PBMR PCU ACOUSTICS

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ABSTRACT

An earlier paper discussing a simple one dimensional acoustic wave equation that represents a Pressurized Water Reactor (PWR) inlet pump and pipe is re-evaluated. The original solution was called into question, however, this work will show the original solution to be correct and that it also has some numerical advantages. The problem described is a homogeneous differential equation with non-homogeneous boundary conditions. A transformation technique is applied changing the form of the problem to a non-homogeneous differential equation with homogeneous boundary conditions by utilizing an auxiliary function. It is shown that, unlike other solutions, an auxiliary function defined on the interior of the media is unnecessary. The resulting models can be utilized to formulate acoustic vibration loads for a Pebble Bed Modular Reactor (PBMR) Power Conversion Unit (PCU). Comparison with other literature is provided.

1. INTRODUCTION

Pumps, fans and compressors are a source of acoustic pulsations in nuclear power plants and other industrial systems. A pump will compress the fluid directly in front of its blades causing the fluid to expand, creating a series of expansions and contractions of the fluid. This acoustic energy will vibrate structures potentially causing material stress and component fatigue and resultant structural failure. Structural components in the PBMR reactor must be evaluated for their structural integrity under acoustic loading.

Acoustic pulsations in a light water reactor core annulus were first examined by Penzes and later by Bowers and Horvay. These two studies utilized an equivalent body force to represent the source of the pump pulsations. However, they differed on the form of this body force. Cepkauskas sought to resolve this discrepancy and demonstrated that the use of a body force is not required. He utilized a technique that transformed the homogeneous differential equation with time dependent boundary conditions to one of a non-homogeneous differential equation with homogeneous boundary conditions. This technique was unique in that the previous authors chose auxiliary functions to make the boundary conditions homogeneous. This was shown to be unnecessary and was expounded upon in a paper with Fisher and Chandra.

For the simple pipe acoustics, Lee & Chandra utilized this same technique, but unlike Cepkauskas, choose unique auxiliary functions to formulate the problem. Kye Bock Lee et al (1992) stated that Lee & Chandra did not meet the boundary condition with their chosen auxiliary functions and proceeded to formulated a different solution using different auxiliary functions. Cepkauskas also addressed this same simple pipe but coupled this solution with the reactor core annulus solution.

Kye Bock Lee et al (1992 & 1994) and Jong-sik Cheong et al called Cepkauskas' analysis into question. Kye Bock Lee et al (1992) stated, incorrectly, that 1) Cepkauskas used a body force 2) Lee and Chandra "missed the constraints on the auxiliary functions to make the boundary conditions homogeneous" 3) they claimed to use the improved technique expounded by Fisher et al. (This improved method was not the transformation of the equations but the fact that no auxiliary function is necessary.) The issue was complicated by the addition of fictitious forcing functions to the problem at the spring end.

This analysis re-evaluates this scenario and resolves any discrepancies and answers any questions in regarding the proper acoustic models needed for the PBMR design.

2. MATHEMATICAL MODEL

2.1 Model

We begin by examining a simple pipe model similar to that of Cepkauskas, Lee & Chandra and Kye Bock Lee et al (1992 & 1994) for steady state conditions. However, no auxiliary functions will be used. A similar model examining the response of the spring mass is found in Cepkauskas & Stevens.

Figure 1 illustrates a pipe of length L and cross sectional area A with a pump having a pressure amplitude P_0 and frequency ω_p at one end. At the other end an acoustic resistance exists represented by a spring end condition having stiffness K. The spring end was first introduced in the work of Penzes and later used by Lee & Chandra as well as Kye Bock Lee et al (1992).

