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SPH MODELLING OF WEIR FLOW THROUGH A FOUR BAY, RADIAL GATED, SUBMERGED SPILLWAY

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ABSTRACT

Spillway models are important for evaluating and improving dam safety, as well as optimising spillway design and economical operation. Traditionally, scaled down physical models have been used for validation and to collect hydraulic data. However the ability to efficiently evaluate a range of different spillway designs using physical models is limited by time, cost and resources. Modelling techniques using computational fluid dynamics (CFD) are able to quickly evaluate different spillway designs. CFD has therefore played an increasing role in spillway modelling, with physical models used more often to supplement and validate simulations. In this work we use the CFD technique of smoothed particle hydrodynamics (SPH) to model weir flow through a four bay, radial gated, submerged spillway system. Advantages of SPH for such modelling include automatic representation of free surface flow behaviour due to the Lagrangian nature of the method, the ability to incorporate complex three dimensional geometries involving complicated spillway operations and the ability to include dynamic obstructions in the flow such as debris. To validate the SPH model, the reservoir water depth predicted in the simulations is compared with a physical model. The effect of SPH resolution on the predicted water depth is evaluated. A range of discharge rates are simulated, with the difference in simulated and experimental water depths found to range from 0.016 to 11.48%.

INTRODUCTION

The ability of a dam to safely pass extreme flood events is affected by the maximum discharge capacity of its spillways and the decisions made regarding the spillway operation. Savage and Johnson (2001) were one of the first to demonstrate the capability of computational fluid models in spillway modelling. They simulated flow over a simple ogee crested spillway using Flow-3D (Hirt and Nicols, 1981), which uses a finite difference method with volume of fluid representation. Simulations were performed by assuming unit thickness along the width direction since the flow was essentially 2D in nature. Simulated pressures on the spillway and discharge rates compared well to physical data. This work was later extended to include the presence of tailwater by Johnson and Savage (2006).

Gessler (2005) used Flow-3D to simulate spillway flow. In this work the discharge rate was used as the control variable and the reservoir elevation was measured. The results were compared to physical experiments for model validation. Chanel and Doering (2007) used Flow-3D in combination with a renormalized group turbulence model to simulate weir flow over three ogee crested spillways, each with a different design head to spillway height ratio. The modelling considered only representative portions of the spillway bays by using reflective boundary conditions. A 0.5 m mesh size was used for all simulations. In the study, simulated discharge rates were compared at a range of fixed headwater levels to rates from a scaled physical model. Results ranged from an over prediction of 9% to an under prediction of 24.4% in the levels as compared to the physical study. The study attributed the larger variations in the discharge predictions to the results being mesh dependent in some cases as well as possible issues with the turbulence model. Chanel and Doering (2008) also simulated orifice flow through these spillways with different gate openings, again comparing simulated discharge to physical model discharge. For a 4 m gate opening even with a 0.25 m mesh resolution (16 elements across the opening) the predicted discharge over-estimated the physical model by as much as 15%. The extent of over-estimation did not change significantly with a mesh resolution variation from 1.0 to 0.25 m. A nested mesh approach with a resolution of 0.5 m away from the gate to a fine mesh of 0.125 m very close to the gate resulted in a significant improvement in the prediction with an over-estimate of less than 2% for all flow rates analysed. This study demonstrated the need for nested meshes to resolve gated spillway flows with high accuracy.

Li et al. (2011) simulated weir flow in 3D through an auxiliary spillway consisting of six gates. Simulations were performed for peak maximum flood using Fluent with a volume of fluid solver and a κ-ε turbulence model. In this study five modifications to the spillway's geometry were investigated to determine which geometry best alleviated areas of large flow separation and recirculation. The selected spillway geometry was evaluated with a physical test. Peak maximum flood (PMF) conditions were validated. This study demonstrated the possibility of using computational fluid models as an alternative to costly scale model experiments for designing spillways for PMF conditions.

