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EXPERIMENTAL AND MODELLING INVESTIGATIONS OF DROPLET DISPERSION IN A TURBULENT JET

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ABSTRACT

The turbulent dispersion of non-evaporating droplets in an axisymmetric round jet issuing from a nozzle is investigated both experimentally and theoretically. The experimental data set has a well-defined inlet boundary with low turbulence intensity at the nozzle exit, so that droplet dispersion is not affected by the transport of nozzle-generated fluctuating motion into the jet, and is influenced solely by gaseous turbulence produced in the shear layer of the jet. This data set is thus ideal for testing algebraic models of droplet fluctuating motion that assume local equilibrium with the gaseous turbulence. A two-fluid turbulence modelling approach is adopted, which uses such an algebraic model and the k - ε turbulence model to predict the droplet and fluid turbulent correlations, respectively. We have shown that the k - ε turbulence model lacks generality for predicting the spread of momentum in jets with and without a potential core. However, in general, the model predicts the radial dispersion of droplets in the considered turbulent jet with reasonable accuracy over a broad range of droplet sizes, once deficiencies in the k - ε turbulence model are taken into account.

INTRODUCTION

A number of two-fluid turbulence models have been developed in the past to simulate the dispersion of particles in a turbulent fluid flow (Elghobashi and Abou Arab, 1983; Pourahmadi and Humphrey, 1983; Chen and Wood, 1986). In the two-fluid modelling approach, both the fluid and the particle phases are treated as separate interpenetrating continua, with each phase represented by a different set of continuity and momentum balances and a mathematical formulation to account for the turbulent nature of that phase. Simonin (1991) has also contributed a number of these models of varying complexity, all derived in the general framework of the Probability Density Function (PDF) approach, each of which uses a different level of sophistication to capture the various mechanisms of interaction between the particles and the gas flow turbulence, and hence to determine the fluctuating motions (or turbulence) of the particles. His most basic model involves the turbulence theory of Tchen (Hinze, 1975), which describes the behaviour of particles in a steady homogenous turbulent flow. In this simplified approach, which is applicable to dilute flows of particles with low inertia, neither the transport of the gas-particle fluctuating velocity correlation nor the transport of particle velocity correlations are modelled directly. Rather, algebraic formulations are derived which relate these particle and gas-particle quantities via inertial drag expressions to the gas flow turbulence, whose transport is modelled directly using the two-equation (k - ε) turbulence model described by Launder and Sharma (1974). Simonin (1991) has applied this model to the problem of particle dispersion in turbulent jets, and has shown that this simplified approach predicts the turbulent dispersion of particles accurately far downstream of the nozzle, but cannot accurately predict particle dispersion close to the nozzle when the particle radial fluctuating velocity at the nozzle exit-plane is significantly affected by the injection method, since the transport of this particle property through the flow domain, and its effect on particle dispersion, is not modelled.

The higher order closure models of Simonin (1991) are more flexible than the basic model in this regard, because the particle and gas-particle fluctuating velocity correlations are modelled more rigorously using transport equations rather than algebraic formulations. In particular, Simonin (1991) has shown that the second moment closure model, based on separate transport equations for the droplet kinetic (or Reynolds) stress components and an eddy-viscosity assumption for the gas-droplet velocity correlations, is able to accurately predict the dispersion properties of particles within jet and shear flows. As pointed out by Simonin et al. (1995), such a model is especially successful in capturing the anisotropy of the particle turbulence, because it separates the production effect due to the mean shear, which acts predominately on the axial fluctuating velocity component, from the turbulence entrainment of the particles, which has a dominating effect on the radial fluctuating velocity component. Nevertheless, the simplified modelling approach based on Tchen's theory remains useful, especially when the additional computational expense of the more complex alternatives is considered. Whilst we recognise that the simple model is limited for use in many flows of practical interest, it provides an important starting point for simulations of droplet evaporation in the simple burners typically used in combustion studies (Masri et al., 1996), which are often carefully designed to produce very little turbulence at the nozzle exit.

