FLOW-ACOUSTIC INTERACTION IN CORRUGATED PIPES: TIME-DOMAIN SIMULATION OF EXPERIMENTAL PHENOMENA

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ABSTRACT

A phenomenological model based on the coupling between a set of "pressure vortex oscillators" and an acoustic field is considered to simulate the dynamical behaviour of a corrugated pipe. The effect of a random Gaussian dispersion of the Strouhal number is investigated by means of non-linear numerical time-domain simulations and also in terms of the modeshapes of the linearized aeroacoustical model. The results demonstrate that such perturbations may qualitatively change the dynamics of the model and might cause localization phenomenon.

1. INTRODUCTION

In the context of aerodynamic noise generation, a corrugated pipe open at both ends emits clear and loud tones when air flows through it at sufficiently high velocities. This tone generation - also called *whistling* - is an example of flow-excited acoustic phenomenon, in which air-flow interacts with the longitudinal acoustic modes of the duct to give rise to self-sustained oscillations. Due to its pratical significance in a large variety of technical applications and its intricate fundamental aspects, the phenomenon is being currently studied by different authors by means of experiments (Belfroid et al. (2007); Kristiansen and Wiik (2007)) as well as numerical simulations (Popescu and Johansen (2008)). To maintain the acoustic oscillations, the central physical feature is a lock-in phenomenon between the resonant acoustic field in the pipe and a vortex shedding process due to the passage of air-flow over the corrugations. The aeroacoustic instability occurs when the frequency of the vortex shedding, characterized by a Strouhal number St, approaches the frequency of one of the natural acoustic modes of the pipe. The whistling frequencies are the natural harmonics stemming from the tube acoustical modes, and the modes which actually become unstable depend on the air-flow velocity, the tube length and corrugation geometry.

In a recent paper (Debut et al. (2007a)), the authors reported preliminary experiments for a number of corrugated pipes with different diameters, length and corrugation lengths. They confirmed most of the qualitative behaviour reported in the literature (Petrie and Huntley (1980); Nakamura and Fukamachi (1991)) and showed that many features observed are not easy to explain. No simple scaling relation was found to characterize the physical phenomenon: the operating Strouhal number based on the corrugation pitch is in the range 0.4∼0.5 for all tested tubes. Moreover, it was observed that the condition for a self-sustained regime to arise concerning the coincidence of the vortex shedding frequency with the frequency of one of the pipe acoustic modes is necessary but not sufficient.

Although the mechanism behind the aeroacoustic instability is not yet clearly established, we are developping conceptual models to simulate the relevant dynamical features observed (Debut et al. (2007b)). Our approach, which is directly inspired by previous studies in vortex-induced vibration (Facchinetti et al. (2003)), is based on the coupling between a set of self-excited "vortex oscillators", standing for the aeroacoustic excitation, and the acoustic pressure field in the pipe. As proposed in (Debut et al. (2007b)), several choices may be considered for the action of the "vortex oscillators" on the pressure field. Here, we assume that self-sustained oscillations are driven by pressure disturbances and thus, a set of "pressure vortex oscillators" uniformly distributed along the pipe axis is considered.

First, the model for the flow-acoustic coupled phenomenon is presented using a representation of the pressure field in terms of the acoustical modes of the open-open pipe. Numerical time-domain simulations are then performed and typical results for the nonlinear computations are presented to illustrate how the model behaves. Because the underlying physics may evolve in an irregular fashion due the rather ill-defined separation point of the vortices, the irregular corrugation geometry and pitch, as well as the presence of highly turbulent pipe flow, it is of interest to consider the case of a random Gaussian dispersion on the Strouhal number. Nonlinear numerical simulations highlight the qualitative changes in the dynamical behaviour. Finally, considering the linearized coupled formulation, the phenomenon of localization of the normal mode shape is discussed.