Lv et al. (2011) simulated weir flow in two dimensions over a broad-crested weir using a hybrid level set and volume of fluid method. The discharge rate for a given overflow depth compared favourably with an analytical solution for discharge rate by Chadwick and Morfett (1998). A convergence study was also presented in which simulations were performed using four meshes of increasing resolution ranging from 0.03 to 0.005 m. The study concluded that a resolution of 0.01 m provided the best balance of accuracy and computational efficiency to resolve headwater levels of between 0.1 to 0.8 m (10 to 80 elements). The percentage error ranged from an under prediction of 10.2% to an over-prediction of 2.3%.

An alternative to mesh based modelling is the particle based method of Smoothed Particle Hydrodynamics (SPH). Developed initially for free surface fluid flows by Monaghan (1994), SPH's scope of application has been demonstrated in studies of dam break flows (Cleary and Prakash 2004, Cleary et al. 2010), flow through porous media (Pereira et al. 2011), suspension of solids in liquids (Prakash et al. 2007), industrial flow modelling (Cleary et al. 2007), particle fluidisation (Xiong et al. 2011) and ship hydrodynamics (Patel et al. 2009).

Compared to mesh based methods, potential advantages of SPH for spillway modelling include; automatic representation of free surface flow behaviour, due to the Lagrangian nature of the method, the ability to incorporate complex three dimensional geometries involving complicated spillway operations and the ability to include dynamic obstructions in the flow such as debris or ice formations. In many spillways, particularly at high discharge rates, splashing and fragmentation of the flow can occur which can be easily captured by SPH. These can affect pressure distributions lower down in the spillway. Ferrari (2010) used the standard weakly compressible SPH in 2D to simulate flow over a sharp-crested weir profile. For a single flow rate, the free surface profile was compared with experimental results of Scimeni (1930). They used an SPH particle spacing of 0.0075 m to resolve a headwater level of 0.55 m with 74 particles. Lee et al. (2010) presented qualitative results from the simulated flow over a simple ski-jump spillway in 3D using the standard weakly compressible SPH formulation. An SPH particle size of 0.2 m was used to resolve a spillway width of 4 m (20 particles).

In this paper, we use the weakly compressible SPH method described in Cleary and Prakash (2004) to model flow in three dimensions through a four bay, radial gated, submerged spillway system including all near field dam components. The 3D modelling is essential since all four bays will have different behaviour depending on their location with respect to the dam. The four gates can also open to varying extents. This contributes to the complexity of the design. This study is also the first to use SPH to model a range of flow rates through a spillway, in three dimensions and compare results to physical model data. This allows for three dimensional flow features to be captured and to observe any variations in the water level across reservoir area. The spillway simulations are performed at the same scaling as a related physical study for comparison and model validation. The results of this paper are presented in two sections. Firstly, a resolution study is presented to understand the variation in the predictions with resolution and to estimate the resolutions required for highly accurate simulation. In the following section, a range of discharge rates are investigated for weir flow conditions.

PHYSICAL MODEL

The Pala Tiloth dam is a 74 m high concrete gravity arch dam, with a crest elevation of 1670 m and crest length of 141 m. For flood control and reservoir management, the dam has a four bay, radial gated, submerged spillway system located at an elevation of 1631.53 m. Each of the spillway bays have dimensions 10.5 m wide by 11.9 m tall, with a combined design discharge capacity of 9,200 m^3/s for peak maximum flood.

Figure 1: A photograph of the four bay radial gated spillway used in the physical model testing.

Physical testing of the spillway design was conducted by SMEC International Pty Ltd (SMEC, 2006) using a scaled model of ratio 1:60, shown in Figure 1. Testing consisted of recording the height of reservoir water, with the gates fully open, for a range of discharge rates. To compare the results from the model to the full scale spillway, Froude number similarity can be used as the spillway flow conditions are governed by the ratio of inertial to gravitational forces. That is:

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_{sp} \qquad (1)$$

where V is a characteristic velocity (m/s), g is acceleration due to gravity and L is the length scale. Subscripts m and sp are used to represent the scaled model and full scale spillway respectively.