The aim of this paper is to test the accuracy of the simple two-fluid turbulence model of Simonin for predicting the turbulent dispersion of droplets in an axisymmetric round jet. This paper extends the model validation work of Simonin (1991) by 1) testing the droplet dispersion model over a broad inertial range, with the smallest droplets following every turbulent fluctuation of the fluid, and the largest droplets being relatively insensitive to these fluctuations, and 2) simulating a jet flow with low turbulence levels at the nozzle exit to enable an effective validation of the simple two-fluid turbulence model. A new experimental data set of a jet flow with low turbulence at the nozzle exit is provided for this purpose.

EXPERIMENTAL

The experimental apparatus consists of a wind tunnel and a nozzle (a long thin tube, 9.8mm in diameter, located centrally at the exit plane of wind tunnel), which produces a round air jet within a co-flow of air with velocities of approximately 23 m/s and 2.4 m/s, respectively, and low turbulence intensities of 2% and 1.4%, respectively. A dispersion of virtually non-evaporating mineral turpentine droplets in the size range from 1 to 90 µm is generated by an ultrasonic nebulizer (Sonotek) and carried by a flow of air through the nozzle tube towards the nozzle exit. The nozzle and wind tunnel are mounted on a traversing mechanism, which allows measurements along the vertical axis and along the two orthogonal transverse directions. A phase-Doppler anemometer or PDA (Aerometrics, RSA 3100) is used to measure the velocity and volume flux of the droplets within the spray. Nijdam et al. (2004) have shown that these PDA measurements are sufficiently accurate that conservation of the total volume flow of the dispersed phase throughout the spray can be demonstrated, so that the volume of droplets in any size class is known everywhere throughout the spray. Details of the experimental apparatus and PDA configuration adopted in this investigation are found in Nijdam et al. (2004).

MODEL DESCRIPTION & NUMERICAL SOLUTION

The gaseous and each droplet phase are treated as separate interpenetrating continua, with the transport of both phases being modelled within an Eulerian framework. The two-fluid model of Simonin (1991) is used to simulate the turbulent dispersion of the droplet phase, while the standard k - ε turbulence model (Launder and Sharma, 1974) is employed to predict the turbulent motion of the gas phase. It is assumed that the droplet number concentration is sufficiently low that the effect of inter-droplet collisions on particle velocity fluctuations is negligible compared with the transfer of momentum from the gaseous turbulence. This is reasonable given that the liquid volume fraction in the jet is less than 10^{-4}. We also assume that no droplet evaporation occurs, because the vapour pressure of mineral turpentine is very low. Details of the modelling approach used here can be found in Nijdam et al. (2006).

A commercially available Computational Fluid Dynamic (CFD) program called CFX4 (ANSYS) is used to solve the equation set. This program employs a structured mesh and a finite volume formulation to solve the partial differential equations. An axisymmetric cylindrical coordinate system is chosen to represent the jet in order to reduce the problem to two dimensions. The grid has 10 evenly spaced nodes across the half-width of the nozzle. The distance between nodes gradually expands in the cross-stream direction towards the edge of the flow domain, which is sufficiently far from the nozzle to not affect the solution significantly. The distance between nodes in the axial direction also expands away from the nozzle. The grid has approximately 2500 nodes, and the converged solution does not change significantly when the number of nodes is quadrupled. The convergence criterion is satisfied when the total sum of the mass residuals for the control volumes falls below the tolerance value of 10^{-10} kg/s, which is approximately 10^{-4} percent of the total droplet inflow.

RESULTS

Radial profiles of the axial and radial mean velocities, axial and radial fluctuating velocities, and the volume flux are measured at normalised axial locations of approximately 1, 5, 10, 20, and 30 nozzle diameters downstream of the nozzle exit for each spray. We separate the droplets measured within the spray into size classes (each size class is 10 µm wide) in order to determine the dependency of the mean and fluctuating velocities and the volume flux on the size of the droplets. The axial mean velocity profiles for each size class at 10, 20, and 30 nozzle diameters are collapsed onto two separate profiles by normalisation, as shown in Figure 1. These profiles are re-dimensionalised using the peak or centre-line excess velocity (U_{eo}) and half-radii ($R_{1/2U}$) data shown in Figures 2 and 3, respectively, where the excess axial velocity U_e is defined as the difference between the local axial mean velocity U and the co-flow axial mean velocity U_c. Similar normalised volume flux profiles, and peak volume flux (F_o) and half-radii ($R_{1/2F}$) data are shown in Figures 4 to 6.

