SUMMARIZE OF METHOD FOR ANALYSIS ON SELF~EXCITED VIBRATION OF GATE

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ABSTRACT

Almost of flow induced vibration of gate comes from the interaction of gate movement and water flow around gate. So its motion must be considered by motion of gate and motion of water flow, the former one is described by the equation of motion of gate vibration and the later is described by the equation of water flow including gate motion.

Author has made the analysis on several type of gate vibration and those vibrations are the type of self-excited vibration. The analysis has been made as instability of gate motion by two methods as analysis on characteristic equation of vibration equation and energy method in one cycle of vibration.

In this paper author introduces the analysis of instability on several gate types by these two methods. The first method is suitable for liner equation of vibration motion and the second is suitable for non-linear equation of motion.

1. INTRODUCTION

Several gates are installed in river intake facilities and dams those are Flap gate, Roller gate, Long span shell roller gate and radial gate for their characteristics. The gate vibrates under the flow related to the interaction between gate motion and water flows. Almost gate vibrations are self-excited vibration with the interaction between water flows and arise around the natural frequency of support system. But some cases it is forced vibration occurred in the gate installed river mouth by the force of waves and the draft gate by the turbulent flow after the turbine in water power station.

The analysis on the self-excited vibration of gate is introduced by the method of minus damping in characteristic equation of vibration or by the energy method in one cycle of gate vibration motion.

In this paper, author introduces the several types of gate vibration and the analyzed results by characteristic equation and energy method of balancing between dissipation term (damping term) and supplied term (external force) of vibration equation.

2. UNSTABLE CONDITION BY CHARACTERISTIC EQUATION

Three types of gate vibration are analyzed by this method; those are flap gate, roller gate under the both over flow and under flow and long span shell roller gate. Former two cases are unstable problems by non-linear external force and the third one is the unstable condition of minus damping.

2.1 Flap gate vibration with nape oscillation

The equation of gate motion is rotation motion and external force comes from pressure changes on gate surface in back side of gate.

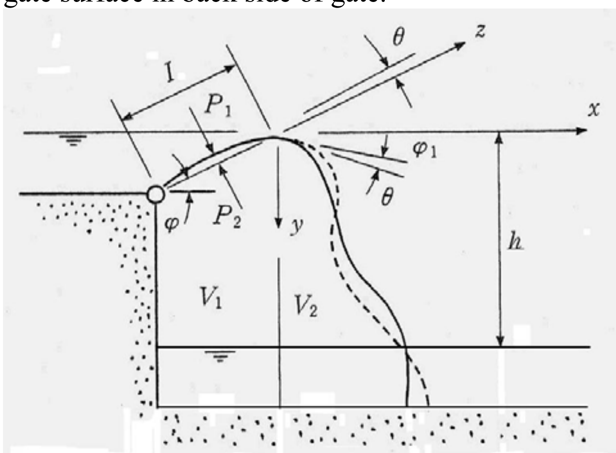

Figure 1 *Coordinate system for flap gate*

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega_n{}^2 = M_1 \int_0^{x_1} x\theta\left(t - \frac{x}{v_0 \cos\varphi_1}\right)dx \quad (1)$$

$$2\gamma = \frac{R_\theta}{I}, M_1 = \frac{1}{I}\frac{RT}{V^2}\frac{B^2 l^2}{2}$$
$$\omega_n{}^2 = \frac{1}{I}\left[kl^2 + \int_0^l B\frac{dP_1}{d\varphi} z\,dz + \frac{RT}{V^2}\frac{B^2 l^2}{4}\right] \quad (2)$$

Pressure changes arise by nape motion of water flow on gate and shock of eater attack on tale water surface. There are interaction between gate motion and pressure from nape motion and equation is written as follows

Here R: Kelvin Constant, T: Kelvin Temperature, I: Moment of inertia of gate, R_θ : Damping constant, B: gate length, l: gate width, V: volume of air under gate and nape, k: spring constant of gate support.

Characteristic equation for Eq. (1) is derived as equation (4) by using following equations.

$$x_1 = \sqrt{\frac{2h}{g}} v_0 \cos\varphi_0 , c = v_0 \cos\varphi_0 \quad (3)$$

Here h is falling height of nape and v_0 is water velocity at top of gate.

$$s^2 + 2\gamma s + {\omega_n}^2 - M_1\left[\frac{c^2}{s^2}\left(1 - e^{-\frac{x_1}{c}s}\right) - \frac{c}{s}x_1 e^{-\frac{x_1}{c}s}\right] = 0 \quad (4)$$

This equation is non-linear equation and there many characteristic roots. The unstable condition can be derived by using complex plane and are shown as figure 2.

