# STIFFNESS OF FLUID LAYER WITH TAYLOR VORTICES

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## ABSTRACT

*The presented paper is focused on the static stiffness definition of fluid layer according to the number of Taylor vortices. There is a gap between two cylinders, where the inner one is rotating and axial flow is not assumed. The stiffness matrix as a function of angular speed is determined too. The fluid layer stiffness is specified for rotor in static balance and problem of damping is not considered. The Taylor vortices' influence is evident in the stiffness matrix, where all the elements are of the same orders of magnitude. In comparison with the stiffness matrix derived from the Reynolds equations, which has contrary the major diagonal elements lower by several orders of magnitude then the others, there is a marked difference. This theory will be used in new design of classical journal bearing using Taylor vortices principle.*

## 1. INTRODUCTION

Classical theory of journal bearing is based on Reynolds equation solution for infinite bearing length. This equation relies on an assumption, that the convective acceleration in Navier-Stokes equations is ignored, so the linear problem is solved. But the flow based on this assumption is characterised by the streamline that lies in the perpendicular plane to the rotation axis (Couette flow). Taylor (Taylor, 1923) established that there is point in concerning with this fluid flow stability. It is proved, that so defined fluid flow is modified by vortices named by Taylor for Taylor number greater than 41,3. Taylor number is defined by term

$$T = \frac{R_0 \omega\, s}{\nu} \sqrt{\frac{s}{R_0}} \qquad (1.1)$$

where $R_0$ is shaft radius, $\nu$ is kinematical viscosity, $\omega$ is angular velocity and $s$ is radial gap. Thus for $T$>41,3 there is impossible to use Reynolds eqs. Taylor vortices can exist in steady or unsteady cases, which is evident in non zero radial and axial velocity components, i.e. wave regime see Fig. 1.1.

Vortex structures are characterised by following:

**Couette flow –** fluid moves in the tangential direction around the rotating cylinder, see Fig. 1.1 a).

**Taylor vortices (TVF) -** basic steady flow forms into axial symmetrical toroidal Taylor vortices and is described by critical value $T_{c1}$ ($T > T_{c1} \geq 41{,}3$), see Fig.1.1 b).

**Wave regime (WVF) –** flow changes with increasing angular velocity of inner cylinder and Taylor number too. For $T_{c2} > T_{c1}$ the wave motion of vortices in tangential direction is observed. Critical number $T_{c2}$ is approximately in interval $T_{c2} \approx (1{,}1 \div 100) * T_{c1}$ and depends on the gap geometry and fluid characteristics, see Fig. 1.1 c).

**Modulated wave regime (MWVF) –** in this state the modulation of the wave vortices motion in circumferential direction is discovered and it is possible to define azimuth wave frequency. Toroidal vortices are tapered and propagate in tangential direction.

**Chaos (TURB) -** chaos regime is observed for $T \approx (100 \div 1000) T_{c1}$. It is very unstable and depends on radius rate and experimental experience (in case of quick revolution change you can get another fluid flow structure than in case of slow regular angular speed increasing). Another revolution increasing causes the turbulent effect formation disturbing the original vortices, see Fig.1.2.

*a) Couette flow b) Taylor vortices c) Wave regime*

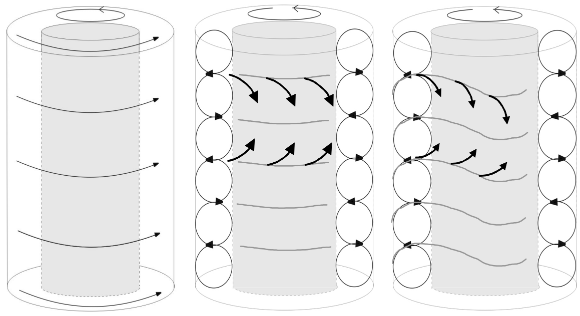

Figure1.1 *Vortex structures*

The value $T_{c1}$ is theoretically defined for infinite length of cylinder. Neither physical experiment nor numerical experiment can accomplish this hypothesis, therefore the critical Taylor value differs. Many of vortex structures were experimentally and numerically solved at VŠB-TU, see Fig.1.2, (Farnik, 2006; Kozubková et al, 2003).