The following is a mathematical model for the development of acoustic pipe loading:

The one dimensional acoustic pressure $P(x,t)$ wave equation with speed of sound C_0 is given by:

$$\frac{\partial^2 P(x,t)}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 P(x,t)}{\partial t^2} = 0 \qquad (1)$$

with corresponding boundary conditions given by:

$$@x = 0 \qquad P(x,t) = P_0 \cos\omega_p t \qquad (2\text{-a})$$

$$@x = L \qquad \frac{\partial P(x,t)}{\partial x} - \frac{A\rho}{K}\frac{\partial^2 P(x,t)}{\partial t^2} = 0 \qquad (2\text{-b})$$

Differential equation 1 with boundary conditions 2-a & 2-b provide a well defined mathematical model. There is no need to define unique auxiliary functions or to modify the forcing function to create a correct response.

2.2 Transformation

A transformation equation is assumed in the form of:

$$P(x,t) = Q(x,t) + g(x)P_0 \cos\omega_p t \qquad (3)$$

The substitution of equation 3 into the differential equation 1 and boundary conditions 2 results in:

$$\frac{\partial^2 Q(x,t)}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 Q(x,t)}{\partial t^2} =$$

$$-[\frac{\partial^2 g(x)P_0}{\partial x^2}\cos\omega_p t + \frac{\omega_p^2}{c_0^2} g(x)P_0 \cos\omega_p t] \qquad (4)$$

$$Q(0,t) = P_0 \cos\omega_p t - g(0)P_0 \cos\omega_p t \qquad (5\text{-a})$$

$$\left.\frac{\partial Q(x,t)}{\partial x}\right|_{x=L} - \frac{A\rho}{K}\left.\frac{\partial^2 Q(x,t)}{\partial t^2}\right|_{x=L} =$$

$$-[\left.\frac{\partial g(x)}{\partial x}\right|_{x=L} P_0 \cos\omega_p t$$

$$+\frac{A\rho\omega_p^2}{K}\left.g(x)\right|_{x=L} P_0 \cos\omega_p t] \qquad (5\text{-b})$$

The transformed equations can now be placed in the form of a non-homogeneous equation with homogeneous boundary conditions by requiring:

$$g(0) = 1 \qquad (6\text{-a})$$

$$\left.\frac{\partial g(x)}{\partial x}\right|_{x=L} + \frac{A\rho\omega_p^2}{K}\left.g(x)\right|_{x=L} = 0 \qquad (6\text{-b})$$

Thus equation (5-a) and (5-b) become:

$$Q(0,t) = 0 \qquad (7\text{-a})$$

$$\left.\frac{\partial Q(x,t)}{\partial x}\right|_{x=L} - \frac{A\rho}{K}\left.\frac{\partial^2 Q(x,t)}{\partial t^2}\right|_{x=L} = 0 \qquad (7\text{-b})$$

2.3 Solution

The free vibration solution is first obtained by setting the RHS of equation 4 to zero, assuming a separation of variables solution and applying boundary conditions (7-a) and (7-b) resulting in:

$$Q(x,t) = Q(x)Q(t) =$$

$$[\sin\frac{\omega_n x}{c_0}][A\cos\omega_n t + B\sin\omega_n t] \qquad (8\text{-a})$$

with frequency equation: $-\frac{K}{A\rho\omega_n c_0} =$

$$\tan\frac{\omega_n L}{c_0} \qquad (8\text{-b})$$

The forced vibration solution is assumed to be of the form:

$$Q(x,t) = \sum C_n Q_n(x)\cos\omega_p t \qquad (9)$$

Required Orthogonality Conditions, as found in reference Lee and Chandra and Kye Bock Lee et al (1992 & 1994) are:

$$\frac{1}{c_0^2}\int_0^L Q_n Q_m dx - \frac{A\rho}{K}Q_n(L)Q_m(L) = 0 \qquad (A)$$

$$\int_0^L \frac{dQ_m}{dx}\frac{dQ_n}{dx}dx = 0 \qquad (B)$$

$$\int_0^L \frac{d^2Q_n}{dx^2} Q_m dx - \frac{dQ_n(L)}{dx} Q_m(L) = 0 \qquad \text{(C)}$$