The profiles for the turbulent kinetic energy k and the square of the radial root-mean-square (rms) fluctuating velocity $\overline{v'v'}$ at 5, 10, 20, and 30 nozzle diameters are presented for each size class in Figures 7 and 8. Comparisons between the predictions of the two-fluid model and the experimental data are shown in the graphs described above where appropriate. Figures 9 and 10 show comparisons between the experimental and simulated half-radii ($R_{1/2U}$ and $R_{1/2F}$) of the velocity and volume-flux profiles for a second jet (Nijdam et al., 2004) having a turpentine droplet flow of 1.86 ml/min, which is approximately one-third of the droplet flow for the first jet (5.33 ml/min).

DISCUSSION

Figure 1 shows experimental and predicted normalised excess axial velocity profiles for different droplet size classes at various axial locations from the nozzle, all collapsed onto a single plot. Both experimental and simulated excess axial-velocity profiles are approximately self-similar in the far-field downstream of 10 nozzle diameters. Figures 2 and 3 show that the experimental and predicted peak excess axial velocity U_{eo} and half-radius $R_{1/2U}$ values that characterise these normalised profiles are in excellent agreement at every axial location measured and over a wide range of droplet sizes. Clearly, all of the major features of the flow are predicted accurately by the model, such as the decay of the centre-line excess axial velocity, and the rate of spread of momentum with axial distance from the nozzle. Reasonable agreement between theory and experiment can also be seen in the normalised volume-flux profiles (Figure 4), and the peak-flux F_o and half radius $R_{1/2F}$ values (shown in Figures 5 and 6, with

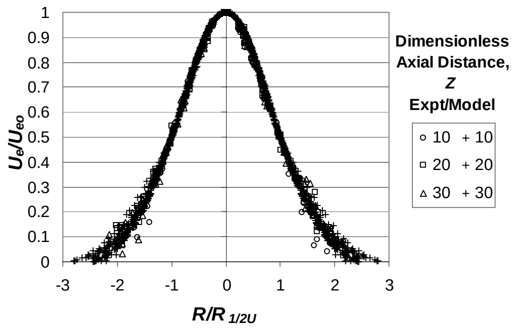

Figure 1: Radial profiles of dimensionless excess axial mean velocity U_e/U_{eo} versus dimensionless radius $R/R_{1/2U}$ for every droplet size class at various axial locations downstream of the nozzle.

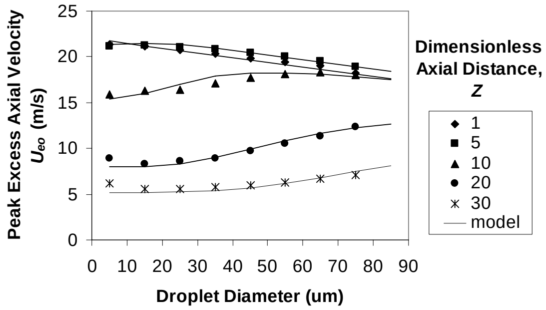

Figure 2: The peak excess axial mean velocity U_{eo} for different droplet size classes at various locations downstream of the nozzle: experimental results (points) and model predictions (lines).

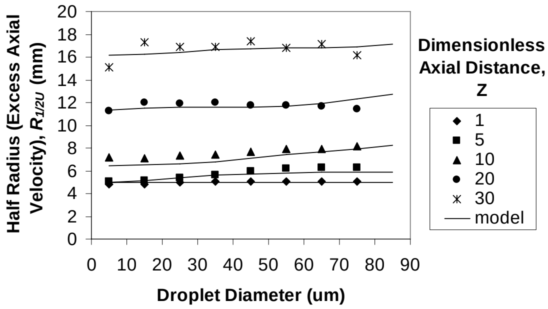

Figure 3: The half-radii $R_{1/2U}$ of the radial profiles of excess axial mean velocity for different droplet size classes at various locations downstream of the nozzle: experimental results (points) and model predictions (lines).