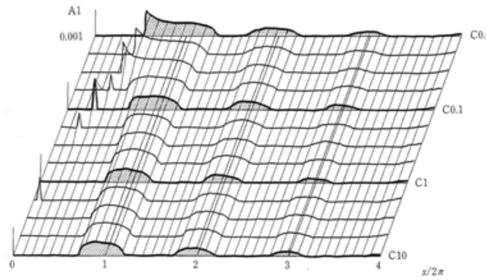

Fig. 2 Unstable zone (Elastic support) Natural frequency

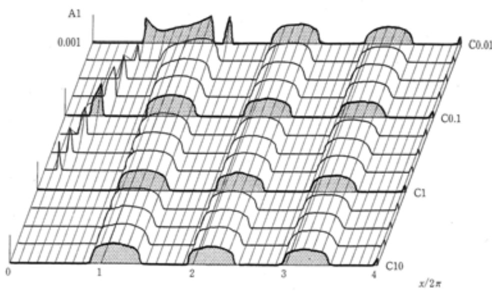

Fig.3 Unstable zone (Fixed support)

Figure 2 *Unstable zones for flap gate vibration*

The analysis by unstable condition on the equation motions drives the results as shown figures 2 and 3. The former one is the case of soft supported system and the latter one is hard supported case. The unstable zones are given as the hill region of the curve in these figures and the vertical axis A shows a parameter for unstable which contains the damping constant of vibration system and falling time. The horizontal axis $x/2\pi$ shows wave number on water curtain. The curves array from front to back are changing by parameter C is 10 to 0.01 which is derived as the combination of gate size, gas constant, air volume under the gate and nappe, water velocity at the top of gate and falling time. These parameters are given as equations in (5).

$$A = 2\gamma C\beta, C = \frac{1}{M_1 c^2 \beta^4}, M_1 = \frac{RT\,B^2 L^2}{2IV^2},$$
$$c = v_0 \cos\phi, \beta = \sqrt{\frac{2h}{g}} \quad (5)$$

Here B, L and I are gate height, gate length and moment inertia of gate around the pin. V is the sum of volume of air under the gate V1 and nappe V2. R and T are gas constant and absolute temperature. β, h and c are falling time interval from gate top to tail water surface defined as the last equation in equation (5), height from tail water surface to gate top and horizontal water velocity at gate top defined as the fourth equation in (5). The term ϕ is angle of direction of water take off at gate top from horizontal axis (Fig.1).

The following results are derived from these figures.

2.1.1 *Soft supported case shown in Fig 2.*

When the vibration occurs by the natural frequency of gate system, the hill region lies at n+3/4 of wave number on water curtain which corresponds to equation (6) is derived by (K. Petricat et al 1965).

$$\frac{\beta}{T} = n + \frac{3}{4} \quad (6)$$

2.1.2 *Hard supported case shown in Fig. 3*

This corresponds to the Sabo Dam and equation (7) shown by (Schwartz 1964). Namely the unstable hill region lies at n+1/4 of wave numbers.

$$\frac{\beta}{T} = n + \frac{1}{4} \quad (7)$$

This type of vibration occurs in some type of Rubber dam.

2.2 Sluice gate under both over and under flows

The sluice gate with rectangular gate lip is significantly vibrated under the flow of both over flow and under flow. The vibration occurs around natural frequency of gate supported system and affected both pressure changes at gate lip.

The model gate and its supported system are shown in figure 3. The basic vibration equation of gate is written as equation (8).

