EFFECT OF DISSIPATION ON LOCAL AND GLOBAL STABILITY OF FLUID-CONVEYING PIPES

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ABSTRACT

This article is focused on the effect of damping on local and global stability of fluid-conveying pipes. Local stability is first considered through the analysis of the dispersion relation for two systems : the fluid conveying pipe on elastic foundation and the tensionned fluid-conveying pipe. The effect of damping is then predicted by calculating the energy of the neutral waves in the system and predictions are drawn regarding stability of the pipe when boundaries are present (finite pipes). Next, numerical computations are presented for finite-length systems and results are compared with predictions. It is finally found that a comparison of lengthscales is enough to know which criterion is able to predict the global stabilty when the non-dimensionnal length $l \gg 1$.

1. INTRODUCTION

The fluid conveying pipe is often considered as a model problem for numerous physical systems where the dynamics of a structure is coupled to an axial flow. This system is known to become unstable at a critical velocity (Païdoussis, 1998), by flutter or buckling, depending on various mechanical parameters and boundary conditions. Two different approaches are used to describe the properties of such one-dimensional systems. When the system is sought infinite, the waves propagating in the medium are considered through the analysis of the *local* wave equation. If temporally or spatially amplified waves are identified the system is said to be *locally* unstable. When the same medium is of finite length, the modes are studied, through the analysis of the same local wave equation, associated with boundary conditions. If a temporally amplified mode is found, the system is said to be *globally* unstable. The comparison of local and global stability properties has been done on various systems by several authors, and one main result is that when the length of the system is increased, the critical velocity for global instability tends to a limit that corresponds to a local criterion. However, no unique local criterion can predict the global instability of these long systems. Depending on the medium and boundary conditions characteristics, various authors found that it can be that of absolute instability (Kulikovskii, 1966), local instability (Doaré and de Langre, 2006) or that of existence of static or dynamic neutral waves (Doaré and de Langre, 2002). This last criterion has for unusual consequence that it is possible to exhibit a system that is locally stable but globally unstable.

This paper if focused on the influence of dissipation on these local and global criteria of stability. Regarding wave propagation, some authors have identified that dissipation can have a stabilizing or -more surprinsingly- a destabilizing effect. Indeed, it is known after Briggs (1964) that neutral waves can be destabilized or stabilized by the addition of damping, depending on the sign of their energy. These authors considered the energy being the work done in building up the wave from rest at time $t = -\infty$. It has been found that elastic plates loaded with mean flow also display negative energy waves (Peake, 2001). Althrough it is expected that the same appens in fluid conveying pipes, no information is available at present time. Conversely, regarding pipes of finite length, destabilization by dampig has been observed (Païdoussis, 1998).

Three points of view will be used in this paper : The infinite case (wave propagation), the finite case, and the finite case, but long enough that a local criterion can predict stability. Two particular systems will be studied, the fluid conveying pipe resting on an elastic foundation, and the fluid-conveying pipe subjected to tension.

Equations of motion will be described in section 2. The section 3 of the article will be devoted to the wave propagation properties in these two media. In absence of dissipation, it will essentially consist in recalling previous results. The addition of damping addition will be then studied through the analysis of neutral waves energy. In section 4, computation results of global stability of these pipes will be presented and analysed through the light of the local stability properties. Finally, some conclusion will be drawn.

2. EQUATIONS

The linearized equation of motion governing the lateral in-plane deflection $Y(X,T)$ of a fluid-conveying pipe is (Bourrières, 1939),

$$EI\frac{\partial^4 Y}{\partial X^4} + (MU^2 - \tau_e)\frac{\partial^2 Y}{\partial X^2} + 2MU\frac{\partial^2 Y}{\partial X \partial T}$$
$$+c\frac{\partial Y}{\partial T} + (m+M)\frac{\partial^2 Y}{\partial T^2} + SY = F(X,T), \quad (1)$$

where EI is the flexural rigidity of the pipe, M the fluid mass per unit length, m the pipe mass per unit length, U the plug flow velocity, S the elastic foundation modulus, τ_e the external tension applied and $F(X,T)$ the external force per unit length. We only consider here the onset of instabilities and nonlinear effects are therefore neglected in the dynamics of the pipe.