2. THE COUPLED NONLINEAR MODEL

2.1. Acoustic pressure field

The acoustic pressure $p(x,t)$ at location x for the case when an external force field $f_x(x,t)$ is considered, is solution of the inhomogenous wave equation given by

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = -\frac{\partial f_x}{\partial x} \tag{1}$$

where c is the sound speed. Considering a line of N discrete "equivalent" pressure source P_n distributed along the pipe at N locations x_n, Eq.(1) becomes

$$\frac{\partial^2 p}{\partial t^2} - c^2\frac{\partial^2 p}{\partial x^2} = -c^2\sum_{n=1}^{N}\mathsf{P}_n\delta'(x-x_n) \tag{2}$$

where $\delta'(x-x_n)$ is the spatial derivative of the Dirac function evaluated at location $x = x_n$.

2.2. "Pressure vortex oscillator" dynamics

The shedding of vortices stemming from a corrugation located at each $x = x_n$ is here described phenomenologically by a non-linear Van der Pol equation given by

$$\ddot{\mathsf{P}}_n + A\left[\left(\frac{\mathsf{P}_n}{B\,V^\alpha}\right)^2 - 1\right]\omega_s\dot{\mathsf{P}}_n + \omega_s^2\mathsf{P}_n = Cp'(x_n,t) \tag{3}$$

and referred to as "pressure vortex oscillator". Parameter A controls the strengh of the nonlinear damping and parameter B is related to the size of the stable limit cycle which is assumed to vary with the flow velocity as V^α. ω_s is the Strouhal frequency. The right-hand side forcing term models the effect of the pressure field on the oscillator at the oscillator location. Note that the pressure field acts via its spatial derivative $p'(x_n,t) = [\partial p(x,t)/\partial x]_{x=x_n}$ proportionnaly to the coupling parameter C. Such coupling is in accordance with Howes analogy, where acoustic energy generation involves the acoustic velocity in the source region (Hirschberg (1995)).

2.3. Computational modal formulation

To solve the coupled problem formed by Eqs. (2) and (3), we use a modal approach in which the pressure field is expanded as the modal series:

$$p(x,t) = \sum_{m=0}^{M} q_m(t)\Phi_m(x) \tag{4}$$

where $q_m(t)$ are the modal amplitudes and $\Phi_m(x) = \sin(m\pi x/L)$ are the pressure modeshapes for a pipe of length L with both ends opened ($m \in \mathbb{N}$). Replacing the modal expansion of the pressure field in Eqs.(2) and (3), multiplying by a modeshape Φ_q, integrating within the domain $[0,\ L]$ and accounting for the modal orthogonality relation, $\int_L \Phi_m(x)\Phi_q(x)\,dx = L/2$ for m=p=1,2,. . . , we finally obtain the set of (N+M) modal ODEs

$$\begin{aligned}
&\ddot{q}_m + 2\zeta_m\omega_m\dot{q}_m + \omega_m^2 q_m = \frac{2c^2}{L}\sum_{n=1}^{N}\mathsf{P}_n(t)\Phi'_m(x_n)\\
&\ddot{\mathsf{P}}_n + A\left[\left(\frac{\mathsf{P}_n}{BV^\alpha}\right)^2 - 1\right]\omega_s\dot{\mathsf{P}}_n + \omega_s^2\mathsf{P}_n =\\
&\qquad = C\sum_{m=1}^{M} q_m(t)\Phi'_m(x_n)
\end{aligned} \tag{5}$$

To investigate the dynamics of the coupled nonlinear model (5), time-domain numerical computations are performed using a time-step integration algorithm presented in (Hart and Wong (1999)). Time histories and spectra of both the pressure field and source variables are computed and evaluation of the instantaneous frequency and amplitude of the dominant mode is done using the Hilbert transform (Oppenheim and Schafer (1998)).