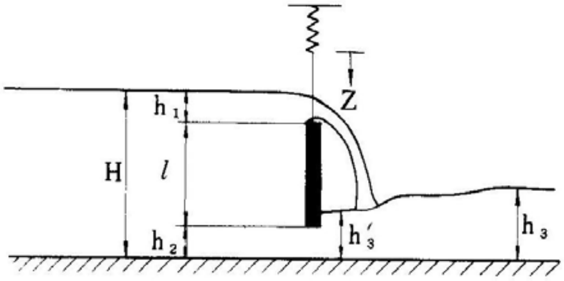

Figure 3 *Sluice gate system*

$$(M+M')\frac{d^2z}{dt^2}+R\frac{dz}{dt}+kz=P'(z,t) \quad (8)$$

Here M and M' are gate mass and added mass of this gate, and external force P' is derived by pressure change of top of gate P_U and bottom of gate P_L.

$$P'(z,t)=A\left[\frac{dP_U}{dh_1}z(t)-\frac{dP_L}{dh_1}z(t-t_0)\right]$$
$$t_0=\sqrt{\frac{2H}{g}}\sqrt{k\frac{h_2}{H}+\frac{l}{H}+C_v\frac{h_2}{H}}+t_1 \quad (9)$$

The pressures between top of gate and bottom of gate have time lag t_0 by falling time of water and transmitted time of t_1. So basic vibration equation is written as equation (10) and parameters for them are given as equation (11).

$$\frac{d^2z}{dt^2}+2\gamma\frac{dz}{dt}+\omega_n{}^2z+\lambda z(t-t_0)=0 \quad (10)$$

$$2\gamma=\frac{R}{M+M'},n^2=\frac{k}{M+M'},$$
$$n_1{}^2=A\frac{dP_U}{dh_1}\frac{1}{M+M'},\omega_n{}^2=n^2-n_1{}^2 \quad (11)$$
$$\lambda=A\frac{dP_L}{dh_1}\frac{1}{M+M'}$$

The characteristic equation for equation (10) is derived as equation (12),

$$s^2+2\gamma s+\omega_n{}^2+\lambda e^{-t_0s}=0 \quad (12)$$

The unstable condition can be shown as figure 4 by two parameters T_0 and Λ.

$$\Lambda=\frac{\lambda}{2\gamma\omega_n},T_0=\omega_nt_0 \quad (13)$$

The unstable condition is given as value Λ is larger than 1.0 at T_0 is given as $\pi/2, 5\pi/2 \ldots (4i+1)\pi/2, i=0,1,2\ldots$ ... (14)

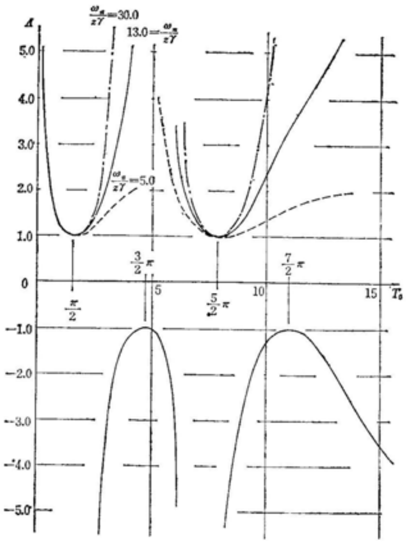

Figure 4 *Unstable zone of sluice gate*

3. UNSTABLE CONDITION BY ENERGY METHOD

When the equation of vibration motion is non-linear type, energy method is better to define the unstable condition. In vibration equation, there two terms as energy dissipation term and energy supply term. The former corresponds the damping term and the latter becomes from external force. Three phenomena are shown here, which are long span shell roller gate, roller gate in outlet from dam under submerged condition and radial gate.

3.1 Long span shell roller gate

This gate is designed under the limit condition as deflection rate is 1/600 (deflection/length of span).

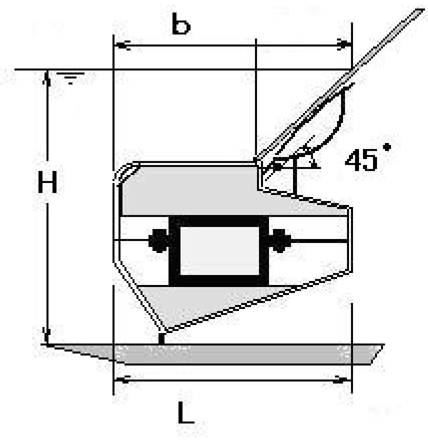

Figure 5 *Shell roller gate*

So in small gate opening, there are gape of opening difference between center part and side part. And when gate moves downward, the center part of gate stops the water flow and it makes momentum force and it becomes external force of gate vibration. This condition makes unstable motion as minus damping in characteristic equation of motion. This type of vibration arises under both submerged flow and free flow.