*Couette flow* *Taylor regime* *Wave vortices* *Modul wave regime* *Turbulence*

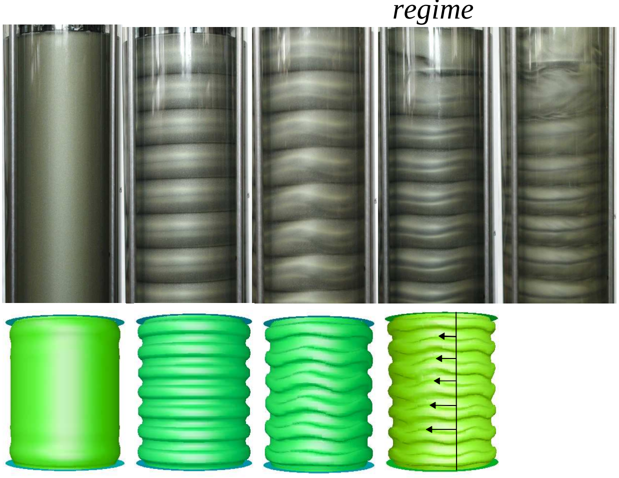

Figure 1.2 *Physical and numerical experiment of vortex structures [5]*

It is not necessary to justify the transition to instability by eigen value analysis, as Taylor did (Taylor, 1923; Chandrasekhar, 1965). There is possible to obtain the general steady flow stability condition analytically, when the Navier Stokes equation and continuity equation for uncompressible fluid and boundary condition is used. Problem solves steady flow disturbance analysis by using eigen values (Pochylý et al, 2003).

## 2. STABILITY OF STATIONARY FLOW

The main goal is to find stability conditions of stationary fluid flow in rotationally symmetric region. Generally the double continuous bounded region with volume $V$ filled by fluid is supposed. Coordinate system $(x_i)$ is inertial and $(y_i)$ is rotating coordinate system with angular velocity $\omega_3 = \Omega$ around axis $x_3$. Because the goal of the work is to define the angular velocity influence on stability, therefore the final equations will be formulated in rotational coordinate system $(y_i)$, using Einstein summation symbolics. Navier Stokes equation and continuity equation are in form

$$\frac{\delta w_i}{\delta t} + \varepsilon_{i3k}\varepsilon_{k3m}\omega_3^2 y_m + 2\varepsilon_{i3k}\omega_3 w_3 + \nu \frac{\partial^2 w_i}{\partial y_j \partial y_j} + \frac{1}{\rho}\frac{\partial p}{\partial y_i} = 0$$

$$\frac{\partial w_i}{\partial y_i} = 0 \qquad (2.1)$$

In equation (2.1) the term $\frac{\delta w_i}{\delta t}$ is so called substantial derivative defined as:

$$\frac{\delta w_i}{\delta t} = \frac{\partial w_i}{\partial t} + \frac{\partial w_i}{\partial y_j} w_j \qquad (2.2)$$

where $w_i$ are the relative velocity components, $p$ is pressure, $\varepsilon_{ijk}$ is Levi-Civit antisymmetrical tensor. Influence of boundary condition will be specified later.

### 2.1 Stability conditions

Stability conditions can be examined using the known principle, defined by Taylor (1923) in case of stability study of rotational fluid motion between two concentric cylinders (Taylor, 1923). Based on this principle the small disturbance on original steady flow ($w_{0i} = w_{0i}(y_j)$, $p_0 = p_0(y_j)$) is superimposed, i.e.:

$$\begin{aligned} w_i &= w_{0i} + v_i(y_j, t) \\ p &= p_0 + \sigma(y_j, t) \end{aligned} \qquad (2.3)$$

