A SIMPLE CONSISTENT METHOD FOR THE TIME-DOMAIN SIMULATION OF TURBULENCE EXCITATIONS APPLIED TO TUBE/SUPPORT DYNAMICAL ANALYSIS UNDER NON-UNIFORM FLOWS

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ABSTRACT

The predictive dynamical analysis of gap-supported tubular bundles subjected to turbulence-induced vibrations is one of practical significance, in particular when addressing nuclear power-plant facilities. It is a multi-disciplinary work which resulted in a number of computational tools, mostly based on time-domain numerical simulations of the flow-excited nonlinear systems. This paper addresses the problem of achieving an adequate model of turbulence excitations when performing such time-domain computations. We propose a simple but consistent method to simulate the continuous space-correlated flow force field, using a finite set of uncorrelated discrete random forces located along the structure. These are computed from the turbulence spectrum and the space correlation of the original turbulence field, accounting for the flow velocity profile. We present illustrative computations which show the effects of non-uniform velocity profiles on the tube dynamical excitations. These examples also highlight how the proposed excitation technique copes with such problems.

1. INTRODUCTION

The predictive analysis of industrial components subjected to flow-induced vibrations constitutes an important step of their design, in order to avoid destructive fluid-elastic instabilities as well as excessive turbulence-induced vibrations. For obvious safety reasons, such concern is of particular significance when addressing nuclear power-plant facilities, in particular when dealing with sensitive components such as steam generator tube bundles and fuel rods. Hence, extensive research programmes have been conducted in several countries during more than two decades, in order to quantify the relevant excitation mechanisms, model the tube/support nonlinear interaction dynamics, characterize the tribology of contacting materials and, finally, relate all these aspects to wear phenomena in order to predict the component life. All this multi-disciplinary work has resulted in a number of computational tools, which perform the dynamical aspects of predictive analysis, based on time-domain numerical simulations of the flow-excited systems - see for instance Sauvé and Teper (1987), Axisa et al (1988), Fisher et al (1989), Haslinger and Steininger (1995) or Hassan et al (2003).

One may fairly state that such computer programs have become sufficiently mature at the present time. Even so, some areas are still open to improvement. This paper addresses one such area: the problem of achieving adequate modelling of flow turbulence excitations when performing time-domain numerical simulations. A literature review shows that, in most papers, the details on time-domain implementations of turbulence excitations shine by their absence, as if such modelling aspects were obvious or non-important. Actually, providing adequate time-domain force functions – which rightly account for the spectral properties, the space correlation, as well as the local magnitude of the flow velocity field – is a nontrivial task, which certainly offers opportunity for some intellectual stimulation. Indeed, more often that not, numerical simulations are performed by exciting the tubes with random forces which account in a crude manner – or not at all – for the partial space correlation of the flow turbulence. For instance, a common approach is to excite the system through a number of random forces, assuming full correlation within each tube span and none beyond – see, for instance, Hassan et al (2003).

In contrast, the approach followed in our previous studies has been to perform excitation through direct random *modal* forces. When computing the time-domain realizations of such modal excitations, which were assumed statistically independent, the spectral content as well as the space correlation of the turbulence field was accounted for (Axisa et al,1988). Even if such approach has produced many useful and realistic results, we have found that it is still open to some criticism. Indeed, recent computations have shown that for some multi-supported systems subjected to strongly non-uniform – e.g., near-local – flows, it tends to produce vibratory responses which spread over the complete system. Such dubious result seemed to be a consequence of the direct modal excitation technique used and therefore triggered our interest in revisiting this issue.

To overcome problems, one may develop a fully fleshed excitation model based on the time-domain simulation of a turbulence force field (described in terms of its local auto-spectrum and spatial coherence function, see section 2), using a suitable (typically large) number of partially correlated random forces computed using the technique developed by Shinozuka and co-workers – see Shinozuka (1971). However, in the present paper we decided to develop an alternative strategy to Shinozuka's interesting approach, in order to obtain satisfying results while avoiding too computer-intensive or over-simplified excitation models.

