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MAPPING OF COLLISION REGIMES IN FLOTATION MODELLING

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ABSTRACT

The kinetics of bubble-particle collision are of particular interest in flotation modelling. Upon collision a mineral particle needs a critical amount of kinetic energy to overcome the energy barrier of bubble-particle attachment when sufficient sliding time is allowed. The nature of the bubble-particle collision is determined by relative velocity between bubble and particle, the turbulent contribution to velocity fluctuation, bubble and particle sizes, and particle density. However, collision kinetics characterization needs a description of a higher order momentum coupling due to the high void fraction of the dispersed phases in flotation systems. The momentum coupling can be obtained through the redistribution of the pressure field. The fluctuations in bubble pressure during bubble-particle collision and subsequent attachment observed in experiments are used to quantify this momentum coupling. In this paper we explore a method to model the efficiency and rate of bubble-particle collision and attachment in flotation systems by bubble pressure as a means of momentum coupling. This method allows for quantitative mapping of bubble-particle collision-attachment, or capture, regimes and can be used to further understand the physico-chemical behavior of flotation systems.

NOMENCLATURE

C_{vm} virtual mass coefficient (set to 0.5)
d_b bubble diameter
d_p particle diameter
g gravitational constant
H correction term in eq.(5)
m (particle) mass
M_a phase indicator function of phase a
p pressure
p_b bubble pressure
p_b^+ bubble over pressure
Re Reynolds number
$\mathbf{u}$ velocity
U fluid velocity
V bubble velocity in Eq.(2)
$\mathbf{u}_s$ bubble slip velocity
$\mathbf{x}$, x location

α_k volume fraction of dispersed phase k
α_{max} maximum volume fraction of air (here set to 0.63)
ρ_c density of the continuous phase
τ shear stress

INTRODUCTION

Mineral froth flotation is a process that is governed by both hydrodynamic and surface-chemical aspects. These two schools of thought are commonly hard to combine in flotation modelling. It is generally understood that the probability of flotation is the product of the probabilities of bubble-particle collision, attachment, and aggregate stability (Derjaguin and Dukhin, 1961). Bubble-particle collision is a key sub-process in mineral froth flotation. Most collision models in literature (see Dai *et al*. (2000) for a review) are of geometric nature, after the work of Von Smoluchowski (1917) and Sutherland (1948). This means that the probability of collision P_c between two spheres of a given diameter is of the form

$$P_c = f\left(d_p, d_b, \mathrm{Re}\right) \tag{1}$$

where *Re* is the Reynolds number. Subsequent bubble-particle attachment process occurs when the sliding time of the mineral particle over the bubble surface exceeds the induction time (Dobby and Finch, 1987). The collision and attachment processes are a balance between resultant forces of bubble and particle hydrodynamics, particle inertia, and DLVO surface forces. The effects of inertia are often neglected because of the assumption of strong bubble surface retardation (Dai *et al*., 1998). An exception is the work of Luttrell and Yoon (1992) (Dai *et al*., 1998), although the collision theory is still geometric and the Stokes flow regime is assumed. Inertial forces play a major role in bubble-particle interaction for particles of medium size (i.e. > ~ 10 μm (Derjaguin and Dukhin, 1961)) and larger.

Recent laboratory observations of bubbles and particles in turbulent flow (Schreithofer *et al*., 2008) show bubble-particle interaction that is of a different nature than commonly assumed in flotation theory and modelling. The description of bubble-particle interaction needs a more fundamental approach. In particular in flotation the dispersed volume fractions of air and solids are so high that higher order coupling between phases is required (Wierink and Heiskanen, 2008). The energy budget available during bubble-particle collision consists of the exchange of momentum in the three-phase interaction. Linear momentum is transferred through redistribution of the pressure field (Davidson, 2004). This leads us to explore a more unified approach by combining bubble-particle collision and attachment in a pressure coupled model. In this paper we quantify pressure fluctuations during bubble-particle collision and formulate the role of local pressure in bubble-particle collision. Pressure data from bubble-particle collision experiments can be used to map different bubble-particle interaction regimes within a flotation cell. Accurate prediction of bubble-particle collision and subsequent attachment probability can be used to analyze particle capture by bubbles in different locations in a flotation cell and modify equipment for the optimal bubble-particle collision regime desired.

