NEW MATHEMATICAL AND COMPUTATIONAL MODEL OF FLUID – STRUCTURE INTERACTION USING FEM

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ABSTRACT

This contribution is focused on the analysis of dynamic behavior of fluid – elastic structure interaction. A blade vibration of centrifugal pump or water turbine in liquid can be typical technical applications and other hydraulic machines. Solution using commercial software packages is very difficult and time consuming. In this contribution is presented new mathematical and computational model of assemblage the local FEM matrices of added liquid affects. This solution is based on the assumption the solution to series expansion of finite number of eigen shape of vibration. As a sample the interaction the cantilever beam with liquid was chosen. Governing equations are the Navier - Stokes and continuity in curvy linear co-ordinates. The Bézier body for the determination of geometrical configuration and its solution was chosen. The MATLAB code for the software performing was chosen. The methodology is based on a transformation which allowed the separation liquid and continuum from each other. Using this is possible to solve nonlinear tasks in the individual frequency or time steps.

Nomenclature

ρ_B - density, S_B - cross-section, E_B - Young's modulus, I_B - second moment of area, y - deflection, R - radius of beam, L_E, l - length, p - pressure, φ - angle, $x_1, x_2, x_3, x = x_3$ - co ordinates, $w_j(t)$ - function, $u_j(x)$ base function (see appendix), q_k - function, m_{ij}, b_{ij}, k_{ij} - elements of local mass finite element matrices for mass, damping and stiffness, p - pressure, η - dynamic viscosity, c_i - velocity, v_{il} - i -th deformation parameter for l - th shape of vibration, $\alpha_k, \alpha_{1_k}, \alpha_{2_k}$ - velocity functions, $h_k, b_{1_k}, b_{2_k}, b_{3_k}, \beta_k$ - pressure functions, δ - Dirac's function, t, τ - time, R_0, R_1, R_2, L_0, L_1 - geometrical properties, $S, \Gamma_1, \Gamma_2, \Gamma_3$ - denotation to the given surfaces enclosing liquid volume

Subscript:
B - beam, E - finite beam element, l - liquid

Keywords:
Fluid - structure interaction, finite element method, added mass, added damping, computational model

1. INTRODUCTION

A solution a problem of fluid - elastic structure interaction belongs to the most difficult problems in mechanics. From the point of view, they are three basic tasks. As the first is the eigen value problem, as the second can be the solution of steady state response do to the harmonic (periodic) excitation and at last the solution of unsteady state response (computational simulation).

It is evident in the last time, that a takeovers and mergers the computational packages, where were interested only in the individual and limited parts of mechanics. As a sample is the merging the ANSYS (solid mechanics) and FLUENT (hydromechanics). This process is inevitable and makes the development and creation a new mathematical and computational models and algorithms for solution.

It is necessary to have two different types of meshes for the solution the fluid – elastic structure interaction. One mesh is for a solid or structure and the second one for a fluid or surroundings. According the solving problem is almost already necessary to do some changing of mesh during the solution. It is evident, that this step leads to increasing the computational time consuming. In substance they are three basic types of changes of mesh:

a. Layering
b. Smoothing
c. Remeshing

When are used the commercial programme codes, especially ANSYS and FLUENT, these approaches are in detail presented in programme system manual (2005). General overview of methods to computational modelling is presented by Axise and Antuncs (2007).

Problem of fluid structure interaction needs the different approach to the computational modelling. In the general, they the two basic problems, in the computational modelling, the first is bad numerical stability and the second is very time consuming. That is why a lot of scientists deal with the idea how to achieve better numerical stability and shorter tome of calculation.

Daneshmand and Niroomandi (2006) presented a new method to simulation fluid – structure interaction. It is based on the use of a meshless technique named as Natural Element Method or natural neighbor Galerkin method in which the natural neighbor interpolation is used for the construction of test and trial function. The eigen value problem arising from the computation of the free vibrations of a coupled fluid – structure system is solved. Displacement variables for both the solid and the fluid domains are used, but the fluid displacements are written as gradient of potential function. One classical example is considered: free vibration of a flexible cavity filled with liquid.