In the following, elastic foundation and external tension will be studied separately. Two different sets of non-dimensionnal numbers, based on two different sets of characteristic length and time will be used. The first set wil be used to study the pipe resting on an elastic foundation, without tension ($\tau_e = 0$). Introducing the non dimensional length $\eta = EI/S^{1/4}$, and time $\tau = \eta^2\sqrt{\frac{M+m}{EI}}$, non dimensional variables and parameters read,

$$x = X/\eta, \quad y = Y/\eta, \quad t = T/\tau,$$
$$\beta = \frac{M}{M+m}, \quad f = \frac{c\eta^2}{\sqrt{EI(M+m)}}. \quad (2)$$

Local wave equation finally reads,

$$\frac{\partial^4 y}{\partial x^4} + v^2\frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}v\frac{\partial^2 y}{\partial x \partial t} + f\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial t^2} + y = 0. \quad (3)$$

The second set of parameters concern the tensionned pipe. Here, using the characteristic length, $\eta = \sqrt{EI/\tau_e}$, the characteristic time τ based on this new characteristic length, and the same parameters as in equations (2), the local non-dimensional wave equation now reads,

$$\frac{\partial^4 y}{\partial x^4} + (v^2-1)\frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}v\frac{\partial^2 y}{\partial x \partial t} + f\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial t^2} = 0. \quad (4)$$

For both systems, the length is L so that $x \in [0,l]$, with $l = L/\eta$. The particular case of a cantilevered fluid-conveying pipe will be considered through the whole paper, so that boundary conditions are,

$$y(x=0) = \left.\frac{\partial y}{\partial x}\right|_{x=0} = \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=l} = \left.\frac{\partial^3 y}{\partial x^3}\right|_{x=l} = 0. \quad (5)$$

3. LOCAL STABILIY

3.1. Pipe on elastic foundation

Looking for solutions in the form of normal modes,

$$y(x,t) = e^{i(kx-\omega t)}, \quad (6)$$

the local equation (3) takes the form of a dispersion relation,

$$\begin{aligned} D(k,\omega) &= k^4 - v^2k^2 + 2\sqrt{\beta}vk\omega \\ &\quad -if\omega - \omega^2 + 1 \\ &= 0. \end{aligned} \quad (7)$$

Local stability is ensured if $\mathrm{Im}(\omega) \leq 0$ for any value of $k \in \mathtt{R}$. When dissipation is absent, the medium is found to stable if (Roth, 1964),

$$v < \sqrt{\frac{2}{1-\beta}}. \quad (8)$$

In the local instability domain of the parameters, absolute and convective instabilities may be distinguished. It has been shown (de Langre, 1999) that for fluid-structure interaction systems without dissipation the transition between absolute and convective instabilities arises when the dispersion relation has a triple root. In the instability domain of the parameters, the medium is absolutely unstable if,

$$v > \left(\frac{12\beta}{8/9-\beta}\right)^{1/4}. \quad (9)$$

In the local stability domain of the parameters, this last criterion is also the criterion of existence of te dynamic neutral range (Doaré and de Langre, 2002). Moreover, a second value of the velocity $v = \sqrt{2}$ give rise to a triple root in the local stability domain. As this root appears at $\omega = 0$, the range of neutral waves that is generated has been referred to as *static*. The criterion of existence of the static range is hence,

$$v > \sqrt{2}. \quad (10)$$

The different domains of stability are plotted on Figure 1a.

As shown by Roth (1964), as soon as dissipation is added in the system, the criterion of stability becomes $v > \sqrt{2}$.