$$EI_y \frac{\partial^4 y}{\partial z^4} + m\frac{\partial^2 y}{\partial t^2} + R\frac{\partial y}{\partial t} = f_y(z,t)$$
$$EI_x \frac{\partial^4 x}{\partial z^4} + m\frac{\partial^2 x}{\partial t^2} + R\frac{\partial x}{\partial t} = f_x(z,t) \qquad (15)$$

$$\omega_{yn} = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI_y}{m}},\ \omega_{xn} = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EIx}{m}} \qquad (16)$$

Basic equations of deformation of shell beam are written as equation (15) for vertical and horizontal directions. And natural angular frequency of both directions is given as equation (16). External forces are derived from the added mass and the momentum force from water stopping by gate motion and also by gate motion itself. So they are written as equation (17)

$$f_y = -\rho VC_{my}\frac{\partial^2 y}{\partial t^2} + \left[\rho\left(b + b_1 \sin\alpha\right)\left(\frac{dy}{dt}\right) - C_y\frac{\rho q}{a}y\right]\left(\frac{dy}{dt}\right)$$
$$f_x = -\rho VC_{mx}\frac{\partial^2 x}{\partial t^2} + C_x\frac{\rho q}{a}y\frac{dy}{dt} - \left[\rho H_1\frac{dx}{dt}\right]\frac{dx}{dt} \qquad (17)$$

Unstable condition can be derived by the relations by the energy of dissipation and the supplied energy by external force in one cycle of gate motion. When the supplied energy is greater than the dissipated energy, the vibration becomes larger, namely it becomes unsteady. The gate motions are given as equation (18), the unstable condition becomes equations (19) and (20).

$$y = a_y \sin\omega t \sin\left(\frac{2\pi z}{L}\right), x = a_x \sin(\varpi t - \varphi)\sin\left(\frac{2\pi z}{L}\right)$$
$$\omega = \omega_{yn} = \omega_{xn}$$
(18)

$$\frac{2a_y}{3\pi}\left[-2\rho(b + b_1\sin\alpha) - C_y\frac{\rho q}{a\omega}\right] > \frac{(m + m_y')\gamma_y}{\omega}$$
$$2\gamma_y = \sqrt{\frac{R}{m + m_y'}}, m_y' = \rho C_{my}V$$
(19)

$$\frac{2a_x}{3\pi}\left[-2\rho H + C_x\frac{\rho q}{a\omega}\left(\frac{a_y}{a_x}\right)^2\cos\varphi\right] + \frac{2\rho gH}{\varpi^2}\left(\frac{a_y}{a_x}\right)\sin\varphi > \frac{(m + m_x')\gamma_x}{\omega} \qquad (20)$$
$$2\gamma_x = \sqrt{\frac{R}{m + m_x'}}, m_x' = \rho C_{mx}V$$

The term $\sin\alpha$ in equation (19) is very important for this phenomenon

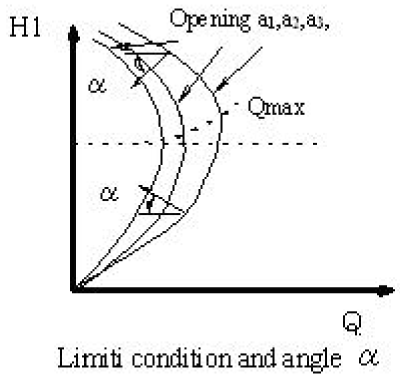

Figure 6 *Definition of angle* $\sin\alpha$ *related discharge, water depth and gate opening*

Namely discharge changes by gate opening depend on the water depth of upstream and it has maximum value at any opening. Definition of $\sin\alpha$ is the inclination of vector as shown in figure 6 and it has minus value at water depth is greater than the point of discharge is maximum.

The region where this inclination is minus, the value of left side of equation (19) becomes plus and becomes greater then right hand side term by dissipation energy.

So the first step for recognize the unstable condition is to make the graph of discharge between gate opening and upstream water depth and to find the maximum discharge point. When the maximum point arises in this graph, the upper region of this point becomes unstable region.

3.2 Roller gate at dam outlet

The gate which has skin plate at upstream side of gate has the self-excited vibration in small gate opening under submerged condition.

The external force of vibration system becomes from the relation of stopping the water flow under the gate by the closing motion of gate and the equation of external force can be derived by applying the momentum equation to this flow zone.

Equation of vibration is written as equation (21), and parameters are shown in equation (22).