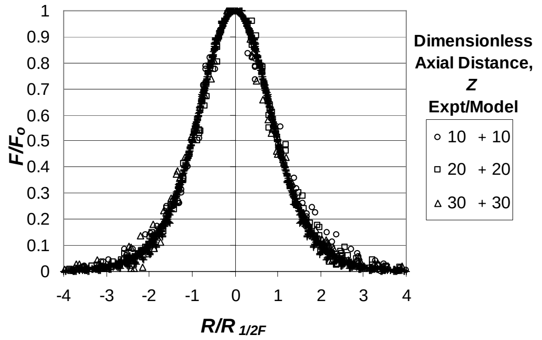

Figure 4: Radial profiles of the dimensionless droplet volume flux F/F_o versus dimensionless radius $R/R_{1/2F}$ for every droplet size class at various axial locations downstream of the nozzle.

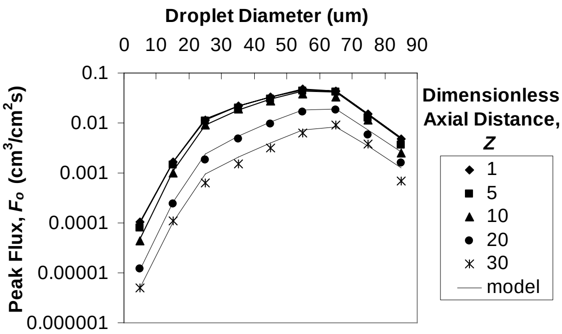

Figure 5: The peak droplet volume flux F_o for different droplet size classes at various locations downstream of the nozzle: experimental results (points) and model predictions (lines).

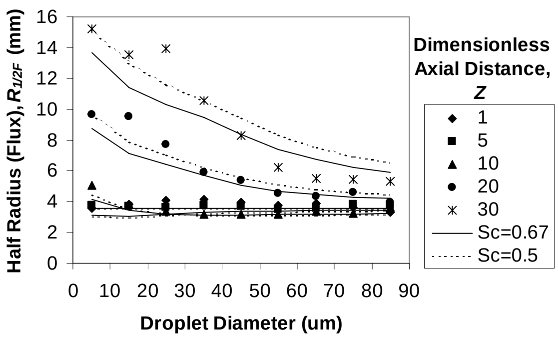

Figure 6: The half-radii $R_{1/2F}$ of the radial profiles of droplet volume flux for different droplet size classes at various locations downstream of the nozzle: experimental results (points) and model predictions (lines) for two values of the turbulent Schmidt number σ_α^t (or *Sc*).

the Schmidt number σ_α^t equal to 0.67) which characterise these normalised profiles. The Stokes number ranges from approximately 0.01 to 10 for droplet diameters ranging from 5 µm to 85 µm, respectively, at 10 nozzle diameters downstream of the nozzle exit plane. Thus, the droplet sizes in the jet investigated in this work cover a broad inertial span, ranging from droplets that follow faithfully every fluctuating motion of the fluid (St <<1) to droplets that are hardly affected by the fluid turbulence (St >>1). The good agreement between theory and experiment demonstrates that the droplet dispersion rates predicted by the simple model adopted in this work are reasonable over a wide range of Stokes numbers. Note that the turbulent Schmidt number σ_α^t for turbulent scalar diffusion in an axisymmetric round jet of 0.67 has been measured experimentally by Antonia and Bilger (1976). Later in the paper, we discuss the consequences of retuning the turbulent Schmidt number σ_α^t from 0.67 to 0.5 in order to match the predicted and experimental spreading rates of the smallest droplets.