Defining the length scale ratio as $L_r = L_{sp}/L_m$ and given the spillway to model ratio is 60:1, we obtain $L_r = 60$. Using Equation 1, the following secondary ratios are then obtained:

Velocity: $V_r = \frac{V_{sp}}{V_m}$ = 7.75:1

Discharge: $Q_r = V_r L_r^2 = \frac{Q_{sp}}{Q_m}$ = 27885:1 and

Reynolds number: $R_r = \frac{R_{sp}}{R_m}$ = 465:1.

These secondary ratios allow for interpretation of the physical results at the full length scale. For example a model discharge of 35.86 L/s is equivalent to a spillway discharge of 1000 m^3/s.

SPH MODEL

The numerical model was constructed at the same dimensions as the physical model. This allows direct comparison of simulated results to physical model results. The numerical model used included a 3D CAD geometry of the spillway constructed from schematics shown in Figure 2 and a digital terrain model (DTM) of the topography surrounding the spillway including the underwater section.

The spillway opening width and height were 0.1750 (actual scale 10.5 m) and 0.198 m (actual scale 11.9 m) respectively. Six SPH resolutions ranging from 0.01 to

0.035 m were used (0.6 to 2.1 m actual scale). The number of SPH particles in the simulation for representing the fluid, terrain and the spillway geometry are given in Table 1 for each resolution. The fluid viscosity and density in the simulations were set to that of water. Discharge rates ranging from 8.96 (250 m^3/s full scale) to 62.76 L/s (1750 m^3/s full scale) were analysed. For consistency, quantities reported will be in the physical model scale.

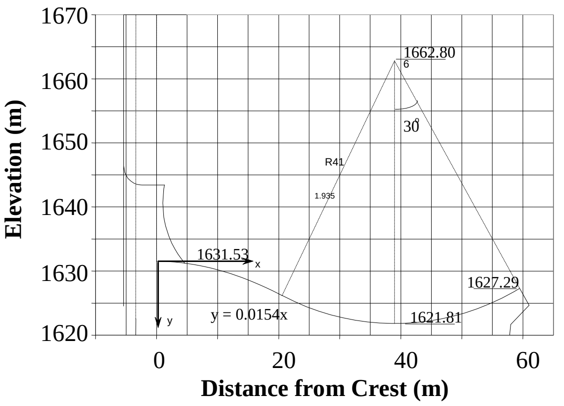

Figure 2: Two dimensional profile of the spillway cross-section provided by SMEC. This schematic and photographs of the physical model were used to produce the CAD geometry of the spillway used in simulations.

Table 1: Number of SPH particles in simulation for each resolution used.

SPH resolution (m)	Fluid ($\times 10^3$)	Terrain ($\times 10^3$)	Spillway ($\times 10^3$)
0.010	5,392	1,937	242
0.015	2,110	878	198
0.020	972	490	61
0.025	589	287	51
0.030	438	224	50
0.035	242	148	25

Figure 3 shows orthographic views of the geometric setup. The reservoir water surface shown in Figure 3 is the level prior to commencement of flow through the spillway gates at time zero seconds. Similar to the physical model, the reservoir area was enclosed by a wall at the back as shown in the top view. The location of this wall was 1.855 m from the spillway gates. A rectangular inflow was positioned along the back wall with the top of the inflow below the bottom of the spillway gates. This was to ensure that the inflow did not interfere with the discharge from the spillway gates. For a fixed inflow rate, when the flow reached steady state, the discharge rate through the spillway was equal to the inflow rate and the reservoir water level was invariant. This steady reservoir water level was compared against physical model results for the range of flow rates mentioned above. For the highest resolution used the simulation took 28 days to reach steady state when running in parallel using a dual Xeon 8-core E5-2650 machine. In contrast, the lowest resolution simulation took 17 hours to reach steady state.