3. LINEARIZED MODAL FORMULATION FOR THE COUPLED MODEL

To derive the linearized modal formulation of the coupled problem, we consider the case of small oscillations around an equilibrium state. Denoting with a bar the mean quantitites and with a hat oscillating quantities, pressure and vortex oscillators displacement quantities are written as

$$p(x,t) = \overline{p}(x) + \hat{p}(x,t) \tag{6}$$

$$\mathsf{P_n}(x,t) = \overline{\mathsf{P_n}}(x) + \widehat{\mathsf{P_n}}(x,t) \tag{7}$$

The subsitution of (6) and (7) into Eqs.(2) and (3) leads to two sets of equations: the zero-order equations which govern the steady state solutions, and the first-order equations which govern the oscillating solutions of the coupled problem. To apply the modal projection method to the obtained equations, both the steady and the oscillating pressure field are expanded as modal series:

$$\overline{p}(x) = \sum_{m=0}^{M} \overline{q}_m \, \Phi_m(x)\,, \quad \hat{p}(x,t) = \sum_{m=0}^{M} \hat{q}_m(t)\Phi_m(x) \tag{8}$$

The first-order equations are in the form of:

$$\ddot{\hat{q}}_m + 2\zeta_m\omega_m\dot{\hat{q}}_m + \omega_m^2\,\hat{q}_m = \frac{2c^2}{L}\sum_{n=1}^{N}\widehat{\mathsf{P}_n}(t)\Phi'_m(x_n)$$
$$\ddot{\widehat{\mathsf{P}_n}} - A\omega_s\dot{\widehat{\mathsf{P}_n}} + \omega_s^2\,\widehat{\mathsf{P}_n} = C\sum_{m=1}^{M}\hat{q}_m(t)\Phi'_m(x_n) \tag{9}$$

Complex eigenvalues and eigenvectors of the linearized formulation are obtained by assuming harmonic solutions to the first-order equations (9) in the usual manner (see Pierce (1981)).

4. BASIC SIMULATIONS

For the presented simulations, the pipe length was set to L=1 m and a modal damping of 0.5% was assumed for all acoustical modes. The self-excited oscillators were uniformely spaced along the pipe. Initial conditions for the vortex oscillators were taken as small random amplitudes of the variables of order 10^{-4}, all the acoustical variables being zero. N=101 "vortex oscillators" and M=10 modes are considered. A reference Strouhal number St=0.4 is assumed.
Figure 1 illustrates the case of *strong* coupling for which the phenomenon of frequency lock-in is clearly observed. The following parameters are used: A=0.005, B=0.0001, C=-200 and α=3. It represents the instantaneous frequencies for the acoustic pressure and oscillators variables when a linear sweep is imposed on the "vortex oscillator" characteristic frequency by means of an increasing flow velocity. Several qualitative observations

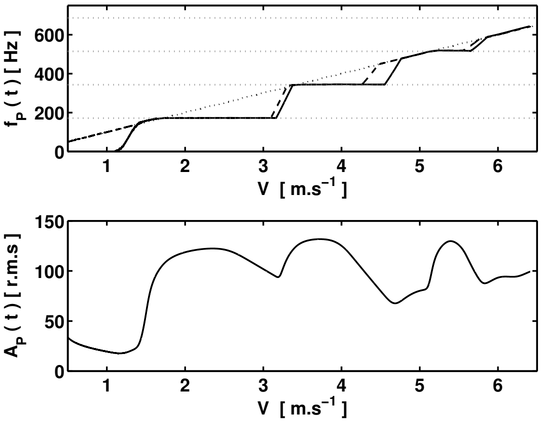

Figure 1: *Increasing linear velocity sweep on the characteristic frequency of the vortex oscillators for a perfectly tuned 101-oscillator pipe. Case for strong coupling. A=0.005, B=0.0001, C=-200, α=3. Instantaneous frequency and amplitude of the internal pressure at the oscillator location x_{10}=0.0982 m. The frequency of the oscillator has been superposed with dashed line (--) to highlight the lock-in stages. The dots in the diagonal represents the Strouhal law. Horizontal dots are for the pipe modal frequencies.*