Above defined terms are put into equations (2.1) and (2.2). Neglecting the small nonlinear terms and subtracting the stationary parts of solution the equations (2.1) a (2.2) for disturbance can be written in form:

$$\frac{\partial v_i}{\partial t}+\frac{\partial w_{0i}}{\partial y_j}v_j+\frac{\partial v_i}{\partial y_j}w_{0j}+2\varepsilon_{i3m}\Omega v_m- \\ -\nu\frac{\partial^2 v_i}{\partial y_j \partial y_j}+\frac{1}{\rho}\frac{\partial \sigma}{\partial y_i}=0 \tag{2.4}$$

$$\frac{\partial v_i}{\partial y_i}=0 \tag{2.5}$$

The stability conditions will be investigated using eigen value and vector analysis. Eigen value can be determined as follows. Let

$$v_i(y_j,t)=u_i(y_j,s)e^{st};\quad \sigma=h(y_j,s)e^{st} \tag{2.6}$$

where $s$ is complex number $s=\alpha+i\omega$, $\alpha,\omega\in \mathrm{Re}$. Stability point is defined by value of $\alpha$, i.e. $\alpha\langle 0$. Inserting (2.6) in (2.4), (2.5), we obtain the following equation for definition the eigen values:

$$su_i+\frac{\partial w_{0i}}{\partial y_j}u_j+\frac{\partial u_i}{\partial y_j}w_{0j}+2\varepsilon_{i3m}\Omega u_m- \\ -\nu\frac{\partial^2 u_i}{\partial y_j \partial y_j}+\frac{1}{\rho}\frac{\partial h}{\partial y_i}=0 \tag{2.7}$$

$$\frac{\partial u_i}{\partial y_i}=0 \tag{2.8}$$

Quantitative analysis of stability conditions is based on using boundary conditions in equations (2.7), (2.8). At first the equation (2.7) is multiplied by function $u_i^*$ conjugate with $u_i$ and integrated (scalar multiplication). So:

$$s\int_V u_i u_i^* dV+\int_V \frac{\partial w_{0i}}{\partial y_j}u_j u_i^* dV+ \\ +s\int_V \frac{\partial u_i}{\partial y_j}u_i^* w_{0j}dV+2\varepsilon_{i3k}\Omega\int_V u_k u_i^* dV- \\ -\nu\int_V \frac{\partial^2 u_i}{\partial y_j \partial y_j}u_i^* dV+\frac{1}{\rho}\int_V \frac{\partial h}{\partial y_i}u_i^* dV=0 \tag{2.9}$$

Because of evaluating $\alpha$ the functions $u_i, h$ will be decomposed in real and imaginary part:

$$u_i=a_i+ib_i;\ a_i,b_i\in \mathrm{Re} \\ h=h_R+ih_I;\ h_R,h_I\in \mathrm{Re}\ . \tag{2.10}$$

Inserting (2.10) in (2.11) we get the relation for $\alpha$:

$$\alpha\int_V |u|^2 dV+\int_V \frac{\partial w_{0i}}{\partial y_j}|u|^2 dV+\frac{1}{2}\int_V w_{0j}\frac{\partial |u|^2}{\partial y_j}dV- \\ -\nu\int_V \frac{\partial^2 u_i}{\partial y_j \partial y_j}u_i^* dV+\frac{1}{\rho}\int_V\left(\frac{\partial h_R}{\partial x_i}a_i+\frac{\partial h_I}{\partial x_i}b_i\right)dV=0 \tag{2.11}$$

From (2.11) it is obvious, that **the stability is not influenced by Coriolis forces**. Equation (2.11) can be simplified using Gauss-Ostrogradskij theorem, continuity equation and non-permeability condition of boundary, i.e. $c_i n_i=0$. After simple adjustments and transformation into cylindrical coordinate system $(r,\Phi,y_3)$ and preconditions $w_{0r}=0$ and $w_{03}=0$ we obtain (2.11) in form:

$$\alpha\int_V |u|^2 dV=\int_V\left\{r\frac{\partial}{\partial r}\left(\frac{w_{0\varphi}}{r}\right)(a_r a_\varphi+b_r b_\varphi)+\eta\frac{\partial u_i}{\partial y_j}\frac{\partial u_i^*}{\partial y_j}\right\}dV \tag{2.12}$$