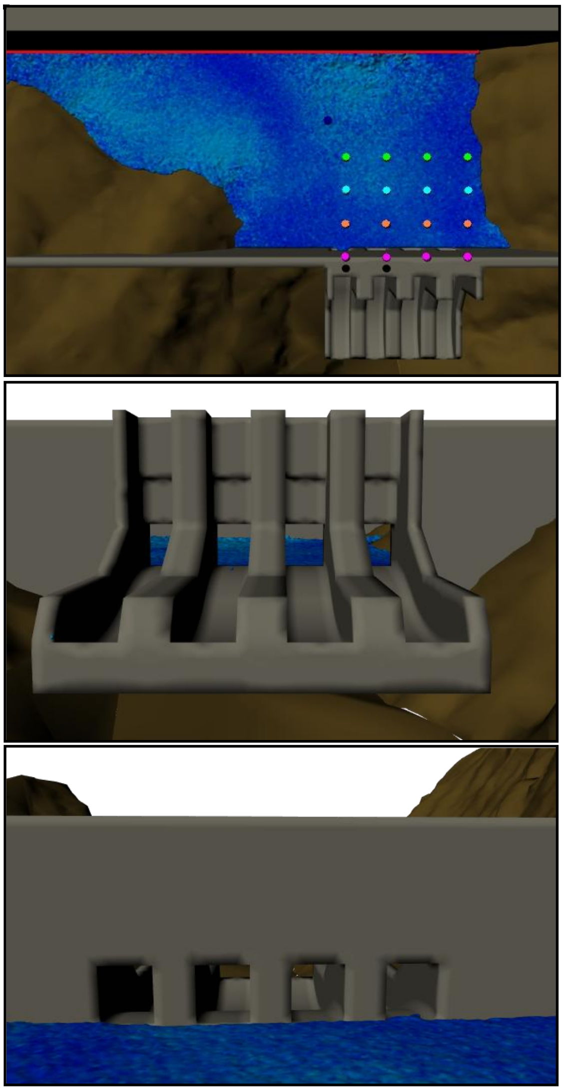

Figure 3: Top, front and back views of the geometric setup. The front and back views show the four spillway gate openings. The red line at the back of the reservoir (in the top view) indicates the position of the inflow. The coloured dots show the locations of the water height sensors which are coloured the same if they have the same orthogonal distance to the spillway.

Nineteen height sensors were used in the simulations to measure the water depth at various locations in the reservoir. These are shown in the top view in Figure 3. Since the terrain is uneven, to compare readings of water depths an appropriate datum was needed. The height of the terrain at the point of intersection with the middle of the spillway base was used as the datum.

FLOW VISUALISATION

Figure 4 presents the flow through the spillway at 10, 18, 26 and 65 s for an inflow rate of 35.86 L/s and particle size of 0.01 m. It shows water exiting the flip bucket portion of the spillway.