One of the possibilities how to achieve this, is presented by Stein at all (2003). In computation of fluid-structure interactions, is used mesh update methods consisting of mesh-moving and remeshing – as - needed. When the geometries are complex and the structural displacements are large, it becomes even more important that the mesh moving techniques are designed with the objective to reduce the frequency of remeshing. To that end, is present here mesh moving techniques where the motion of the nodes is governed by the equations of elasticity, with selective treatment of mesh deformation based on element sizes as well as deformation modes in terms of shape and volume changes. It is also presented some results from application of these techniques to a set of two-dimensional test CASE.

Legay and Kölke (2006) presented new approach to the solution, where velocity and pressure are solved on base the weak formulation of the governing equations of viscous and incompressible fluid flow (Navier – Stokes equations) is discretized by finite space – time elements using discontinuous Glerkin scheme for time integration. To capture the occuring moving discontinuities from embedding a thin solid body into the flow field, a locally enriched space time finite element method is applied to ensure a fluid mesh independent from the current configuration of the structure. Based on the concept of the extended finite element method, the space – time approximation of the pressure is enriched to present strongly discontinuous solution at the position of the structure. The similar approach is presented by Kölke and Legay (2006). A numerical method for investigation challenging interaction phenomena of viscous fluid flow and flexible structures of negligible thickness like membranes and plates on a topologically fixed fluid discretization is presented. Since the formulation of fluid, structure and coupling conditions uniformly uses velocities as unknown and the integration of the governing equations is performed on the deformed space – time mesh, the realization of a strong coupling of the physical domains becomes very comfortable and results in a monolithic system.

Sigrist at all (2004) presented approach to he solution of the fluid structure interaction with a finite element discretization or with modal approach. The structure problem is modeled in the CFD code with various Fortran subroutines. Fluid is solved using finite volume discretization. For the achieving better numerical stability the special algorithm for the discretization in time and spatial domains is suggested.

Giannopapa and Papadakis (2004) presented the first stage of development of such a method, in which the solid equations are formulated so as to be solved for velocity and pressure i.e. for the same unknowns as the ones for the liquids equations.

In many cases the governing of the fluid are expressed in an Arbitrary – Lagrangian – Eulerian (ALE) frame reference that in a natural way treats the complex movement of the interface between the fluid and the structure without the need for surface tracking procedures. Lund et all (2004) is presented approach for analysis and semi – analytical design sensitivity analysis of time dependent fluid – structure interaction problem discretized by finite element methods. The aim of the method is to provide a general design tool than can be used for both analysis and synthesis of fluid - structure interaction where the dynamic interaction of a flexible structure and a viscous flow is in focus.

In immersed interface methods, solid in a fluid are presented by Shung and Wang (2007), by singular forces in the Navier – Stokes equations, and flow jump conditions induced by the singular forces directly enter into numerical schemes. The article is focused on the implementation of an immersed interface method for simulation fluid – solid interaction in the 3D space. The method employs the method of control volumes for the spatial discretization and method of Runge Kutha the 4 order for the time integration. The FFT – based Poisson solver for the pressure Poisson equation is used. A fluid – solid interface is tracked by Lagrangian markers.

A Lagrangian model for the numerical simulation of fluid – structure interaction problems is proposed by Antoci et all (2007). In the method both fluid and solid phases are described by smoothing particle hydrodynamics: fluid dynamics is studied in the inviscid approximation, while solid dynamics is simulated through an incremental hypoelastic relation. The interface condition between fluid and solid is enforced by a suitable term, obtained by an approximate smoothed particle hydrodynamics evaluation of a surface of fluid pressure. The method is validated by comparing numerical results with laboratory experiments where an elastic plate is deformed under the effect of a rapidly varying fluid flow.

The newly developed immersed object method is presented by Tai et all (2007). Parallel computation of unsteady incompressible viscous flows around moving rigid bodies using an immersed object method with overlapping grids is solved. Approach to parallel calculation is presented by Tai et all (2005). Newly is extended for 3D unsteady flow simulation with fluid – structure interaction, which is made possible by combining it with a parallel unstructured multigrid Navier – Stokes solver using a matrix – fee implicit dual time stepping and finite volume method. An object mesh is immersed into the flow domain to define the boundary of the object. The advantage of this is that bodies of almost arbitrary shapes can be added without grid restructuring, a procedure which is often time – consuming and computationally expensive.