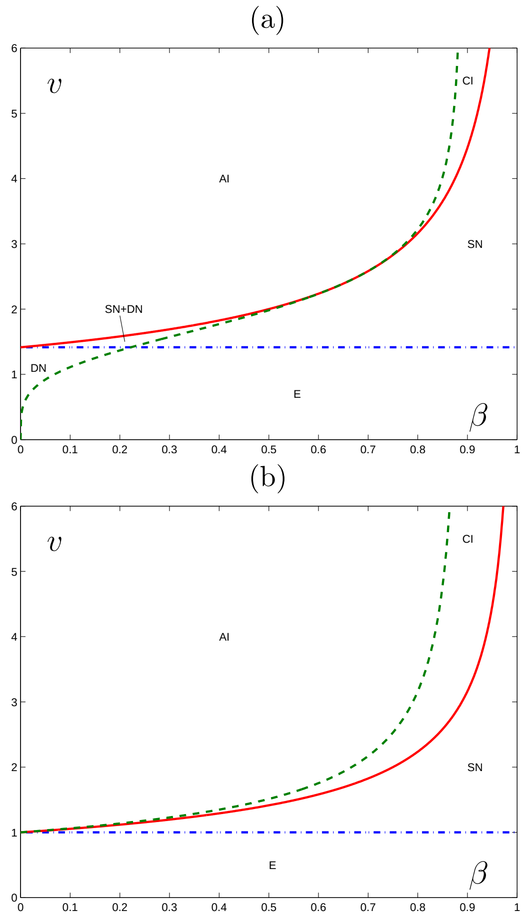

Figure 1: Infinite pipe without dissipation, schematic view of the different domains of wave properties in the (β, v) space; AI, absolute instability; CI, convective instability; SN, stability with existence of the static range of neutral waves; E, no neutral waves range exists, there are evanescent waves at any real frequency; (a), with elastic foundation; (b), with tension.

3.2. Tensionned pipe

The dispersion relation is here,

$$\begin{aligned} D(k,\omega) &= k^4 + (1-v^2)k^2 + 2\sqrt{\beta}vk\omega \\ &\quad -if\omega - \omega^2 \\ &= 0. \end{aligned} \tag{11}$$

Using similar analysis as in section 3.1, the following criteria apply when dissipation is absent [see de Langre (1999)] :

- Criterion of stability :

$$v < \sqrt{\frac{1}{1-\beta}} \tag{12}$$

- Criterion for the existence of the static neutral range :

$$v > 1 \tag{13}$$

- Criterion for absolute instability :

$$v > \frac{2\sqrt{2}}{\sqrt{8-9\beta}}. \tag{14}$$

There is no crossing of stability transition and convective to absolute instability transition criteria. So a dynamic range cannot be observed here.

The presence of dissipation modifies the stability criterion. An analysis of the dispersion relation, not presented here allows to show that as soon as $f > 0$, the medium is unstable if,

$$v > 1. \tag{15}$$

These criteria are all plotted on Figure 1b. The main difference between this case and the previous case - pipe on elastic foundation - is that here, no region of existence of the dynamic range exists in the parameter space.

3.3. Effect of dissipation on neutral waves ranges

The concept of wave energy, introduced by Briggs (1964), allows to predict de the effect of damping on the stability. Wave energy is calculated on neutral waves, *i.e.* $k \in \mathtt{R}$, $\omega \in \mathtt{R}$ and corresponds to the work done in slowly building up the wave starting from rest at time $t = -\infty$. It is predicted that a wave for which this energy is negative will be destabilized by the addition of damping. Cairns (1979) showed that the wave energy E of a neutral wave $y = Ae^{i(kx-\omega t)}$ is given by,

$$E = e|A|^2 = \frac{\omega}{4}\frac{\partial D}{\partial \omega}|A|^2. \tag{16}$$

For both pipes (tensionned and resting on an elastic foundation), the wave energy reads,

$$e = \frac{\omega}{2}\left(\sqrt{\beta}kv - \omega\right). \tag{17}$$

This energy is plotted for two typical cases. On Figure 2a, it is plotted in the case of the pipe on elastic foundation, at $\beta = 0.15$, $v = 1.2$, $\omega \in [0, 1.5]$. This set of parameters is chosen to display both static and dynamic neutral ranges. It appears on this figure that, for two waves in the static range, the energy defined in equation (17) is negative. Conversely, waves in the dynamic range are all positive. Hence, neutral waves in the dynamic range are stabilized by damping while two waves in the static range are destabilized. The criterion of instability of the infinite, damped