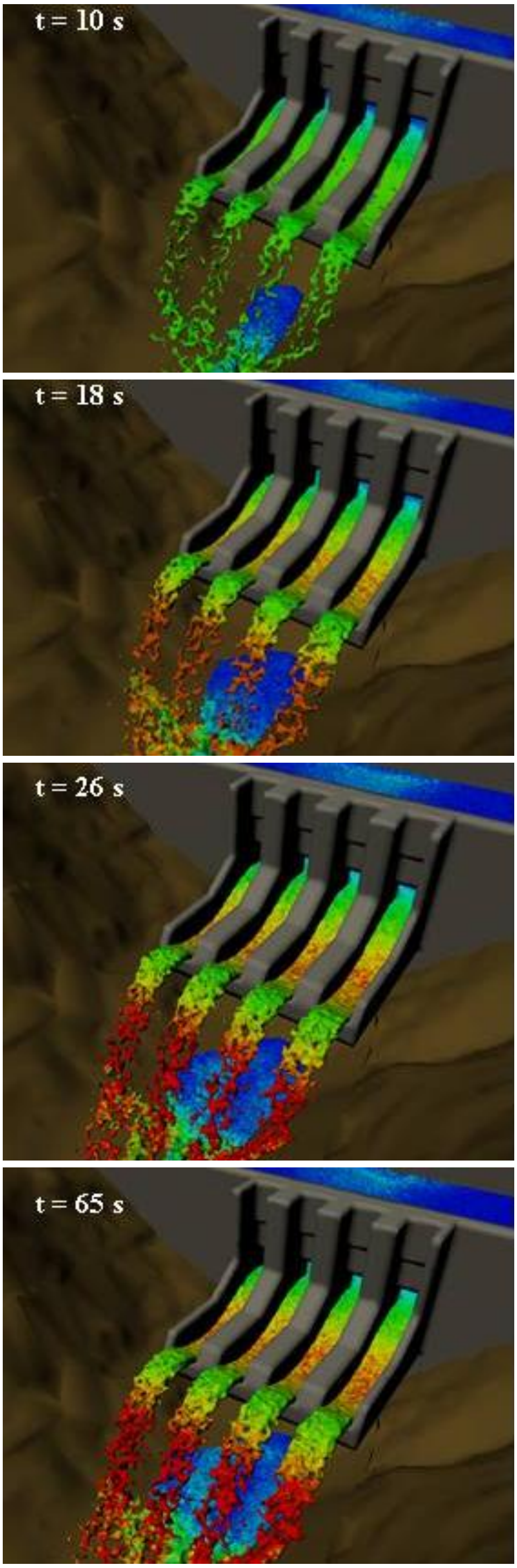

Figure 4: Flow development through the spillway for a flow rate of 35.86 L/s and a particles size of 0.010 m. The shading on the water surface represents speed, with blue being 0 m/s, green intermediate and red being 2.5 m/s.

At 10 s, the water height above the spillway crest is 0.079 m. Water has just begun to exit through the spillway gates with only a thin layer of water on the flip bucket part of the spillway. There is some fragmentation of water in this area. The speed of the water flowing down and exiting the flip bucket is approximately 1.25 m/s. At 18 s, the water height has increased to 0.109 m and the flip bucket has filled leading to a continuous stream of water with no fragmentation. The speed of the water on the flip bucket is no longer constant, with a velocity profile that changes proportionally to changes in the slope of the flip bucket. The water exit speed has increased to around 1.8 m/s. By 26 s, the water height has increased to 0.124 m and the velocity profile of the water in the flip bucket has almost stabilised. In contrast to the flow at 10 s, the water exiting the flip bucket at 26 s is propelled upward by momentum instead of falling. A continued widening of the flow exiting the flip bucket is also seen as compared to previous times. The maximum speed of water is close to 2.5 m/s. At 65 s, steady state is reached and the water height above the spillway crest stabilises at approximately 0.139 m. The velocity profile of water has also been clearly established in the spillway.

RESOLUTION STUDY

The accuracy of the simulated spillway flows was found to be dependent upon the particle size used to resolve the flow behaviour. A resolution study was therefore conducted to understand the trade-off between cost and accuracy for spillway flow prediction using SPH. A fixed inflow rate of 35.86 L/s was used for the analysis. The water depth is resolved at least by 15 particles for the lowest resolution of 0.035 m.

Since the inflow rate is specified, the simulated reservoir depth is the key prediction to be compared to the physical model results. For the resolution study, the simulated reservoir water depth was measured at the sensor furthest from the spillway (see Figure 3 top view). The water depth recorded in the physical testing for a discharge rate of 35.86 L/s was 0.570 m. Comparison of the simulated reservoir water depths for the different resolutions are presented in Table 2. Two quantities are used for comparison namely:

a) Percentage difference in depth which compares the simulated reservoir depth with physical data, and
b) Relative percentage difference comparing the depth obtained from lower resolution simulations with the highest resolution of 0.01 m.

From Table 2, results for the particle size of 0.015 m are comparable to the highest resolution simulation of 0.01 m mm, with a relative difference of only half a percent. The particle size of 0.015 m was therefore used for the simulations of weir conditions at the spillway, as this resolution provides the desired balance of computational efficiency and accuracy.

Figure 5 shows the percentage difference between the simulated water depth and the physical model depth for the range of SPH resolutions. The percentage difference converged to a resolution independent value beyond a resolution of 0.02 m.

Table 2: Simulated water depth for different resolutions. Percentage difference is between simulated reservoir depth and physical data. Relative percentage difference compares lower resolution simulations with the highest resolution of 0.01 m.