THEORY

Pressure, velocity and momentum

The momentum transfer between unit volumes and different phases consists of a linear and an angular momentum. When taking the divergence of the equation of motion for fluids and using the Biot-Savart inversion we obtain (Davidson, 2004)

$$p(\mathbf{x}) = \frac{\rho}{4\pi} \int \frac{[\nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u})]'}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}' \tag{2}$$

where $\mathbf{x}$ and $\mathbf{x'}$ are locations in the fluid domain. From Eq.(2) we can see that momentum $\rho\mathbf{u}$ is transferred through the flow domain via the pressure field. A turbulent eddy at $\mathbf{x}$ causes a pressure wave that creates another turbulent eddy at $\mathbf{x'}$ (Davidson, 2004). For ideal incompressible fluids the pressure field is redistributed instantaneously. This, however, may commonly not be the case in multiphase systems.

For multiphase systems with dispersed phase a with volume fraction α the momentum balance equation can be written as

$$\frac{\partial\left(\alpha_a \langle\rho\rangle_a \langle U^j\rangle_a\right)}{\partial t} + \frac{\partial\left(\alpha_a \langle\rho\rangle_a \langle U^j\rangle_a \langle U^i\rangle_a\right)}{\partial x^j} =$$
$$\frac{\partial\left(\alpha_a \langle p\rangle_a\right)}{\partial x^i} - \frac{\partial\left(\alpha_a \langle\tau^{ij}\rangle_a\right)}{\partial x^j} + \alpha_a \langle\rho\rangle_a g^i$$
$$+ \overline{\rho U^i \left(U^i - V_i^j\right)\frac{\partial M_a}{\partial x^j}} + \overline{\tau^{ij}\frac{\partial M_a}{\partial x^j}} + \overline{p\frac{\partial M_a}{\partial x^i}} \tag{3}$$

where $\overline{f}$ is the ensemble average of quantity f, $\langle f\rangle_a$ is the phase-weighted ensemble average of quantity f over phase a, and M_a is the phase indicator function. The first and last term on right hand side represent the coupling of linear momentum through the pressure field. The last three terms on the right hand side are the phasic interaction terms and need to be modeled. In this paper we aim to quantify the pressure coupling term for the flotation process by experiment.

Bubble pressure

Bubble-particle attachment and particle recovery from the flotation cell can occur when the solid particle overcomes the energy barrier caused by repulsive forces in the three-phase system. The energy barrier is schematically shown in Figure 1.

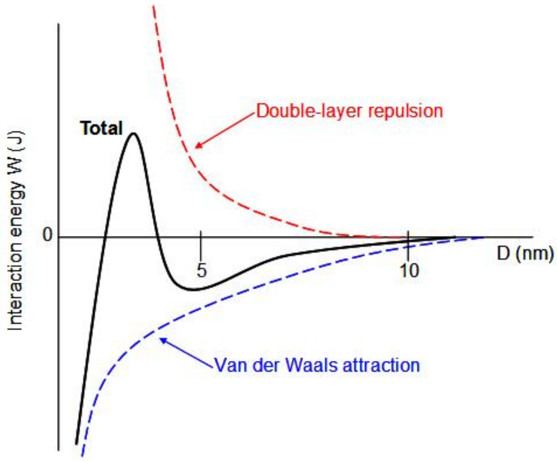

Figure 1: Attractive and repulsive forces between air bubble and mineral particle (after Israelachvili, 1992).