Substitution of equation (9) into equation (4) and multiply both sides by $Q_m(x)$ and integrate over the length leads to:

$$\sum_{n=1}^{\infty} C_n \int_0^L Q_m(x) \frac{d^2Q_n(x)}{dx^2} dx + \frac{\omega_p^2}{c_0^2} \sum_{n=1}^{\infty} C_n \int_0^L Q_m(x) Q_n(x) dx =$$

$$-P_0 [\int_0^L Q_m(x) \frac{d^2 g(x)}{dx^2} dx + \frac{\omega_p^2}{c_0^2} \int_0^L Q_m(x) g(x) dx] \qquad (10)$$

Similarly, substitution of equation (9) into equation (7-b) and multiply both sides by $Q_m(L)$ results in:

$$\sum_{n=1}^{\infty} Q_m(x) \frac{dQ_n(x)}{dx} + \frac{A\rho\omega_p^2}{K} \sum_{n=1}^{\infty} [Q_m(x) Q_n(x)]_{x=L}$$

$$= 0 \qquad (11)$$

The subtraction of equations (11) from (10) gives:

$$\sum_{n=1}^{\infty} C_n [\int_0^L Q_m(x) \frac{d^2Q_n(x)}{dx^2} dx - Q_m(x) \frac{dQ_n(x)}{dx}]_{x=L}$$

$$+ \frac{\omega_p^2}{c_0^2} \sum_{n=1}^{\infty} C_n [\int_0^L Q_m(x) Q_n(x) dx$$

$$- \frac{A\rho c_0^2}{K} Q_m(x) Q_n(x)]_{x=L}$$

$$= -P_0 \int_0^L Q_m(x) \frac{d^2 g(x)}{dx^2} dx$$

$$+ P_0 \frac{\omega_p^2}{c_0^2} \int_0^L Q_m(x) g(x) dx \qquad (12)$$

Note that the first term in brackets [ ], is the orthogonality condition, equation (A) and the second term in brackets [ ] is the othogonality condition, equation (C). Thus all terms on the left hand side of equation (12) are zero except for the terms n=m, thus:

$$C_n (I_1 + \frac{\omega_p^2}{c_0^2} I_2) = -P_0 (I_3 + \frac{\omega_p^2}{c_0^2} I_4) \qquad (13)$$

The required integrals will now be evaluated:

$$I_1 = \int_0^L Q_n(x) \frac{d^2Q_n(x)}{dx^2} dx - \left[Q_n(x) \frac{dQ_n(x)}{dx} \right]_{X=L}$$

$$= -\frac{\omega_n}{c_0} [\frac{L}{2} (\frac{\omega_n}{c_0}) + \frac{\sin 2(\frac{\omega_n}{c_0}) L}{4}] \qquad \text{(14-a)}$$

The integral I_2 is similar to I_1 and results in:

$$I_2 = -\frac{c_0^2}{\omega_n^2} I_1 \qquad \text{(14-b)}$$

$$I_3 = \int_0^L Q_n(x) \frac{d^2 g(x)}{dx^2} dx$$

Integrate I_3 by parts twice results in:

$$I_3 = Q_n(L) [\frac{dg(x)}{dx}]_{x=L} - Q_n(0) [\frac{dg(x)}{dx}]_{x=0}$$

$$- g(L) [\frac{Q_n(x)}{dx}]_{x=L} + g(0) [\frac{Q_n(x)}{dx}]_{x=0} + \int_0^L g(x) \frac{d^2Q_n(x)}{dx^2} dx$$

The second term of this integral goes to zero due to the mode shape being zero at x = 0. Note that g(0)=1 in the fourth term due to equation (6-a). Thus this reduces to:

$$I_3 = (\frac{\omega_n}{c_0}) - \frac{\omega_n^2}{c_0^2} I_4 +$$

$$A\rho \frac{g(L)}{K} \sin \frac{\omega_n L}{c_0} (\omega_n^2 - \omega_p^2) \qquad \text{(14-c)}$$