The experimental and predicted turbulence kinetic energy (k) profiles of the 5µm droplet size class, which

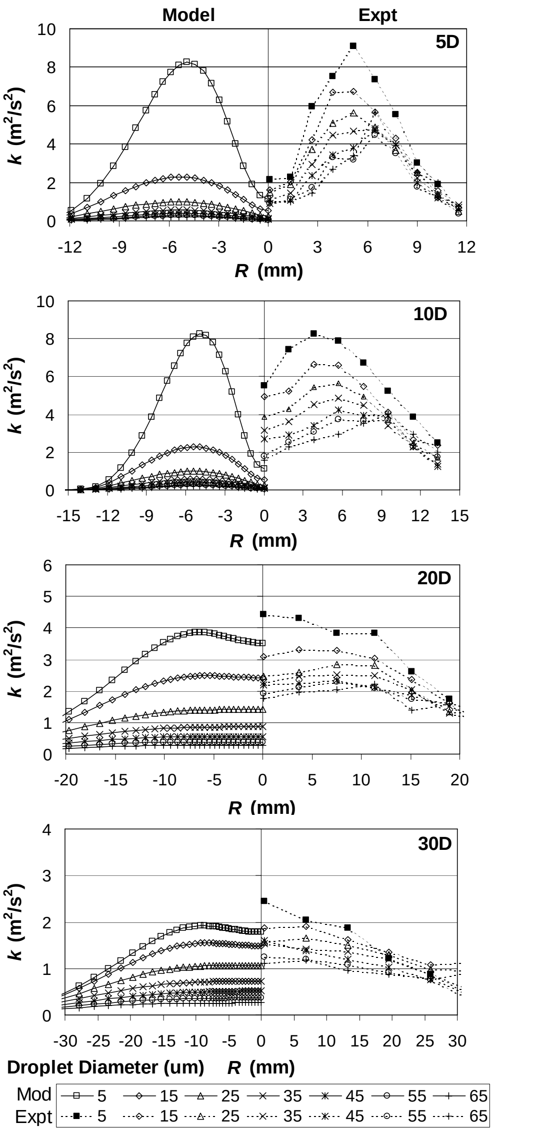

Figure 7: Turbulence kinetic energy k for different droplet size classes at various locations downstream of the nozzle: theory and experiment.

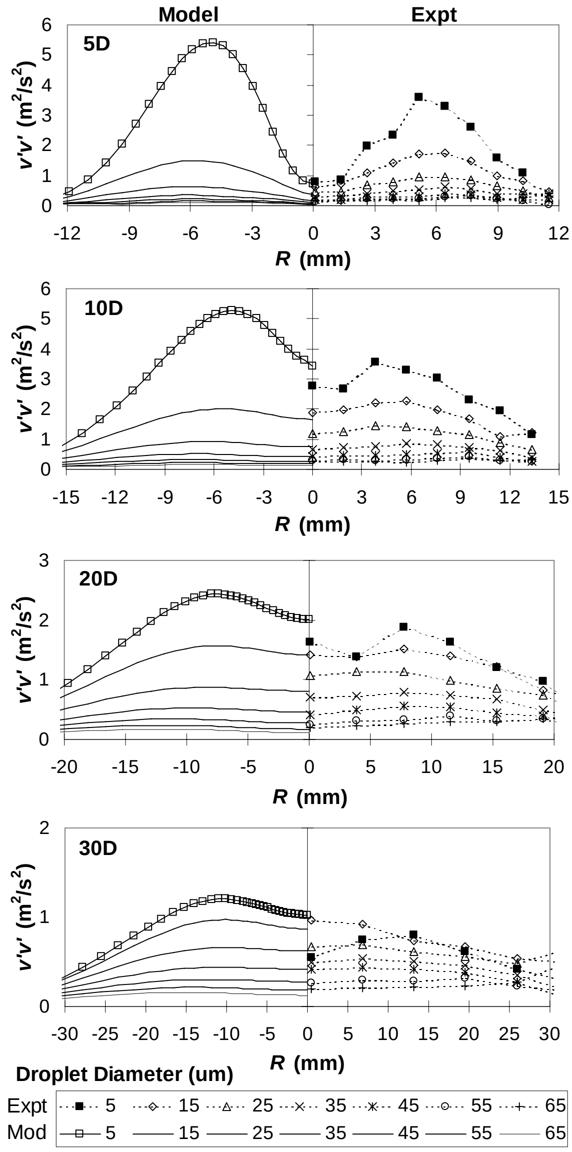

Figure 8: Radial kinetic stress $\overline{v'v'}$ for different droplet size classes at various locations downstream of the nozzle: model predictions and experimental results.