where $|u|=u_i u_i^*$. From the expression (2.12) it is obvious, that instability arises in case of negative value of integral (2.12) and this is condition of Taylor vortices arising too. This situation comes up at specific rotor angular velocity, when convective acceleration forces exceed the effect of viscous forces. This instability appears on the first eigen mode shape, when $(a_r a_\varphi+b_r b_\varphi)\rangle 0$, i.e. Taylor number T$\geq$ 41,3.

## 3. NUMERICAL RESULTS

### 3.1 Physical model and boundary conditions

Numerical experiment is focused on stable Taylor vortices corresponding to Fig. 1.1 b). Other vortex structures are from point of journal bearings applications unsuitable and are not in this work investigated. Computational region was defined on the gap between two cylinders of following geometry:

- inner radius r=0.025 m
- gap width s=0.0003 m

- cylinder length	l=0.003 m
- eccentricity in $y$ direction	e=0, 0.00003, 0.00006, 0.00009 m

Inner cylinder revolved with speed:

- revolutions per minute	n=200, 500, 600, 700, 800, 900, 1000, 2000, 3000, 5000, 7000, 9000, 11000 min$^{-1}$

Water was chosen as fluid with following physical properties:

- density	ρ=1000kgm$^{-3}$
- dynamical viscosity	η=0.001Pa.s
- kinematical viscosity	ν=0.000001m$^2$s$^{-1}$

### 3.2 Turbulent models and results

The length of the cylinder was defined with respect to reasonable number of grid elements, boundary planes were chosen as symmetry planes. Moreover every vortex must be covered at least by ten grid elements in every coordinate direction. Grid elements number was around 600000.

Taylor number according to given speed lied in interval $T \in \langle 0;1500 \rangle$, Reynolds number $\mathrm{Re} \in \langle 150;11000 \rangle$. Reynolds number characterizes type of fluid flow, i.e. laminar, transient between laminar and turbulent and turbulent flow. Based on values of Reynolds number and on physical and numerical experiments (Kozubková et al, 2003; Pochylý et al, 2002) laminar model was chosen. Standard two-equation $k-\varepsilon$ turbulent models underestimate vortex structures or number of vortices tends to zero. Only LES model is for computation available too, but it takes much more time due to time dependent equations and many testing cases. Therefore laminar model was selected. For illustration some results (speed n=3000 min$^{-1}$ and eccentricity e=0.00006m) are shown in Fig. 3.1.

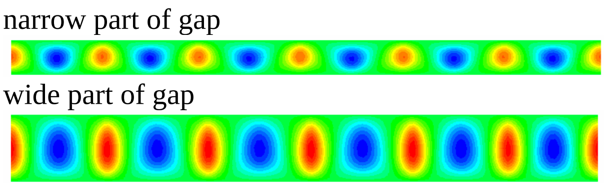


Figure 3.1 *Radial velocities in the narrowest and widest part of gap.*

In Fig. 3.1 we can see number of vortices, which was the same in both parts of the gap. Differences are evident in vortex shape. Vortices distribution is regular. It is better observed in schematic figure of radial velocity in the same cut plane, see Fig. 3.2. In this case the numerical convergence was satisfactory.