Thus the required constants are given by:

$$C_n = -\frac{P_0}{c_0} \frac{\omega_n^3}{I_1 (\omega_n^2 - \omega_p^2)} +$$

$$\frac{1}{I_1} \frac{P_0 \omega_n^2}{c_0^2} [I_4 - A\rho \frac{g(L) c_0^2}{K} \sin \frac{\omega_n L}{c_0}] \qquad (15)$$

Substitution of 15 into equation 9 and finally into equation 3 results in:

$$P(x,t) = -\sum_1^{\infty} \frac{P_0}{c_0} \frac{\omega_n^3}{I_1 (\omega_n^2 - \omega_p^2)} \sin \frac{\omega_n x}{c_0} \cos \omega_p t$$

$$+ \sum_1^{\infty} \frac{1}{I_1} \frac{P_0 \omega_n^2}{c_0^2} [I_4 - A\rho \frac{g(L)}{K} \sin \frac{\omega_n L}{c_0}]$$

$$\sin \frac{\omega_n x}{c_0} \cos \omega_p t + g(x) P_0 \cos \omega_p t$$

It can be demonstrated that the second term is the negative Fourier expansion of the third term and thus cancel. This is achieved by utilizing orthogonality condition (A) and assuming that g(x) can be expanded into a generalized Fourier series according to equation 7 of Kreyszig, page 474. The denominator of this expansion is I_2 and the use equation 14-b provides the cancellation. This results in the final solution of

$$P(x,t) = -\sum_1^{\infty} \frac{P_0}{c_0} \frac{\omega_n^3}{I_1 (\omega_n^2 - \omega_p^2)} \sin \frac{\omega_n x}{c_0} \cos \omega_p t \qquad (16)$$

Thus it is demonstrated that no auxiliary function is needed, only the restriction on the auxiliary function at its boundaries. This solution can now be simplified for the two bounding cases of an open end pipe and a closed end pipe. As stated in references Lee & Chandra and Kye Bock Lee et al, these solutions can be obtained by letting K = 0 and K = ∞, respectively.

Thus the open end solution is given by:

$$P_{open}(x,t) = -\sum_{i=1,2}^{\infty} \frac{2P_0 c_0 \omega_i}{L(\omega_i^2 - \omega_p^2)} \sin\frac{i\pi x}{L} \cos\omega_p t \quad (17)$$

with $\omega_i = i\pi C_0 / L$ and note that the integral I_1 reduced to $I_1 = -(\frac{i\pi}{L})^2 \frac{L}{2}$

Thus the closed end solution is given by:

$$P_{close}(x,t) = -\sum_{j=1,3}^{\infty} \frac{2P_0 c_0 \omega_j}{L(\omega_j^2 - \omega_p^2)} \sin\frac{j\pi x}{2L} \cos\omega_p t \quad (18)$$

with $\omega_j = j\pi C_0 / 2L$ and note that the integral I_1 reduced to $I_1 = -(\frac{j\pi}{2L})^2 \frac{L}{2}$

3. NUMERICAL RESULTS

A 10 m pipe filled with helium having a speed of sound of 2100 m/sec and a pump exit pressure of .01 MPa and forcing frequency $\omega_p = 2\pi 100$ is examined. The density and area are taken as 1.0. Figure two gives the extreme open end and closed end pressure along the length of the pipe provided by equations 17 & 18. In addition the response is approximated using equation 16 with K=2 for open end and K=8E10 for the close end. It should be noted that, at the pump end, a numerical struggle exists. The boundary condition given by equation 2-A requires a finite value while the mode shape in equation 8-A requires the results to be zero while ω_n is approaching infinity in the series. This produces a local numerical convergence at the pump as discussed in Fisher et al, where it is shown by taking more and more terms of the series results in the solution approaching the required P_0 at x = 0.