are sufficiently small to be representative of the airflow, are in reasonable agreement at every axial location measured, as shown in Figure 7. Clearly, the generation of airflow turbulence in the shear layer of the jet and subsequent transport of this turbulence throughout the jet is predicted with reasonable accuracy by the standard k - ε turbulence model. However, the predicted turbulence of any droplet size class larger than 5µm deviates significantly from experiment, which indicates that the droplet-phase turbulence model based on Tchen's theory is not applicable for this type of flow. Simonin (1991) has demonstrated computationally and Prevost et al. (1996) have shown experimentally that, while Tchen's theory is reasonable to describe the radial fluctuating motion of droplets in a jet because it is dominated by the local interaction with the gas turbulence, the axial velocity fluctuations of droplets in the jet may be largely under-predicted, because such an approach does not account for production by the droplet mean velocity shear. This conclusion is confirmed in this paper. The experimental and predicted radial kinetic stresses $\overline{v'_\beta v'_\beta}$ of the droplets Figure 8) are of similar magnitude which, given that the turbulent kinetic energy of the droplets is under-predicted, implies that the axial kinetic stress $\overline{u'_\beta u'_\beta}$ of the droplets is also under-predicted.

The radial turbulent dispersion of the droplets is modelled with reasonable accuracy despite the intrinsic shortcomings of the droplet turbulence model to predict the turbulent kinetic energy of the droplets, as shown in Figure 6. Under-predicting the axial kinetic stress $\overline{u'_\beta u'_\beta}$ does not affect the accurate prediction of the radial mean velocity of droplets, which is dominated by both the fluid-particle turbulent drift velocity and the radial kinetic stress component contributions. Modelling the radial kinetic stress $\overline{v'_\beta v'_\beta}$ properly in the framework of Tchen's theory, which we have confirmed in Figure 8, appears to be crucial and ensures that the correlation between the gas and droplet fluctuating velocities in the radial direction $\overline{v'_\alpha v'_\beta}$ and hence the radial component of the fluid-particle turbulent drift velocity are modelled accurately.

In addition to comparing the predictions of the model with the experimental data presented in this paper, a similar comparison has also been conducted for the jet flow measured experimentally by Nijdam et al. (2004), which has a droplet flowrate approximately one-third of the value tested in this work. We have found that different values for the turbulence parameter $C_{1\varepsilon}$ in the k - ε turbulence model are required for each jet in order to match the experimental and simulated decay of the centreline axial mean velocity, which casts doubt on the generality of the gas-flow turbulence model used. It is unlikely that the required alteration in $C_{1\varepsilon}$ is due to the effect of airflow turbulence modulation by the presence of droplets or the effect of droplet collisions on the mean transport equations, since the mass loading ratio m_L of each jet is less than 0.04. Rather, this alteration is attributed to a difference in the axial mean velocity profiles generated at the exit-plane of the nozzle for each jet, which is probably related to the observed formation of a thin axially-moving liquid annulus produced by droplet-wall collisions on the inside wall of the nozzle. This liquid annulus differed in thickness and velocity depending on the droplet flowrate used, which affected the shape of the axial mean velocity profiles at the exit plane of the nozzle. In the case of the jet with the higher droplet flow, an axial mean velocity profile somewhat similar to a fully-developed velocity profile in a pipe was generated at the exit plane of the nozzle, while the jet with the lower droplet flow developed a top-hat axial mean velocity profile, indicating the presence of a potential core. A sensitivity analysis has shown that a value for $C_{1\varepsilon}$ of 1.6 is best for a jet with a "fully developed" nozzle velocity profile, as demonstrated for the first jet in Figure 3, whereas this value results in an under-prediction of the momentum spreading rate of a jet with a top-hat nozzle velocity profile (the second jet), for which a lower value for $C_{1\varepsilon}$ of 1.55 is more appropriate, as shown in Figure 9.