Here, we propose a simple but consistent method to simulate the continuous space-correlated axial flow force field, using a finite set of uncorrelated discrete random forces located along the structure. These are computed from the turbulence spectrum and the space correlation of the original turbulence field, accounting for the flow velocity profile. We establish the necessary conditions which such force set must fulfil, depending on the number of excited structure modes and on the flow velocity field. We furthermore demonstrate the robustness of this excitation method. Aspects related to the azimuthal correlation of the force field are not addressed in the present paper – an aspect which certainly also deserves further attention. Here, following the common practice, we will simply assume that the orthogonal turbulence flow force fields $f_y(x,t)$ and $f_z(x,t)$ are fully uncorrelated.

Then, using the improved excitation technique, we reappraise the direct modal excitation approach, by looking at the modal forces computed from the force set which simulates the turbulence excitation. Interestingly, such analysis demonstrates that the modal excitations are statistically independent only for *uniform* flow velocity profiles, while for *localized* flow excitation profiles the corresponding modal forces become partially time-correlated.

We present two computational examples which demonstrate the consistent behaviour of the proposed excitation technique and highlight how it copes effectively with non-uniform flow velocity fields. Computations also numerically validate the theoretical developments presented concerning the correlation of turbulence-induced modal forces.

2. TURBULENCE EXCITATION

Here we will briefly review the main aspects of turbulence excitation, as thoroughly discussed in several papers, in particular Axisa et al (1990) and De Langre and Villard (1998).

2.1 Random response of linear structures

Let us consider a tubular structure with length L, external diameter D and modal properties m_n, ω_n, ζ_n and $\phi_n(x)$, which is subjected to a random force field $f(x,t)$ stemming from turbulence excitation. The flow is described by its density ρ and the velocity profile $V(x)=\bar{V}\,u(x)$, where $\bar{V}$ is a representative length-averaged flow velocity and $u(x)$ the normalized velocity profile. Then, linear tube responses are given by the modal equations $(n=1,2,...,N)$:

$$m_n\ddot{q}_n + 2m_n\omega_n\zeta_n\dot{q}_n + m_n\omega_n^2 q_n = f_n(t) \qquad (1)$$

while the physical responses $y(x,t)$ and modal forces $f_n(t)$ are computed as:

$$y(x,t)=\sum_{n=1}^{N}\phi_n(x)\,q_n(t) \;\; ; \;\; f_n(t)=\int_0^L \phi_n(x)\,f_y(x,t)\,dx \qquad (2)$$

and similarly for the orthogonal motion $z(x,t)$ and the corresponding physical and modal forces.

Then, following the well established theory of random vibrations, the cross-spectra of the modal responses are obtained in the frequency domain as $(m=1,2,...,N\,;\,n=1,2,...,N)$:

$$S_{q_nq_m}(\omega)=H_n(\omega)H_m^*(\omega)\times$$
$$\times\int_0^L\int_0^L \phi_n(x_1)\,\phi_m(x_2)\,S_f\left(x_1,x_2,\omega\right)dx_1dx_2 \qquad (3)$$

where $H_n(\omega)=\left[m_n\left(\omega_n^2-\omega^2+2i\omega\omega_n\zeta_n\right)\right]^{-1}$ and the excitation field $f_y(x,t)$ is entirely described in terms of its cross-spectrum $S_f\left(x_1,x_2,\omega\right)$. Then physical response is given by:

$$S_{yy}(x,\omega)=\sum_{n=1}^{N}\sum_{m=1}^{N}\phi_n(x)\,\phi_m(x)\,S_{q_nq_m}(\omega) \qquad (4)$$

Often cross-terms are much smaller than the diagonal terms in (4), which then simplifies to:

$$S_{yy}(x,\omega)=\sum_{n=1}^{N}\left[\phi_n(x)\right]^2 S_{q_nq_n}(\omega) \qquad (5)$$

with:

$$S_{q_nq_n}(\omega)=\left|H_n(\omega)\right|^2\int_0^L\int_0^L \phi_n(x_1)\phi_n(x_2)\,S_f\left(x_1,x_2,\omega\right)dx_1dx_2 \qquad (6)$$

2.2 Turbulence modelling

As it is well known, the turbulence excitation $S_f\left(x_1,x_2,\omega\right)$ may be conveniently modelled in terms of a local auto-spectrum $\Phi\left(x,\omega\right)$ and a spatial correlation function $\gamma\left(x_1,x_2,\omega\right)$:

$$S_f\left(x_1,x_2,\omega\right)=\left[\Phi(x_1,\omega)\Phi(x_2,\omega)\right]^{1/2}\gamma\left(x_1,x_2,\omega\right) \quad (7)$$

where $\left|\gamma\left(x_1,x_2,\omega\right)\right|\leq 1$ is real or complex.