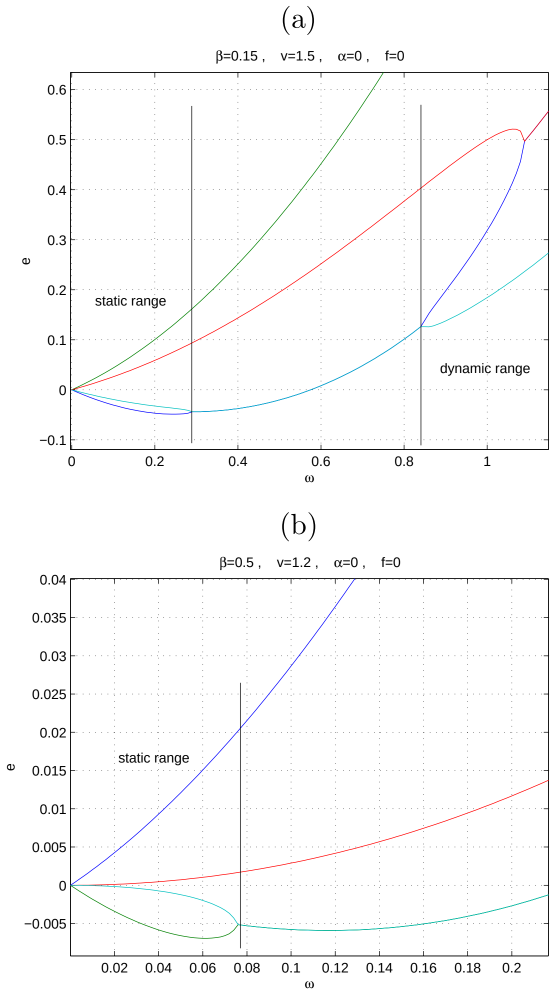

Figure 2: Wave energies of waves as function of real frequency; (a) pipe on elastic foundation; (b), tensionned pipe.

fluid-conveying pipe on an elastic foundation is thus $v > \sqrt{2}$.

On figure 2b, the energy is plotted in the case of the tensionned pipe for $\beta = 0.5$, $v = 1.1$ and $\omega \in [0, 0.22]$. As in the previous case, the static range displays two negative energy waves and the medium is destabilized by disspation. No dynamic range exists in the case of tensionned pipe. The infinite, damped, tensionned fluid conveying pipe is hence unstable when $v > 1$.

These two criteria confirm the results given in the previous section and show that the instability caused by damping originates from the destabilization of waves in the static range.

4. GLOBAL STABILITY

A Galerkin numerical method is used to compute the eigenfrequencies of the system. Up to 100 modes have been used to compute the eigenfrequencies of the longuest pipes. Instability is predicted when an eigenfrequency has a positive imaginary part.

4.1. Pipe on elastic foundation

Figure 3 shows the evolution of the marginal stability curve in the (β, v) plane as the non-dimensional length l is increased. This last parameter can be sought as a measure of the length compared to the wavelengths of waves propagating in the system. When $l \gg 1$ the global riterion of stability can be well approximated by a local criterion. As shown in a previous paper (Doaré and de Langre, 2002), the long system limit is the criterion of existence of the dynamic neutral range. When dissipation is added, the marginal stability curve approaches a different limit when the length is increased. This limit is the local stability transition, $v = \sqrt{2}$. As shown in the previous section, this criterion is that of existence of the static neutral range, which exists when there is no damping.

4.2. Tensionned pipe

Figure 4 shows the evolution of the marginal stability curve as the length is increased. As no dynamic range exists in this medium, the limit for long system is the local stability transition criterion. When damping is added, the marginal stability curve tends to a different criterion when length is increased. This limit is again the local stability criterion of the damped medium, $v = 1$. Again, this criterion corresponds to the criterion of existence of the static neutral range, appearing when there is no damping.