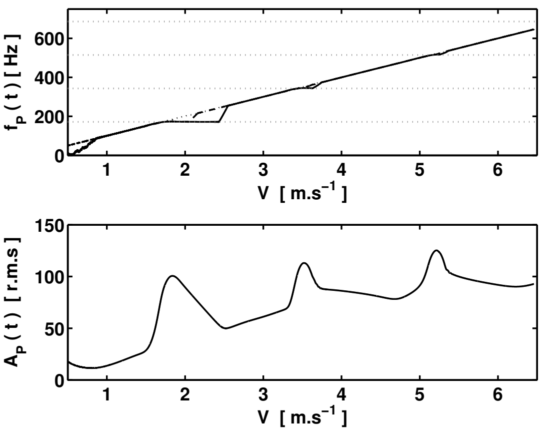

Figure 2: *Increasing linear velocity sweep on the characteristic frequency of the vortex oscillators for a perfectly tuned 101-oscillator pipe. Case for weak coupling. C=-30. Same legend as in Figure 1.*

assert the presence of frequency lock-in: first, the entrainment of the "pressure vortex oscillators" by the pressure field; then, the mutual adjustment of the frequencies in a given range of flow velocities; finally, the presence of large pressure pulsations in these regions. Moreover, by comparison with the plot in Figure 2 where the case of *weak* coupling is considered, one observes that the strength and the extent of lock-in are strongly controlled by the coupling parameter C.

5. DISPERSION ON THE VORTEX STROUHAL NUMBER

Departure from regularity in the geometrical configuration might be expected in corrugated pipes. In such spatially extended periodic structures, disorder might strongly influence the vortex generation, as well as the propagation of waves, by causing the phenomenon of localization (Hodges (1981)). Here, we allow some disorder in the system by assuming that the vortex shedding process differs slightly for each corrugation. Disorder is simulated as a random variation of the Strouhal number defined with a Gaussian distribution. Looking at Eqs.(5), we are now concerned with an ensemble of self-excited oscillators with slightly different characteristic frequencies interacting via the acoustic field. The question of self-organization of the model to evolve coherently with the same frequency has to be reconsidered. Intuitively, it may be that at least some oscillators lock on a common frequency (and thus contribute to the pressure field) whereas others are nonentrained and oscillate at their own frequency. Thus, the onset of a collective lock-in, if possible, may be less predictable than for the tuned case - depending on the distribution of the Strouhal frequencies, the oscillator locations and the coupling strength. In the next section, we provide a first examination of the effect of disorder in the nonlinear model (5) by means of numerical simulations. Finally, we examine the occurence of localization in terms of the normal modeshapes of the coupled linearized formulation (9).

5.1. Time domain simulations

From the nonlinear dynamical point of view, we present results of numerical simulations in Figures 3 and 4 when considering a random perturbation with a standard deviation $\sigma = 0.02$ added to the reference Strouhal number St=0.4. In comparison with the plot at Figure 1, Figure 3

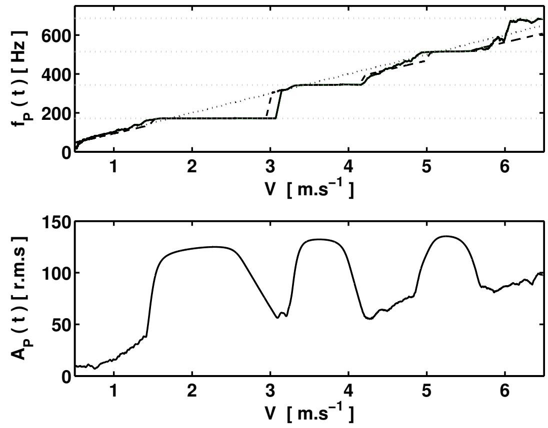

Figure 3: *Increasing linear velocity sweep on the characteristic frequency of the vortex oscillators for a mistuned 101-oscillator pipe. Strong coupling C=-200. Standard deviation of St, σ=0.02. Same legend as in Figure 1.*