How is evident from this research study, all tasks the fluid - elastic structure interaction are solved as coupled. To achieve better numerical stability and shorter time computing they are used special algorithms.

Another of possibilities for achievement these two problems is application of new type of boundary conditions for contact between continuum and liquid. Approach is based on the application the expansion to the solution according the finite number of eigen shapes of continuum vibration. The authors are many years interested in the possibilities, how to separate the continuum and liquid from each other. They proved, that this is possible for the solid body. The summarizing results more then sixth years research are presented by Pochyly and Malenovsky (2004) or Malenovsky and Pochyly (2004). The possibility for separation is based on the approximation of solution for velocity and pressure functions in the form of convolutory integrals. In this contribution is presented application on an elastic continuum. For testing of this possibility and proving the validity of this approach, was chosen cantilever beam with circular cross section vibrating in water. The objective is to determine the expressions for local matrices of added mass and damping of liquid and suggest algorithm for solution the elastic structure – liquid interaction.

Similar model sample was presented by Levy and Wilkinson (1976). Vibration a shaft in water is solved as a coupled problem. Authors are mainly focused only on the determination of added mass. Only the potential flow is taken into account and the water is considered as ideal. The finite element method for the both structure and liquid is used.

2. IDEAL LIQIUD AND BEAM

The linear eq. of motion for beam has form

$$\rho_B S_B y^{\bullet\bullet} + E_B I_B y'''' = -R\int_0^{2\pi} p(\varphi)\sin\varphi d\varphi \qquad (1)$$

The FEM is applied for solution. For a displacement of beam is valid

$$y(x,t) = w_j(t)u_j(x) \qquad (2)$$

After the substitution to (1) is obtained

$$\rho_B S_B u_j w_j^{\bullet\bullet} + E_B I_B u_j'''' w_j = -R\int_0^{2\pi} p(\varphi)\sin\varphi d\varphi \qquad (3)$$

Using integration over the length of finite element which is submerged in liquid is

$$\rho_B \int_0^{L_E} S_B u_j u_i dl w_j^{\bullet\bullet} + E_B \int_0^{L_E} I_B u_j'''' u_i dl w_j = \\ = -R\int_0^{L_E} u_i \int_0^{2\pi} p(\varphi)\sin\varphi d\varphi dl \qquad (4)$$

From this equation are evident the expressions for local mass and stiffness matrices beam element.

Other assumption is the approximation of solution w_j into series of eigen shapes of vibration v_{jk}

$$w_j = v_{jk} q_k \tag{5}$$

where $k = 1,...n$ for n shapes of vibration. Pressure is defined using variable h_k and $q_k^{\bullet\bullet}$

$$p = h_k q_k^{\bullet\bullet} \tag{6}$$

From here for pressure is evident

$$p = h_k v_{jk}^{-1} w_j^{\bullet\bullet} \tag{7}$$

After the treatment into do (4) is obtained

$$m_{B_{ij}} w_j^{\bullet\bullet} + k_{B_{ij}} w_j = = -R \int_0^{L_E} u_i \int_0^{2\pi} h_k(\varphi) \sin\varphi d\varphi dl v_{jk}^{-1} w_j^{\bullet\bullet} \tag{8}$$

The local added mass matrix for ideal liquid is given by

$$m_{l_{ij}} = R \int_0^{L_E} u_i \int_0^{2\pi} h_k(\varphi) \sin\varphi d\varphi dl v_{jk}^{-1} \tag{9}$$

3. REAL LIQIUD AND BEAM

The linear eq. of motion for beam including the influence of liquid has form

$$\rho_B S_B y^{\bullet\bullet} + E_B I_B y'''' = -R \int_0^{2\pi} \left(p + \eta \frac{\partial c_2}{\partial x_2} \right) \sin\varphi d\varphi \tag{10}$$

whereas is assumed the beams vibration in the direction 2. Also in this case according application the FEM the eq. of motion has form

$$m_{B_{ij}} w_j^{\bullet\bullet} + k_{B_{ij}} w_j = -R \int_0^{L_E} u_i \int_0^{2\pi} \begin{pmatrix} p + \\ +\eta \dfrac{\partial c_2}{\partial x_2} \end{pmatrix} \sin\varphi d\varphi dl \tag{11}$$