It is generally understood that the main means of a solid particle to overcome this energy barrier in flotation systems is its kinetic energy content, relative to that of the bubble. A particle needs a minimum impulse $m\mathbf{u}$, relative to the bubble, to overcome the repulsive energy barrier and to allow for three-phase contact. The consequence is that smaller particles and particles with a lower density need a higher relative velocity to reach the critical impulse for successful bubble-particle collision to occur. The source of relative velocity is local pressure gradient. Therefore, local pressure, or pressure gradient, may provide a means to map the probability for successful collision between bubbles and particles of certain sizes.

Fielden *et al.* (1996) and Hirata *et al.* (1990) provide values for the energy barrier shown in Figure 1 for colloidal systems. The solid particle sizes in these studies were 3 μm and 10-1000 nm, respectively. Schreithofer (2003) reports that this "jump-in force" is in the order of 20 mN/m for flotation-like systems (d_p = 15 μm). The complexity of forces however makes it difficult to use these values in a straight manner in computational modelling. In this paper, therefore, we focus on the transfer of linear momentum through the pressure field.

Bubble pressure is a function of the dispersed volume fraction α and the bubble slip velocity $\mathbf{u}_s$ and can be modeled as (Spelt and Sangani, 1998)

$$p_b = \rho_c C_{vm} \alpha \mathbf{u}_s \cdot \mathbf{u}_s \tag{4}$$

where C_{vm} is the virtual mass coefficient. For spherical bubbles $C_{vm} = 0.5$ (Monahan *et al.*, 2005). Monahan *et al.* (2005) point out that when α goes to zero the bubble pressure remains non-zero in Eq.(4) and propose to use the bubble pressure model of Biesheuvel and Gorissen (1990).

$$p_b = \rho_c C_{vm} \alpha \mathbf{u}_s \cdot \mathbf{u}_s H \tag{5}$$

where (Batchelor, 1988)

$$H = \frac{\alpha}{\alpha_{\max}} - \frac{\alpha^2}{\alpha_{\max}^2} \tag{6}$$

with α_{max} the maximum volume fraction of air.

BUBBLE PRESSURE EXPERIMENTS

Experimental set-up

A differential bubble pressure device is built to measure the pressure shock inside the air bubble during bubble-particle collision. These measurements quantify the bubble pressure p_b in Eq.(5). Figure 2 shows a schematic overview of the experimental setup.

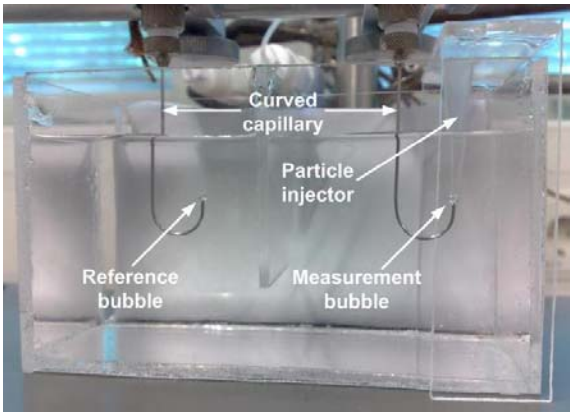

Figure 2: Bubble pressure experiment (bubble diameter 1 mm).

The differential bubble pressure device consists of two curved capillary needles submerged in water. On each capillary an air bubble of known volume can be grown using a pair of precision syringes. The measurement and reference bubbles are separated by a glass plate so that only hydrostatic and atmospheric pressure are equal for both bubbles. Solid particles are introduced through the particle injector and collide with the measurement bubble. At this stage of development of the experimental set-up 150-180 µm quartz particles are used. These particles are relatively large for typical flotation conditions and development of a particle injector for smaller particles is ongoing.

During the bubble-particle collision the bubble pressure is measured at a rate of 3 kHz using a SensorTechnics HCLA analogue pressure sensor. The accuracy of the pressure sensor is 0.06 Pa. The pressure sensor data is collected using a National Instruments (NI) data acquisition card and is processed in NI LabView. In LabView the pressure data is filtered using a fast Fourier transform and converted to an output signal in Pa.