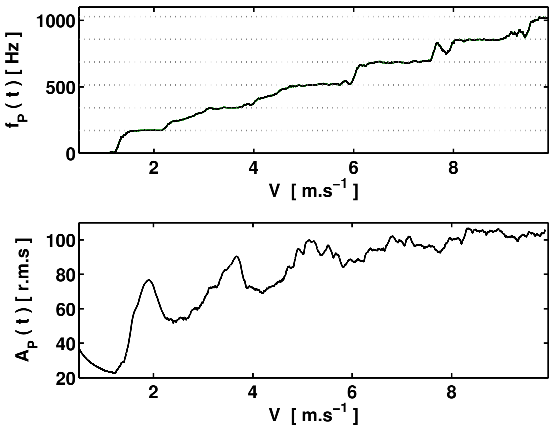

Figure 4: *Increasing linear velocity sweep on the characteristic frequency of the vortex oscillators for a mistuned 101-oscillator pipe. Weak coupling C=-30. Standard deviation of St, σ=0.02. Same legend as in Figure 1.*

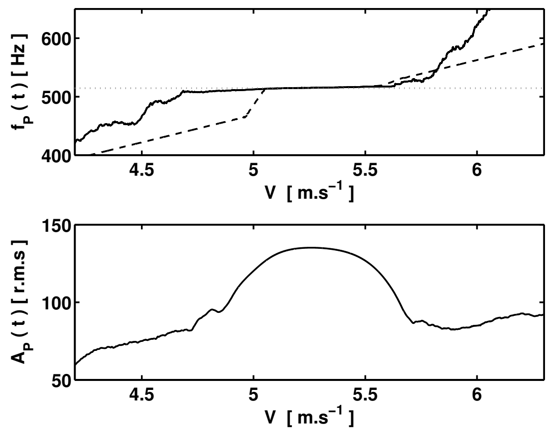

Figure 5: *Increasing linear velocity sweep on the characteristic frequency of the vortex oscillators for a mistuned 101-oscillator pipe around the frequency of the third acoustic pipe mode. Strong coupling C=-200. Standard deviation of St, σ=0.02. Same legend as in Figure 1.*

clearly shows some quantitative changes. Particularly, some lock-in ranges are shortened and the threshold velocities defining the stable/unstable character of each acoustic mode are altered. It is observed that the velocity ranges for which an acoustic mode is unstable are broadened (see Figure 5) and that the transitions between modal jumps are less marked. An important decrease in the amplitude of the acoustic pressure is also observed between two successive stages although amplitude maxima remains large within lock-in. Figure 4 is obtained for the case of *weak* coupling with a standard deviation σ=0.02.
Contrary to the case with identical Strouhal number oscillators, computed time-history signals are not always stationnary. Strickly speaking, the frequencies of the "vortex oscillators" *nearly* adjust with the pipe natural frequencies. In addition, several oscillators might have uncorrelated movement during an aeroacoustic instability and thus the all-to-all complete time-space correlation between the "vortex oscillators" does not appear necessary to explain the excitation of the acoustic field. Indeed, it is quite reasonable to state that lock-in still exists, because the ratio of the instantaneous frequencies of the interacting oscillators and pressure field remains approximatively constant in a range of flow velocities. These remarks are illustrated in Figure 4.
Figure 6 shows the time envelopes and corresponding spectra for tuned and mistuned cases under constant excitation. Looking at the spectrum for the mistuned case, one notices the presence of closely located distinct peaks distributed in a narrow band region about a central frequency, as reported in (Elliot (2005); Debut et al. (2007a)). In the time-domain, the corresponding amplitude modulation effect is observed and can be clearly heard from the corresponding computed sound. This suggests that modeling random phenomena may be of importance. Looking at the pressure amplitude, note the strong attenuation for the mistuned case. It has also been observed that the entrainment of the oscillators may be prevented for large mistuning ($\sigma \backsim 0.1$) causing a strong attenuation for the acoustic pressure field. As a result, mistuning appears as a stabilizing factor for the coupled system. This observation is supported by the qualitative remarks provided by Petrie and Huntley (Petrie and Huntley (1980)) who noticed that very small differences in corrugation shape eliminate the whistle.