Also in this case the solution w_j is approximated into the series of finite number of eigen shapes of vibration v_{jk}

For the possibility to achieve the separation liquid and beam from each other is suitable to assume the solution for velocity and pressure in form

$$p = \int_0^t \beta_k (t-\tau) q_k^{\bullet}(\tau) d\tau \tag{12}$$

$$c_2 = \int_0^t \alpha_k (t-\tau) q_k^{\bullet}(\tau) d\tau \tag{13}$$

After the treatment into (13)

$$m_{B_{ij}} w_j^{\bullet\bullet} + k_{B_{ij}} w_j = = -R \int_0^{L_E} u_i \int_0^{2\pi} \begin{bmatrix} \int_0^t \beta_k (t-\tau) q_k^{\bullet}(\tau) d\tau + \\ +\eta \int_0^t \dfrac{\partial \alpha_k (t-\tau)}{\partial x_2} q_k^{\bullet}(\tau) d\tau \end{bmatrix} \sin\varphi d\varphi dl \tag{14}$$

It is possible the solution for velocity and pressure function assume in form

$$\alpha_k = a_{1_k} \delta(t) + a_{2_k}(t) \tag{15}$$

$$\beta_k = b_{1_k} \delta^{\bullet}(t) + b_{2_k} \delta(t) + b_{3_k}(t) \tag{16}$$

If is the influence of functions a_{2_k} and b_{3_k} small, eq. (14) has form

$$m_{B_{ij}} w_j^{\bullet\bullet} + k_{B_{ij}} w_j = = -R \int_0^{L_E} u_i \int_0^{2\pi} \left[b_{1_k} q_k^{\bullet\bullet} + b_{2_k} q_k^{\bullet} + \eta \frac{\partial a_{1_k}}{\partial x_2} q_k^{\bullet} \right] \sin\varphi d\varphi dl \tag{17}$$

With thinking of eq. (7), eq. (20) has the form

$$m_{B_{ij}} w_j^{\bullet\bullet} + k_{B_{ij}} w_j = = -- R \int_0^{L_E} u_i \int_0^{2\pi} \begin{bmatrix} b_{1_k} v_{jk}^{-1} w_j^{\bullet\bullet} + \\ + b_{2_k} v_{jk}^{-1} w_j^{\bullet} + \\ + \eta \dfrac{\partial a_{1_k}}{\partial x_2} v_{jk}^{-1} w_j^{\bullet} \end{bmatrix} \sin\varphi d\varphi dl \tag{18}$$

From eq. (18) are evident the expressions for local added mass and damping matrices of real liquid

$$m_{l_{ij}} = R \int_0^{L_E} u_i \int_0^{2\pi} b_{1_k} \sin\varphi d\varphi dl v_{jk}^{-1} \tag{19}$$

$$b_{l_{ij}} = R \int_0^{L_E} u_i \int_0^{2\pi} \left(b_{2_k} + \eta \frac{\partial a_{1_k}}{\partial x_2} \right) \sin\varphi d\varphi dl v_{jk}^{-1} \tag{20}$$

It is evident, that real liquid has influence on mass and damping.

4. MODEL SAMPLE

The model sample is a cantilever bar in liquid. This model was chosen with regard to the possibility of comparing with an experiment.

Scheme of this is on the Figure 2. The geometrical properties: $R_0 = 16.85$ mm, $R_1 = 17.85$ mm, $R_2 = 50$ mm, $R_1 = 17.85$ mm, $L_0 = 1100$ mm, $L_1 = 1000$ mm.

The results of the model task are only for matter – of - fact purposes and that is to present the possibilities the computational modelling. The whole analysis is carried out only for the first shape of vibration. The real and ideal liquids were assumed for computational analysis. On Figure 3 is drawn velocity distribution for both liquids. The dependence of the bar eigen frequencies in liquid with the different heights are drawn in Figure 4.