McGuirk and Rodi (1979) have explained that $C_{1\varepsilon}$ is only an empirical constant, which does not take a universal value for every type of flow. Indeed, some researchers have devised empirical correction functions for $C_{1\varepsilon}$, which have no physical interpretation or justification, to account for any variations between flow types (Pope, 1978), while Pope (1978) has formulated an additional term for the turbulence energy dissipation transport equation, based on a vortex stretching mechanism, to avoid any lack of generality. Moreover, McGuirk and Rodi (1979) have stated that the value for $C_{1\varepsilon}$ of 1.6 is only applicable for round jets well away from any potential core and for jets issuing into stagnant surroundings. However, the jet with the top-hat axial mean velocity profile clearly has a potential core, and both jets investigated in this work issue into a co-flow. Clearly, the k - ε turbulence model lacks generality, and more sophisticated gas-flow turbulence models are needed to resolve this. However, such complex turbulence models are likely to be computationally very expensive, and simpler formulations are preferable when other phenomena, such as combustion or droplet evaporation, are the main focus of an investigation.

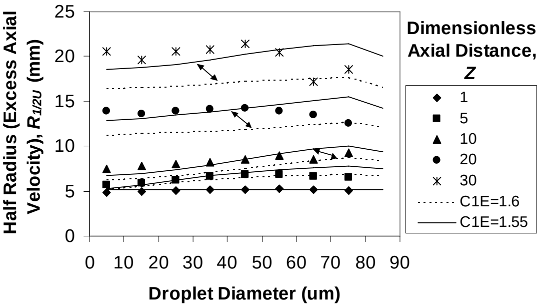

Figure 9: The half-radii $R_{1/2U}$ of the radial profiles of excess axial mean velocity for the second jet: experimental results (points) and model predictions (lines) for two values of the turbulence parameter $C_{1\varepsilon}$.

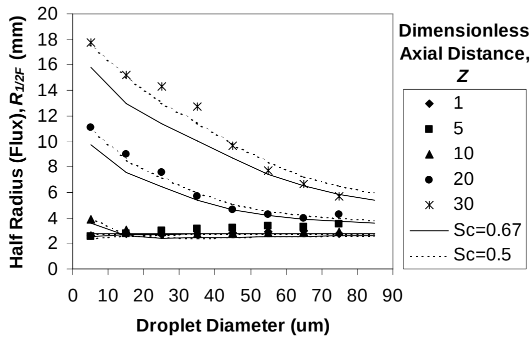

Figure 10: The half-radii $R_{1/2F}$ of the radial profiles of droplet volume flux for the second jet: experimental results (points) and model predictions (lines) for two values of the turbulence Schmidt number σ_α^t (or Sc).

Figure 10 shows that the spreading rates of the smaller droplets in the second jet are slightly underestimated, which can also be seen in the predictions of the first jet (Figure 6). The Schmidt number σ_α^t controls the dispersion rate of the smallest droplets, which spread in a similar fashion to a scalar quantity. Thus, the standard value for turbulent Schmidt number σ_α^t of 0.67 (Antonia and Bilger, 1976) appears to underestimate scalar dispersion in both jets. A lower value for the turbulent Schmidt number σ_α^t of 0.5 results in better agreement between the predicted and experimental spreading rates of the scalar (or smallest droplets), as shown in Figures 6 and 10. However, the predicted spreading rate of the larger droplets is consequently over-estimated, which implies that the cross-trajectory effect, when large droplets pass through turbulent eddies due to their high relative inertia, is underestimated by the current droplet dispersion model. The required alteration of the Schmidt number σ_α^t from the standard value may be caused by different forms being taken by the velocity and volume fraction profiles at the nozzle exit (top-hat for velocity and bell-shape for volume fraction in the second jet), such that the dispersion of each of these quantities is not entirely analogous, which is assumed when using the classical scalar dispersion model.

CONCLUSIONS

The aim of this paper is to compare experimental measurements of a poly-disperse axisymmetric round jet spray with the predictions of a simple two-fluid turbulence model for dilute gas-droplet turbulent flows developed by Simonin (1991). Here, the key condition is that sufficiently low levels of droplet velocity fluctuations are generated within the nozzle so that the downstream transport of this turbulence has a negligible effect on the dispersion of droplets in the jet. The modelling approach used here is based on the k - ε turbulence model for the gas phase and an algebraic model for the droplet fluctuating motion, which assumes local equilibrium with the gas-phase turbulence.