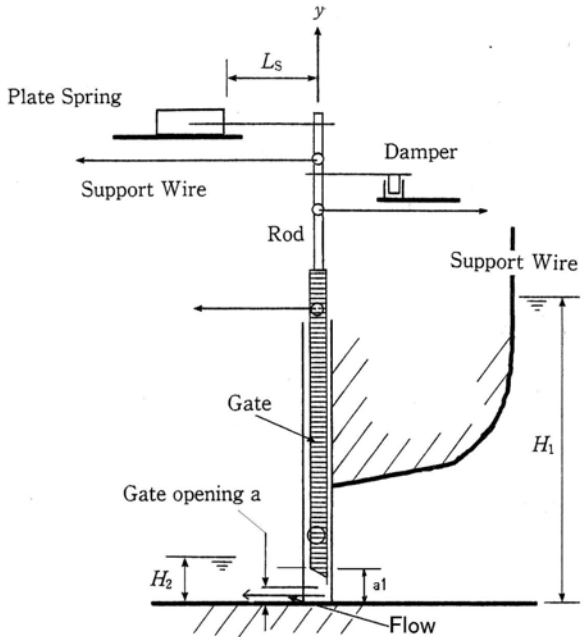

Figure 7 *Roller gate in dam outlet*

$$\frac{d^2y}{dt^2}+2\gamma\frac{dy}{dt}+\omega_n{}^2y=\left(\beta y-\delta\frac{dy}{dt}\right)\frac{dy}{dt}$$

$$2\gamma=\frac{R}{m},\omega_n{}^2=\frac{k}{m},\beta=\rho Q_0\left(\frac{1}{a}-\frac{1}{a_1}\right)\frac{1}{m}\quad(21)$$

$$\delta=\frac{\rho BL}{m}$$

Here a and a1 are gate opening and opening at down stream side under the gate. Unstable condition can be derived by energy relations between dissipation term and supplied term.

$$\gamma<\frac{2\varepsilon}{3\pi}(\beta-2\delta\omega_n).\ \text{(22)}$$

Here ε is amplitude of initial gate vibration.

3.3 Radial gate

The instability analysis on vibration of radial gate has been done in 1999 under the research project In JPGA (Japan penstock and water gate association) and also the model test of gate vibration has been done. The almost theoretical results have been satisfied of the model test results, but there is one point not to be able to explain the relation of natural frequency of gate support system.

The instable condition is given by the forth power of frequency of gate system in model test, and the fact that the region of instable becomes smaller as the gate opening larger, is same as the results of theoretical analysis.

Author develop the new approach which is considering the pressure change by stopping the water flow same as water hummer in pipe line.

For this self-exited vibration, two centers as center of tranion and center of force is not coincided and center of force must lie under the center of tranion.

Equation of vibration motion is written as the rotation motion around the tranion pin and external force comes from the pressure change as water hammer by stopping the water flow under gate closing motion.

The external force by gate closing motion is derived as pressure by stopping the water flow transmitted to upstream side of gate and it acts on gate surface. The resultants of this pressure make the momentum force to rotate the gate.

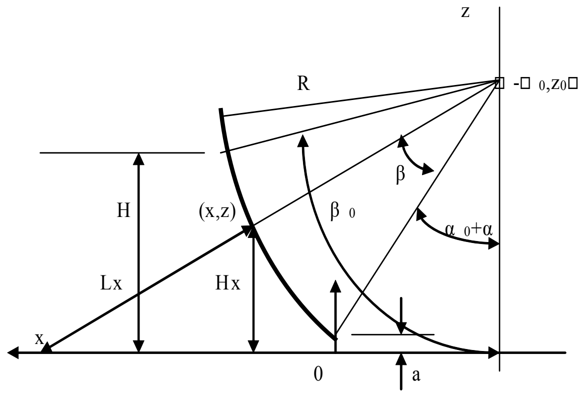

Figure 8 *Coordinate for analysis*

$$\frac{d^2\theta}{dt^2}+2\gamma\frac{d\theta}{dt}+\omega_n{}^2\theta$$

$$=\frac{\rho a_y v_0{}^2\delta BR}{RI}\left(\frac{c}{\omega R}\right)^2 e^{i\omega t}\left[F\left(\frac{c}{\omega R},z_1\right)-F\left(\frac{c}{\omega R},z_0\right)\right]$$

(23)

Here z0 and z1 are given as equation (24). And c is celerity of pressure wave and δ is the distance of two centers.