4.3. Length scales

It appears on Figure 3b that before tending to the local criterion of stability with dissipation, the curve approaches the local criterion without dissipation. This indicates that there exists an intermediate length where the local criterion without damping still dominates the global behavior of the system. To analyze this phenomenon, let us define a characteristic length of dissipation effects,

$$\eta_f = \left(\frac{EI(M+m)}{c^2} \right)^{1/4}. \qquad (18)$$

A non dimensional length can then be defined as,

$$l_f = \frac{L}{\eta_f} = l f^{1/2}. \qquad (19)$$

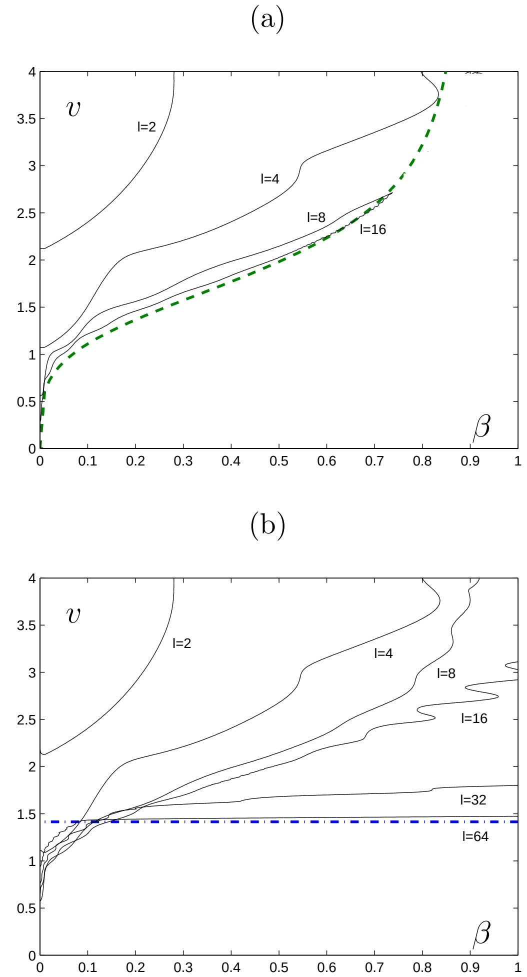

Figure 3: Fluid-conveying pipe on elastic foundation, global stability curves for different values of the non dimensional length l and of the damping parameter f. (a), $f = 0$; (b) $f = 0.01$.

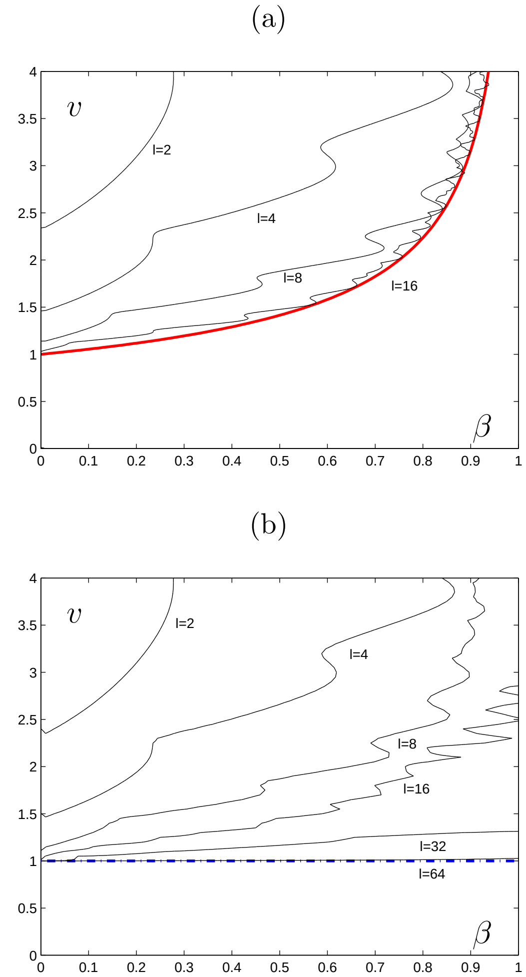

Figure 4: Tensionned fluid-conveying pipe, global stability curves for different values of the non dimensional length l and of the damping parameter f; (a), $f = 0$; (b) $f = 0.01$.