We have found that the model underestimates the turbulent dispersion of a scalar quantity, here represented by the volume fraction of the smallest droplets, in the axisymmetric round jet investigated in this work. A turbulent Schmidt number σ_{α}^{t} of 0.5 produces better predictions of the scalar spreading rate than the commonly used value of 0.67. However, this correction leads to an overestimation of the spreading rate of the largest droplets, which implies that the cross-trajectory effect is underestimated by the current droplet dispersion model. The standard value for the turbulence parameter $C_{1\varepsilon}$ in the k - ε turbulence model of 1.6 appears to be valid for jets without a potential core, while this value must be reduced to 1.55 in the case of jets with a well-defined potential core in order to match the experimental and predicted decay of centreline axial mean velocity. However, in general, the model predicts the radial dispersion of droplets over a broad inertial range of droplet sizes with reasonable accuracy, once the deficiency in the k - ε turbulence model is taken into account.

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REFERENCES

ANSYS CFX, www.ansys.com/cfx.

ANTONIA, R.A. and BILGER, R.W., (1976), "The heated round jet in a co-flowing stream", *AIAA J.*, **14** (11), 1541-1547.

CHEN, C.P. and WOOD, P.E., (1986), "Turbulence closure modelling of the dilute gas-particle axisymmetric jet", *AIChE J.*, **32** (1), 163-166.

ELGHOBASHI, S.E. and ABOU-ARAB, T.W, (1983), "A two-equation turbulence model for two-phase flows", *Phys Fluids*, **26** (4), 931-938.

HINZE, J.O., (1975), "Turbulence", McGraw-Hill, New York.

LAUNDER, B.E. and SHARMA, B.T., (1974), "Application of the energy dissipation model of turbulence to the calculation of flow near a spinning disk", *Lett. Heat and Mass Transfer*, **1**, 131-138.

MASRI, A.R., DIBBLE, R.W., BARLOW, R.S., (1996), "The structure of turbulent non-premixed flames revealed by Raman-Raleigh-LIF measurements", *Progress Energy Combust. Sci.*, **22** (4), 307-362.

MCGUIRK, J.J. and RODI, W., (1979), "The calculation of three-dimensional turbulent free jets", in *Turbulent Shear Flows* (edited by Durst et al.), Springer-Verlag, **1**, 71-83.

NIJDAM, J.J., STÅRNER, S.H., LANGRISH, T.A.G., (2004), "An experimental investigation of droplet evaporation and coalescence in a simple jet flow", *Exp. Fluids*, **37**, 504-517.

NIJDAM, J.J., GUO, B., FLETCHER, D.F., LANGRISH, T.A.G., (2006), "Lagrangian and Eulerian models for simulating turbulent dispersion and agglomeration of droplets within a spray", Accepted for publication in *Applied Mathematical Modelling*.

POURAHMADI, F. and HUMPHREY, J.A.C., (1983), "Modelling solid-fluid turbulent flows with application to predicting erosive wear", *PhysicoChemical Hydrodynamics*, **4** (3), 191-219.

SIMONIN, O., (1991), "Prediction of the dispersed phase turbulence in particle-laden jets", in 4th Int. Symposium on Gas-Solid Flows, *ASME FED*, **121**, 197-206.

SIMONIN, O., DEUTSCH, E., BOIVIN, M., (1995), "Large eddy simulation and second-moment closure model of particle fluctuating motion in two-phase turbulent shear flows", in *Turbulent Shear Flows* (edited by Durst et al.), Springer-Verlag, **9**, 85-115.

PREVOST, F., BOREE, J., NUGLISCH, J., CHARNAY, G., (1996), "Measurements of fluid/particle correlated motion in the far field of an axisymmetric jet", *Int. J. Multiphase Flow*, **22** (4), 685-701.

POPE, S.B., (1978), "An explanation of the turbulent-jet/plane-jet anomaly", *AIAA J.*, **16** (3), 279-281.