Experimental results

The pressure shocks that have been measured during bubble-particle collisions are in the range of 0.28 ± 0.06 Pa for 150-180 µm untreated quartz particles. After bubble-particle collision attachment occurs. The collision and attachment processes cause a pressure fluctuation in the bubble. Figure 3 shows an example of bubble pressure during bubble-particle collision and subsequent attachment. Bubble-particle collision followed by attachment is used to map the capture of particles by bubbles. The temperature of the ultra-pure water during the experiments was between 22.5 and 23.0 °C.

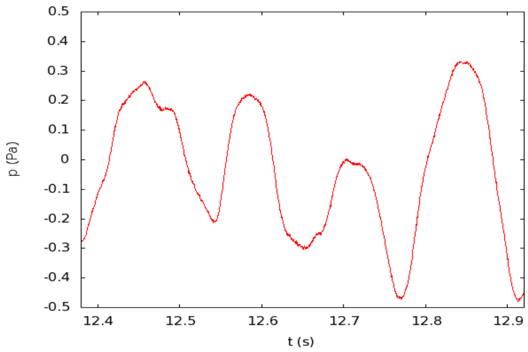

Figure 3: Bubble pressure versus time during bubble-particle collision and attachment (d_b = 1 mm, d_p = 150-180 µm, quartz).

MODEL DESCRIPTION

The simulations are carried out for a laboratory scale flotation cell of 45 litres with an Outotec FloatForce mechanism. The flotation rotor is 102 mm in diameter and rotates with 700 rpm (v_{tip} = 3.7 m/s). The superficial gas velocity is 0.5 cm/s. A 300,000 cell hybrid structured-unstructured computational mesh was built in Gambit. Figure 4 shows the computational mesh projected on the rotor-stator system.

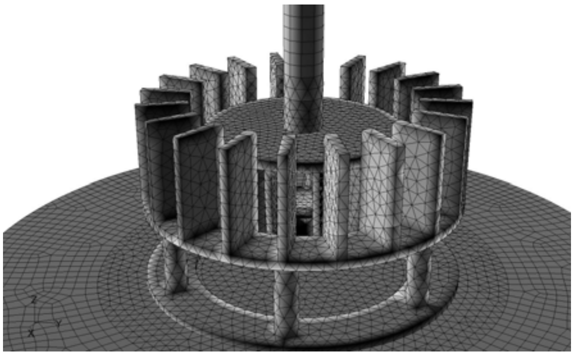

Figure 4: The structured-unstructured computational mesh on the 102 mm Outotec FloatForce rotor-stator system.

The two-phase CFD simulations were carried out in Fluent 6.3 with a modified version of the mixture model of Manninen and Taivassalo (1996) as a set of user defined functions. The bubble size in the mixture model was 1 mm and the steady-state k-ε model was used.

The experimental results show that successful collision and attachment of the quartz particles to the air bubble are accompanied by a bubble pressure peak of 0.28 Pa. To map the collision regime of the quartz particles an additional user defined function was written for the bubble "over pressure" p_b^+ for successful collision-attachment as

$$p_b^+ = \rho_c C_{vm} \alpha \mathbf{u}_s \cdot \mathbf{u}_s H - 0.28 \quad (7)$$

In this work the CFD simulations are used as a vehicle to explore the applicability of the proposed method of direct pressure coupled flotation modelling.

RESULTS

Figure 5 shows the results for volume fraction of air of two-phase CFD simulations of the flotation cell. The shape and values of the air volume fraction distribution in the flotation cell are in good agreement with the experimental results of Grau (2006), Rudolphy *et al*. (2005) and Rudolphy *et al.* (2006) for air hold-up in a larger (265 litre) Outotec laboratory flotation cell. In this paper CFD simulation of the pulp phase with a direct two-phase mixture model is used. The profile in Figure 5 is asymmetric despite steady-state simulation because the cut plane is the *z*-*y*-plane and the plane of flow symmetry is helical in this type of flotation system (Wierink, 2006).

