Re = 1660$. The base flow is from left to right of the figure and the vertical span indicates the top and bottom walls of the channel. The instability of the streamwise streaks in the form of waviness can be observed. When the injection rate is slightly increased (in this case to $V_{inj} = 0.29$), hairpin vortices are generated from the crests of the wavy streak and can be seen in Fig. 2(b). The head once formed, grows along the flow direction and attains a steady position along the center of the channel. The distance between two adjacent hairpins' heads increases as they move towards the center of the channel and is on the order of the channel half height. The critical injection rate was monitored for various Reynolds numbers of the base flow.

The normalized critical injection velocity is plotted against various values of the Reynolds number on a logarithmic scale in Fig. 3. The slope of this graph, fitted by the least squares method is 1.53. The agreement between this exponent and that predicted theoretically by Chapman (2002) is very good.

## 4. Conclusions

The scaling law of threshold amplitude as function of $Re$ predicted for subcritical transition in plane Poiseuille flow by Chapman (2002)[3] is confirmed experimentally. Accordingly, streamwise vortices, produced experimentally by long streamwise (almost independent) injection of vertical velocity become unstable to secondary instability which in turn triggers nonlinearity. This process of nonlinearity is well captured by the initiation of hairpin vortices, suggesting them to be a key element in transitional and post-transitional flows.

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