4. COMPARISON WITH OTHER SOLUTIONS

It was suggested by Kye Bock Lee et al (1992 & 1994) that the solution found by Cepkauskas for region I, inlet pipe, does not contain physical meaningful natural frequencies and thus is not a valid solution. The inlet pipe solution obtained by Cepkauskas, hence forth referred to as SMIRT5 is given by:

$$P_{SMIRT5}(x,t) = -\sum_{l=1,3}^{\infty} \frac{c_0 \omega_l P_0 + A^* c_0^{\,2}}{(\omega_l^2 - \omega_p^2)L/2} \sin\frac{l\pi x}{2L} \cos\omega_p t \quad (19)$$

with $\omega_j = l\pi C_0 / 2L$ (A* notation is used to distinguish from Area, A)

This solution is plotted in figure 4. It is seen that this solution does, although it has non physical frequencies (those of a closed end), match the results obtained above. Thus the solution is mathematically correct. It has some advantages over the present solution in that the eigenvalues are not found via a transcendental equation 8-b, but a more manageable equation $\omega_j = j\pi C_0 / 2L$.

The question arises as to why the SMIRT 5 solution is valid. In SMIRT 5 the boundary condition at x = L was based on physical reasoning, in that if the pump has a harmonic frequency that the gradient of the pressure is a constant times the forcing frequency. That is:

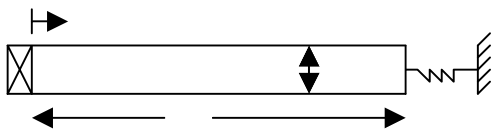
$$\left.\frac{dP(x,t)}{dx}\right|_{@x=L} = A^* \cos\omega_P t \quad (20)$$

Comparison of equation 7-b with equation 20, and recognizing the present solution results in $A^* = -A\rho\omega_P^2 Q(L)/K$. Therefore it is not surprising the solutions are mathematically identical.

Lee and Chandra utilized the same problem statement as found in the differential equations (1), boundary conditions (2-a & 2-b) and transformation (3) as found in this manuscript. However, they chose a very simple auxiliary function whose second derivative is zero. Thus their equivalent I_3 term is zero. There exists no mathematical requirement that states the auxiliary function needs to have higher order derivatives. Their choice of auxiliary function will result in a correct solution; in fact it has some advantage in showing the time dependent boundary condition is satisfied. However, the choice of auxiliary function required to demonstrate that no auxiliary function is needed on the interior of the problem relies on I_3 being non-zero. Thus the present solution and Lee and

Chandra's solution are numerically identical when one corrects two typographical errors in their manuscript; Lee and Chandra equation 17 should have ω_n in the denominator and the term c_0^3 / L, appearing twice in equation 24 should be replaced with $\beta\omega_p^2$.

It was stated in Kye Bock Lee et al (1992 & 1994) that Lee & Chandra did not meet all the required boundary conditions. Kye Bock Lee et al (1992 & 1994) claimed to have the complete solution. However, comparing Lee and Chandra and the SMIRT 5 solutions with the solution above, it is seen that it is a cross between the two solutions. At first glance this appears to be a possible solution.