Particle size (m)	**Simulated water depth (m)**	**Percentage difference in depth**	**Relative percentage difference**
0.010	0.610	7.02	n/a
0.015	0.612	7.37	0.33
0.020	0.611	7.19	0.16
0.025	0.623	9.30	2.13
0.030	0.635	11.40	4.10
0.035	0.653	14.56	7.05

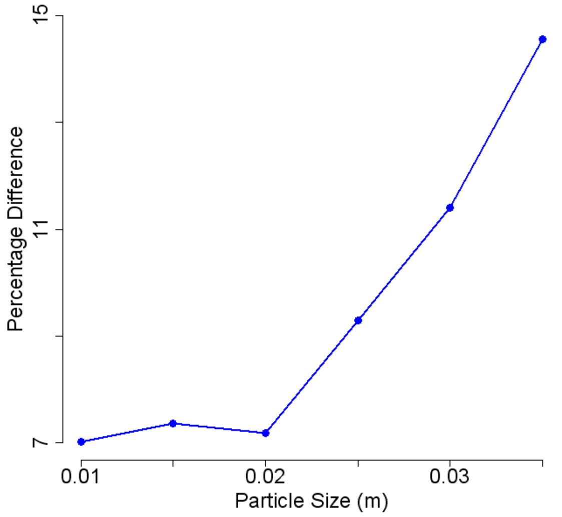

Figure 5: Percentage difference in simulated reservoir water depth for different SPH resolutions at a discharge rate of 35.86 L/s.

WEIR FLOW

In this section, simulations of weir flow conditions at the spillway are presented. Weir flow occurs when the water flowing through the spillway does not contact the top of the gate opening. The spillway operates at orifice flow conditions above these flow rates.

Water depth in the reservoir is variable. The level is lowest close to the spillway crest because the flow follows the tapering spillway profile as it exits from the gate. Figure 6 shows the variation in water depth with distance from the spillway crest. The difference in the water depth in the reservoir is 0.016 m (0.96 m full scale) orthogonal to the spillway. This compares with Gessler's (2005) Flow-3D predictions which showed a variation in water depth of around 0.3 m (full scale) in the reservoir. Closer to the spillway crest the water depth also shows transverse variations of as much as 0.003 m as seen from the four magenta coloured sensor points. This is because near the spillway crest, flow behaviours become increasingly three dimensional as the water discharges through the spillway gates. From Figure 6 at a distance of around 1.0 m from the spillway crest the water depth begins to stabilise. The measurement location used for sampling the water depth in the physical model is unknown. Given this and due to significant variations in water depth across the reservoir, we compare the simulated results averaged across transverse locations from the spillway against the physical data.

Figure 7 compares the water depths as a function of discharge rate at different distances from the spillway. The predicted discharge rates are shown in blue, orange, magenta and black, while the physical model discharge rate is shown in red. The colouring in Figure 7 corresponds to sensor locations in Figure 3, with readings at similarly coloured sensors averaged for each discharge rate.

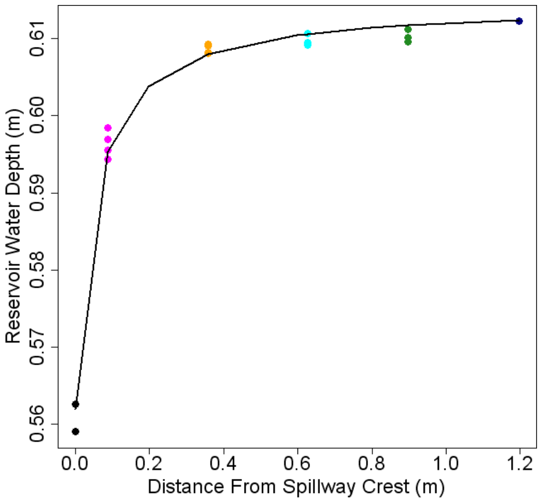

Figure 6: Variation of water depth in the reservoir with distance from the spillway crest using sensors marked in Figure 3. The discharge flow rate used was 35.86 L/s and SPH resolution was 0.015 m.