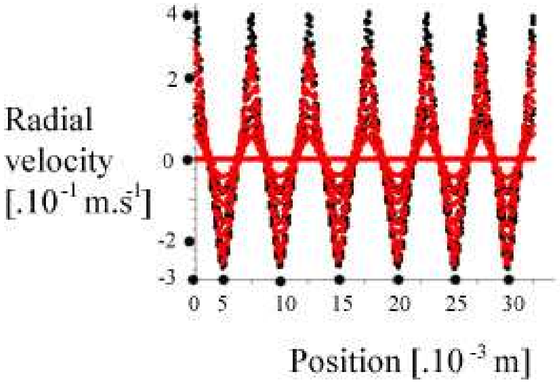


Figure 3.2 *Radial velocities in the most thin and most wide part of gap*

At high speed ($n \geq 9000\,\mathrm{min}^{-1}$) in given geometry tasks have bad convergence, which indicates previously described wave regime. In this case tasks must be solved as time dependent. Because vortices number does not change in defined regime, the value of moments and stiffness matrix is nearly constant. But there are not many problems of this kind. In different geometries this problem can be more significant (Farnik, 2006). Number of vortices changes between three to seven according to revolution and eccentricity, see Fig. 3.4. For lower revolution the eccentricity influence is lower and for higher revolution is higher.

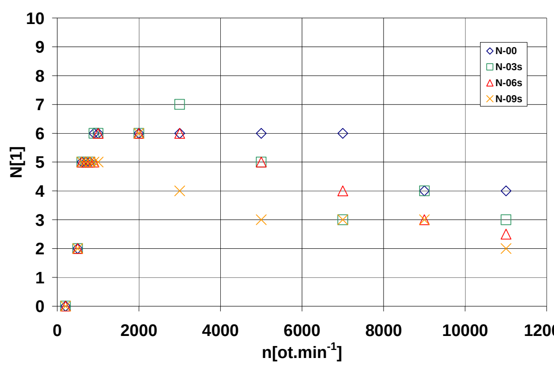


Figure 3.4 *Vortices number vs. revolution and eccentricity*

## 4. STIFFNESS OF FLUID LAYER

Stiffness of fluid layer is defined in dependence on region created by two cylinders. The inner cylinder rotates with constant angular velocity, the outer one is stationary. Axis of inner cylinder is shifted by the eccentricity specified by vector components $\mathbf{e} = (e_1, e_2)$. Force caused by fluid impacting on rotor depends on eccentricity and is given by:

$$\mathbf{F=-Ke} \qquad (4.1)$$

where the stiffness matrix $\mathbf{K}=\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$ with elements in following relations $K_{11} = K_{22}$, $K_{12} = -K_{21}$. The stiffness matrix value of fluid layer with Taylor vortices was computed by applying the numerical results of forces and moments obtained by software Fluent. Moments were specified for rotor center, see Fig. 4.4. The stiffness matrix **K** has all elements of the same orders. The stiffness matrix derived from Reynolds equation has vice versa the major diagonal elements lower by several orders of magnitude then the others. So the two matrices differ considerably and stiffness in case of used Taylor vortices is higher and increases according to angular velocity. Wave moment values $M_x, M_y$ are negligible. Twist moment value $M_z$ characterizes input defining dissipation of mechanical energy.

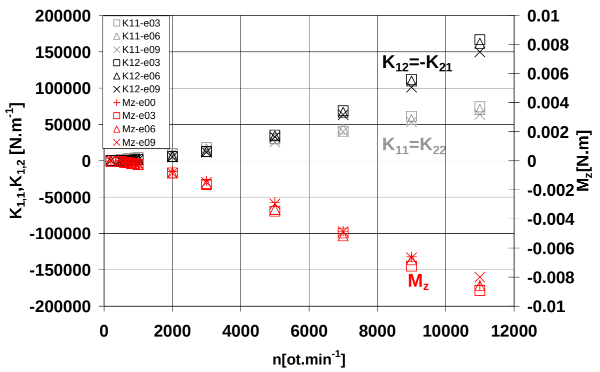


Figure 4.4 *Stiffness and moment vs. revolution and eccentricity*

## 5. CONCLUSION

Numerical modeling demonstrates existence of stable Taylor vortices region for wide angular speed interval in case of given gap width. Theory can be applied for design of journale bearings. Unlike classical theory of journal bearings based on Reynolds equation the stiffness matrix has all elements of the same orders. It will significantly influence rotor dynamics based on Taylor vortices principle. There was only static stiffness presented in this paper. The theory is being developed for the added mass and damping evaluation as well.


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## 7. ACKNOWLEDGEMENT

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