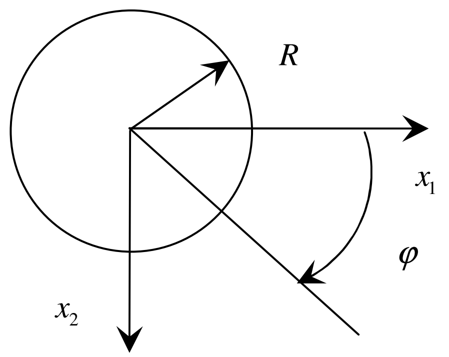

Figure 1: Scheme of co ordinate system

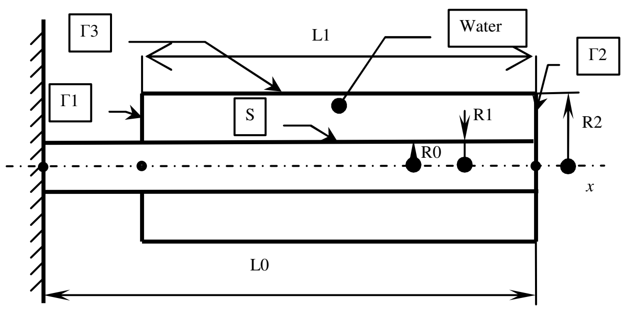

Figure 2: *Scheme of model task*

5. CONCLUSION

In this contribution is presented approach to the composition of local added matrices of liquid. They are assumed two models of liquids, ideal and real. It is possible to use, the presented approach to the solution, for the continuum with large displacement and large constrains. For the general, the algorithm is as follows:

1. All range in frequency or time domains is divided into finite number of steps.
2. It is provided the modal behavior analysis of individual continuum for finite number of steps for geometry configuration.
3. Analysis of individual liquid with the boundary conditions which are given by the chosen eigen shape of vibration. This step is repeated until the finite number of eigen values is achieving. From each step, the velocity and pressure field for given continuum position, is obtained.
4. On behalf of velocity and pressure field on continuum surface are calculated the forces which impacted the continuum. After this are calculated the added matrices from liquid influence. In this step are created the global added matrices for given shape of vibration and given vibrating position.
5. Interpolation analysis of individual continuum with including the global matrices from analysis of individual liquid (see step 4).

It is necessary the comparison with experiment, because this approach to analysis is new. This time are prepared 5 types of vessels with different outer diameter with centered beam submersed into the water with different high of water.

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Appendix

Base functions

$$u_1(x) = 1 - \frac{3x^2}{L_E^2} + \frac{2x^3}{L_E^3}, \quad u_2(x) = x - \frac{2x^2}{L_E} + \frac{x^3}{L_E^2},$$

$$u_3(x) = \frac{2x^2}{L_E^2} - \frac{2x^3}{L_E^3}, \quad u_4(x) = -\frac{x^2}{L_E} + \frac{x^3}{L_E^2}$$

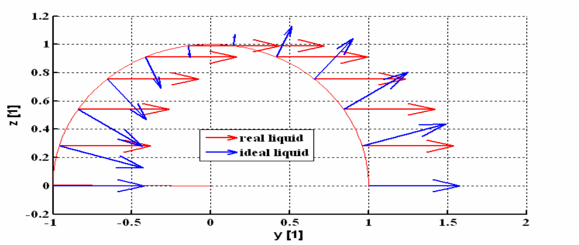

Figure 3: *Velocity distribution on the beam*

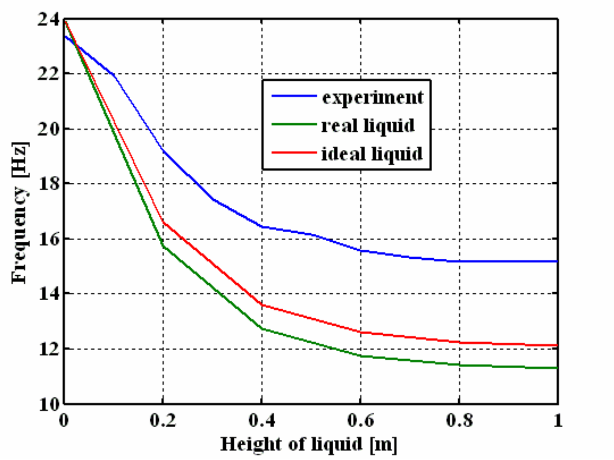

Figure 4 *Dependence of eigen frequency on the height of liquid*