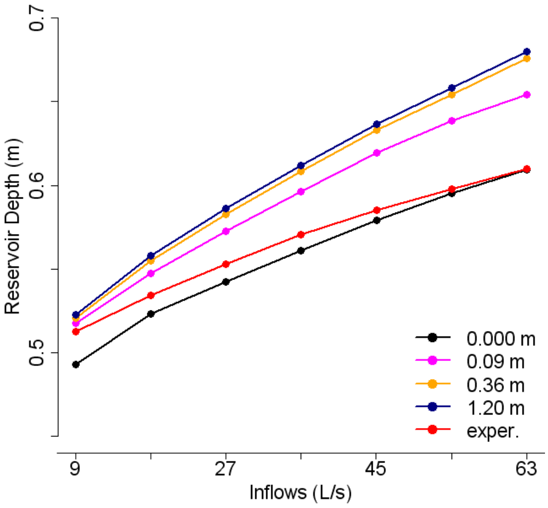

Figure 7: Change in reservoir water depth with discharge rate at different distances from the spillway. The predicted discharge rates are shown in blue, orange, magenta and black, while the physical measurements are shown in red.

The discharge rate predicted at the spillway crest is the closest to the experimentally measured values with a maximum difference of only around 2.4% occurring at a flow rate of 8.96 L/s. The discharge rate predictions are furthest away from the experimental values 1.2 m from the crest (blue line in Figure 7). At this location the maximum difference between the simulated and experimental values is 11.50% and occurs at a discharge rate of 62.76 L/s.

Table 3 compares the simulated water depth with the physical model data at the spillway crest and 1.2 m from the crest. At the spillway crest the simulated values are always below the measurements with the smallest difference of 0.16% occurring at a discharge rate of 62.76 L/s and a maximum of 2.34% at 8.96 L/s. At 1.2 m from the crest the predictions are always above the measurements with the difference ranging from 1.95% at 8.96 L/s rising to 11.48% at 62.76 L/s. Gessler (2005) has suggested that physical models of spillways are only accurate up to 5% in estimating water depths in the reservoir.

Table 3: Comparison between simulated reservoir depth and physical model reservoir depth at the spillway crest and 1.2 m away from the spillway crest.

Rate (L/s)	Model water depth (m)	Sim. water depth (m) At crest / 1.2 m from crest	Percent diff. depth At crest/1.2 m from crest
8.96	0.512	0.500/ 0.522	-2.34 / +1.95
17.93	0.534	0.523 / 0.558	-2.06 / +4.49
26.89	0.553	0.542/ 0.586	-1.99 / +5.97
35.86	0.570	0.561 / 0.612	-1.58 / +7.37
44.83	0.585	0.579/ 0.637	-1.03 / +8.89
53.79	0.598	0.595 / 0.658	-0.50 / +10.03
62.76	0.610	0.609 /0.680	-0.16 / +11.48

CONCLUSION

Weir flow conditions through a four bay, radial gated submerged spillway, were modelled using SPH. These simulations were conducted for a range of inflow rates using the entire spillway domain. Modelling the entire spillway domain allowed variations in reservoir water level to be observed relative to the spillway gates. This was particularly important when comparing the simulated reservoir water depth to the physical model results, as there was uncertainty in the location of the experimentally measured water depth. The maximum difference between the simulated and experimental water depth was 11.5%, which occurred at a discharge rate of 62.76 L/s. This depth was recorded in the centre of the reservoir area. At the spillway crest the maximum difference was only 2.4% for a discharge rate of 8.96 L/s. However, to provide further confidence in simulated results, the exact location of the sensor used in physical testing is needed for accurate comparison.

For the discharge rate of 35.86 L/s, the effect of resolution on simulated results was investigated. With an increase in resolution from 0.035 to 0.01 m (5 to 17 particles across the width of the spillway opening) the percentage difference in the simulated and experimental water depth reduced from 14.56 to 7.02%. These percentage differences were obtained from simulated results recorded in centre of the reservoir. The solution became insensitive to resolution for an SPH particle size of 0.02 m.

This work forms the preliminary research for a future study that will investigate pressures on the flip bucket, forces on the gate structures and the effect of dynamically moving gate structures.

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