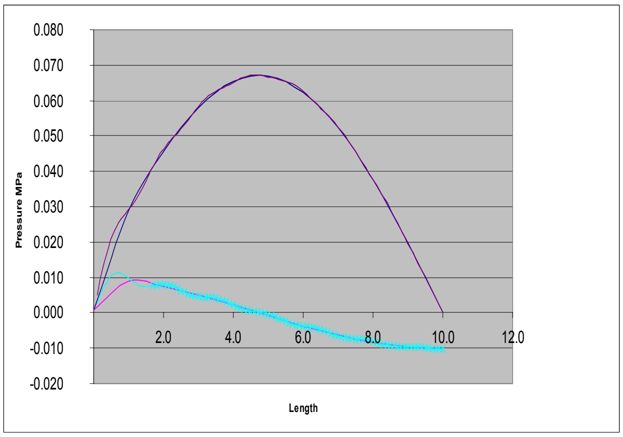


Figure 2
Present Solution for open end and closed end

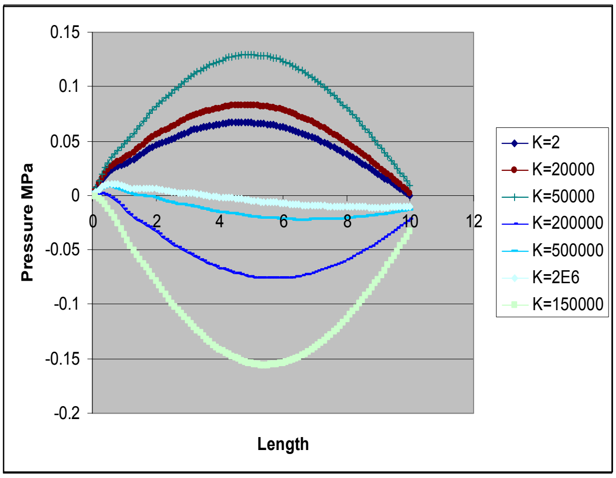


Figure 3
Present Solution for various spring constants

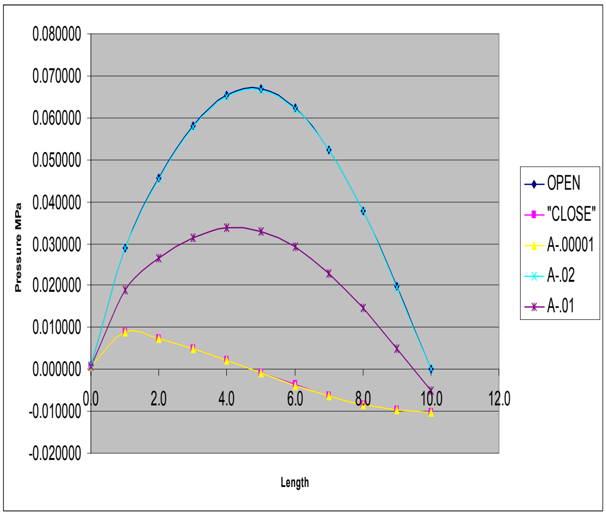


Figure 4
SMIRT5 Solution

In order to determine if their solution is correct, it was re-derived using their assumptions, but with the present technique. Their differential equation and boundary conditions are also identical to the present analysis; however their transformation equation added a second time dependent forcing function and thus required two auxiliary functions. Typically, each auxiliary function should have two restrictions, one at x=0 and one at x=L. However, they added an additional restraint (their equation 9) to make the boundary condition homogeneous.

The steps found in this paper were followed assuming Kye Bock Lee et al two auxiliary functions were continuous and had higher order derivatives. Equation 13 of this manuscript resulted in the evaluation of two I_3 and I_4, one for each auxiliary function. The resulting solution showed that one of the two auxiliary functions is eliminated, but not both. The original premise is that if the problem is well defined, no choice of auxiliary function is required for the interior of the medium. Since both auxiliary functions do not disappear in the solution it is concluded the problem is ill-defined.

Recall that Kye Bock Lee sited the use of the Fisher et. al. publication where it is clear that only one auxiliary function exists for each time dependent forcing function. In the Kye Bock Lee solution there are two auxiliary functions but only one time dependent boundary condition. It is possible that the problem Kye Bock Lee wanted to solve was that the second boundary condition (equation 2-b) should have an unknown amplitude with pump harmonic time dependency on the right hand side of equation 2-b. This appears to be the case since their transformation includes this term and based on Fisher et al, this would also appear in the boundary condition. If this was the case, two auxiliary functions would be required. The first would be identical to the one found in this document. The second would be zero at x= 0 and $\left.\frac{\partial g_2(x)}{\partial x}\right|_{x=L} + \left.\frac{A\rho\omega_p{}^2}{K}\, g_2(x)\right|_{x=L} = 1$.

5. CONCLUSION

It is concluded that the model utilized by Cepkauskas in SMIRT 5 is complete and accurate, has some improved numerical benefits and can be utilized to pursue PBMR acoustic vibrations. The Lee and Chandra solution is numerically equivalent. The Kye Bock Lee solution needs some refinement. Details of the proper coupling of the pipe with the annulus will be considered in a future paper.

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