$$z_0=R\cos(\alpha_0+\alpha),z_1=R\cos(\alpha_0+\alpha+\beta)\ (24)$$

The external force in equation (23) is so complex the gate. Function F( ) is given by equation (25) which is the explanation of a part of momentum force around tranion pin. It is derived by integration of pressure distribution on gate surface considering transmission of pressure wave by gate closing motion.

$$F(\xi)=\int \frac{P_a \xi}{\sqrt{1-\left(\frac{c}{\omega R}\right)^4 \xi^2}}$$

$$\times \exp\left[-\frac{\left[\cos\alpha_0-\sqrt{1-\left(\frac{c}{\omega R}\right)^4 \xi^2}\right]\left(1-\left(\frac{c}{\omega R}\right)^4 \xi^2\right)}{\left(\frac{c}{\omega R}\right)^2 \xi \cos(\alpha_0+\alpha)}\right]$$

$$\times\left(\xi-\left(\frac{\omega R}{c}\right)^2 \sin(\alpha_0+\alpha)\right)d\xi$$

(25)

And ξ is defined as equation (26) and also Pa is defined as equation (27).

Unstable condition is derived as equation (28) by the relations of dissipated energy by damping term and energy supply by external term.

$$\xi=\left(\frac{\omega R}{c}\right)^2 \sin(\alpha+\alpha_0+\beta) \quad (26)$$

$$P_a=\sqrt{\left\{\left(\frac{a}{R}\right)^2\left(\frac{\cos(\alpha_0+\alpha)}{\left(\cos(\alpha_0)-\sqrt{1-\left(\frac{c}{\omega R}\right)^4 \xi^2}\right)^2}\right)^2 + \left(\left(\frac{R\omega}{v_0}\right)\frac{1-\left(\frac{c}{\omega R}\right)^4 \xi^2}{\left(\frac{c}{\omega R}\right)^2 \xi\cos(\alpha_0+\alpha)}\right)^2\right\}}$$

(27)

$$\frac{a_y}{\theta_a}\frac{\rho v_0^{\,2}\delta BR}{2I\,c^2}\left(\frac{c}{R\omega_n}\right)^4\left[F(\xi_1)-F(\xi_0)\right]\sin\varphi>h$$

(28)

The term of ξ_1 *and* ξ_0 are corresponding to z1 and z0. And h is damping ratio of vibration system and φ is the phase of gate motion and pressure. Equation (28) contains the forth power of angular frequency of gate vibration and so self-excited vibration occurs in lower frequency.

4. CONCLUSION

Two methods for analyzing on unstable problem of self-excited gate vibration are introduced in this paper, namely by characteristic equation and by energy method. As almost of self-excited vibration of gate are written as non-linear equation, method by characteristic equation is difficult for analysis. The energy method gives the results in such cases when the integration in one cycle of vibration can be done. Two cases by characteristic methods and three cases by energy method are introduced in this paper.

5. REFERENCES

Kunihiro Ogihara, Hiroya Emori and Yukihiko Ueda, Flow induced vibration of radial gate under small gate opening, FIV2000, Flow Induced Vibration, Ziada & Staubi (eds), Balkema, Rotterdam, 2000 (proceeding seventh international conference on flow induced vibrations)

Kunihiro Ogihara, Unstable condition of self-excited oscillation of flap gate, IAHR 21st Congress, August 1985, Melbourne, Australia

Kunihiro Ogihara, Theoretical analysis on radial gate vibration, Proceeding of PVP 2006□ICPVT-11, 2006 ASME Pressure Vessels and Piping Division Conference, July 23□27, 2006, Vancouver, BC, Canada, PVP2006-ICPVT-11-93743

Kunihiro Ogihara, Self-excited vibration of radial gate related to the shape of gate lip. Water power XIII, 2003

Kunihiro Ogihara and Yukihiko Ueda, Self-excited oscillation of roller gate under small gate opening, International Conference Flow Induced vibration, Bowness-on Windermere, England, 12-14 May, 1987

Kunihiro Ogihara, A fundamental study of the vibration phenomena in a sluice gate, Transaction of JSCE, no.141, May 1967, pp31-41

Kunihiro Ogihara, Review on gate vibration related to the gate lip and its theoretical analysis, Hydrovision 2004

Kunihiro Ogihara and Kanenori Yudou, Design by considering flow induced vibration and the results of prototype test, XXVIII IAHR Congress, August 22-27, 1999, Graz, Austria