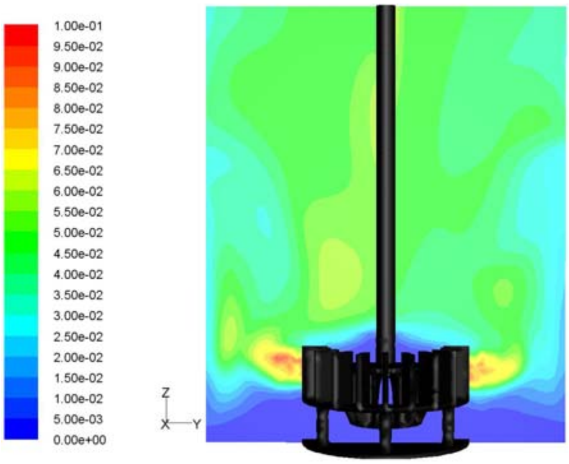

Figure 5: CFD simulation result for volume fraction of air in the Outotec flotation cell.

The results for bubble overpressure p_b^+ are shown in Figure 6. The results show agreement in spatial distribution of the collision regime with flotation theory and simulation results of Koh and Schwarz (2006). The value of 0.28 Pa in Eq.(7) comes from experimental results with 150-180 µm quartz particles. The authors are aware that this particle size is fairly coarse for typical flotation practice, however, at the time of writing reliable bubble pressure results for smaller particles are not yet available.

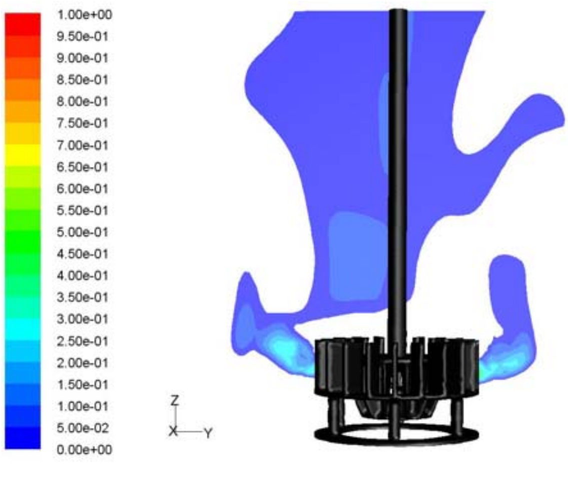

Figure 6: The region in the flotation cell where the critical bubble pressure (0.28 Pa) is exceeded (p_b^+ in Pa).

This method is under development and can provide a powerful tool for accurate modelling of mineral froth flotation. Coupling of velocity, pressure, and momentum in two-phase CFD combined with experimental bubble pressure data is a more direct and holistic representation of the process. In these simulations second order momentum coupling is used and is directly related to the energy budget available to bubbles and particles to form aggregates.

CONCLUSION

The models used to simulate mineral froth flotation commonly are developed for dilute multiphase flow. The dispersed volume fraction of air and solids in flotation systems is such however that higher order momentum coupling and a more holistic modelling approach is required. In this light a new method to flotation modelling is taken. Bubble pressure during bubble-particle collision is directly measured and a critical bubble pressure determined. The critical bubble pressure is then combined with a modified two-phase mixture model as a user defined function in CFD. This new method of coupling of velocity, pressure, and momentum in two-phase CFD combined with experimental bubble pressure data is a more direct and holistic representation of the flotation process. The results for collision regimes show good agreement for flotation theory and practice. This method of including linear momentum coupling and inertia into flotation modelling is under development and shows promising results. Future work is geared to bubble pressure measurements for smaller particles, polydisperse bubble and particle populations and higher order momentum coupling.

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