When $f^{1/2} \gg 1$, $l_f \gg l$, dissipation effects are dominant in the pipe and the critical velocity is only determined by the local stability criterion of the pipe with dissipation. Conversely, when $f^{1/2} \ll 1$, $l_f \ll l$, two situations arise :

- $l \gg 1 \gg l_f$, the pipe is long enough to let the local conservative instability to developp but not enough to let dissipation to affect the system. The global criterion of instability is hence that of the undamped system.

- $l \gg l_f \gg 1$, the pipe is long enough to let the dissipation affect the system. The global criterion of instability is that of the dissipative system.

The critical velocity of global instability is plotted in Figure 5 for two values of the mass ratio β and for typical values of the ratio η_f/η :$\infty, 60, 30, 10$ and 0.5. When $\eta_f/\eta = \infty$, $f = 0$, the asymptotic criterion of global instability is $v > v_{ac}$, the criterion of existence of the dynamic range of neutral waves. For the other cases presented on Figure 5, damping is present and the asymptotic criterion of global instability is $v > v_n$ the criterion of instability of the damped medium, which is also the criterion of existence of the static neutral range when there is no dissipation in the medium. However, as predicted above, when $\eta_f/\eta \neq \infty$, the critical velocity approaches a different limit for intermediate lengths, that is when $1 \ll l < \eta_f/\eta_s$. This limit is the global instability criterion of the system without damping.

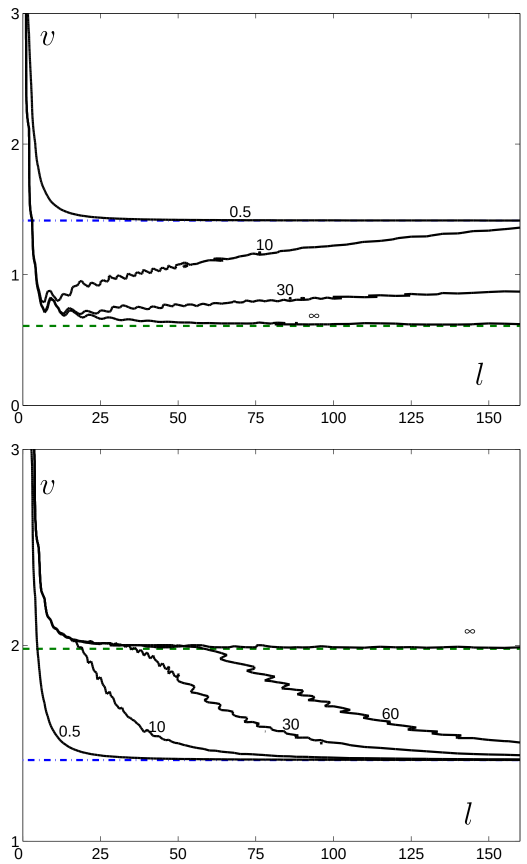

Figure 5: Fluid conveying pipe on elastic foundation with dissipation : non dimensional critical velocity for global instability as function of the non dimensional length l for different values of the ratio η_f/η_s; (a), β=0.01; (b), $\beta = 0.5$; (- -) criterion of existence of the dynamic neutral range; (- ·) criterion of existence of the static neutral range which is also the criterion of local instability for the pipe with dissipation.

5. CONCLUSION

In this article, the effect of damping on local and global stability of two systems has been analysed, the cantilevered fluid-conveying pipe on elastic foundation and the cantilevered fluid-conveying pipe subjected to tension. Regarding local stability, in the elastic foundation case, it has been found that dissipation can have a stabilizing or destabilizing effect, depending on the mass ratio β. In the tensionned pipe case, damping has a destabilizing effect for any value of β. These results have been found to affect the global stability of the pipe as its length reaches infinity while for intermediate lengths, the criteria with or without damping can predict the global stability, depending on the respective values of non dimensionnal lengths l and l_f. These results are expected to be applicable in other systems where a slender structure interacts with an axial flow.

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