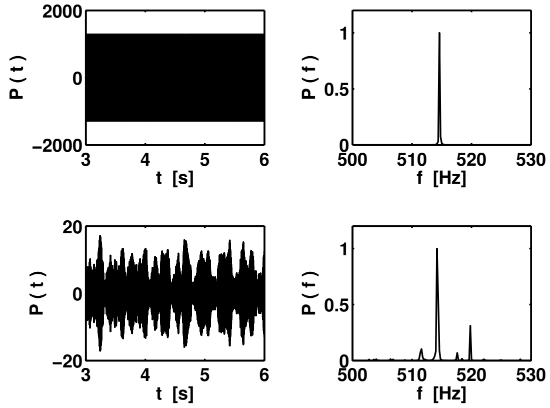

Figure 6: *Time envelopes and corresponding normalized autospectrum for a constant excitation of the third mode ($f_s = f_3$). Tuned (up) and mistuned (down) cases for a 101-oscillator pipe. Strong coupling C=-200. Standard deviation of St, σ=0.02.*

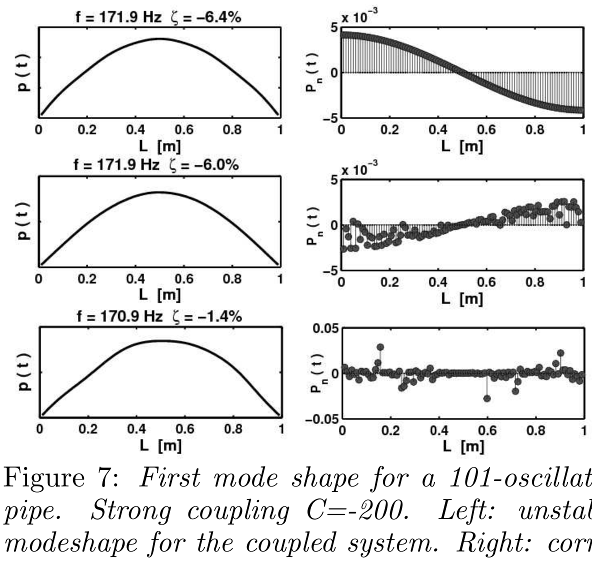

Figure 7: *First mode shape for a 101-oscillator pipe. Strong coupling C=-200. Left: unstable modeshape for the coupled system. Right: corresponding oscillator displacements along the pipe axis. Increasing standard deviation of St from top to bottom.* $\sigma = 0$ *(up) ;* $\sigma = 0.01$ *(middle);* $\sigma = 0.05$*(bottom).*

5.2. Effect on the modeshapes

The linearized modal equations for the coupled problem (9) enable to study the system stability through the computation of its complex eigenvalues and eigenvectors. As illustrated on Figure 7 for the first mode of a perfectly tuned 101-vortex oscillators pipe during lock-in, a global coherent vibration of the vortex oscillators can easily synchronize with the acoustic modes specifically with the space derivative of the pressure field (see Eq.(3)). As the degree of mistuning increases, its effect is clearly seen on the vortex oscillators responses, localizing their action in a small region of the pipe. For the corresponding unstable mode of the coupled system, we observe that the negative modal damping decreases as the standard deviation σ increases. Notice also the slight distorsion for the pressure coupled modeshape as the mistuning becomes stronger.

6. CONCLUSIONS

Based on a phenomenological model dealing with the aeroacoustic coupling between a line of "pressure vortex oscillators" with an acoustic field, numerical time-domain simulations for a corrugated pipe were performed. It was found that including random perturbation on the Strouhal number might qualitatively change the dynamical behaviour of the nonlinear model. Under certain circumstances, the threshold values for the instability of an acoustical mode are altered and the model succeeds in simulating the "noisy" sound observed experimentally for high flow velocities. Considering the linearized formulation for the coupled system, localization has been observed, the acoustical modeshapes being then slightly distorded in comparison with those for the unperturbed case. However, it is important to note that all our observations clearly depend on the relative strength between the random perturbations and the coupling parameter magnitude. It is interesting to note that, in Figure 4, the qualitative evolution of the amplitude is quite well reproduced compared to experiments. Finally, because of the high level of turbulence in the pipe flow and in the light of these results, it will be interesting to include a model for the turbulence disturbance acting on the "vortex oscillators".

7. ACKNOWLEDGEMENTS

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