For *cross-flow excitations*, the spatial correlation function is real and may be described in the simple form:

$$\gamma_{Tr}\left(x_1,x_2,\omega\right)=\exp\left(-\frac{\left|x_2-x_1\right|}{\lambda_c(\omega)}\right) \quad (8)$$

where λ_c is a correlation length of the force fluctuations, which for tube bundles is of the order of the tube diameter – see Inada et al (2007). For *axial-flow excitations*, the correlation function is complex, as it also depends on the axial convection velocity V_{AC} of the turbulence fluctuations:

$$\gamma_{Ax}\left(x_1,x_2,\omega\right)=\exp\left(-\frac{\left|x_2-x_1\right|}{\lambda_c(\omega)}+i\frac{x_2-x_1}{V_{AC}/\omega}\right) \quad (9)$$

In the present paper we will postulate, for simplicity, that effects of V_{AC} may be neglected. The formulation presented hereafter is therefore assumed adequate for both transverse and axial flow excitations.

For obvious reasons it is convenient to express the turbulence spectra in dimensionless form. For single-phase flows, adequate collapsing of experimental data is achieved by scaling Φ in terms of the flow pressure head and using the reduced frequency $f_R=fD/\bar{V}$, so that the following *reduced spectrum* $\bar{\Phi}$ is obtained:

$$\bar{\Phi}(f_R)=\frac{V/D}{\left(\frac{1}{2}\rho V^2 D\right)^2}\Phi(f) \quad (10)$$

2.3 Tube responses

Using (7) and (8), the modal tube responses (6) are given by:

$$S_{q_nq_n}(\omega)=\left|H_n(\omega)\right|^2\int_0^L\int_0^L\phi_n(x_1)\phi_n(x_2)\left[\Phi(x_1,\omega)\Phi(x_2,\omega)\right]^{1/2}\times \\ \times\exp\left(-\frac{\left|x_2-x_1\right|}{\lambda_c}\right)dx_1dx_2 \quad (11)$$

and, accounting for (10), we obtain after some simplifying assumptions (Axisa et al, 1990):

$$S_{q_nq_n}(f)=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\left|H_n(f)\right|^2\bar{\Phi}\left(\frac{fD}{\bar{V}}\right)L_{cn}^2 \quad (12)$$

where L_{cn} is the so-called *joint-acceptance* integral, which encapsulates the combined effects of the spatial correlation of the turbulence fluctuations λ_c, the flow velocity profile $u(x)$ and the structural modeshapes $\phi_n(x)$:

$$L_{cn}^2=\int_0^{L_f}\int_0^{L_f}u(x_1)^2u(x_2)^2\phi_n(x_1)\phi_n(x_2)\exp\left(-\frac{\left|x_2-x_1\right|}{\lambda_c}\right)dx_1dx_2 \quad (13)$$

where L_f is the tube length subjected to the flow. Because in general $\lambda_c/L_f\ll 1$, integration (13) may be shown to simplify drastically:

$$L_{cn}^2\approx 2\lambda_c\int_0^{L_f}\left[u(x)^2\phi_n(x)\right]^2dx \quad (14)$$

so that (12) and (13) reduce to:

$$S_{q_nq_n}(f)=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\left|H_n(f)\right|^2\left[\frac{\lambda_c}{L_f}\bar{\Phi}\left(\frac{fD}{\bar{V}}\right)\right]\times \\ \times 2L_f\int_0^{L_f}\left[u(x)^2\phi_n(x)\right]^2dx \quad (15)$$

Furthermore, to avoid any possible difficulty connected to an insufficient knowledge of λ_c (or a dependence on frequency of the correlation length), an *equivalent spectrum* $\bar{\Phi}_E$ has been defined which embeds the effects of this parameter – see Axisa et al (1990) or De Langre and Villard (1998):

$$\bar{\Phi}_E\left(f_R\right)=\frac{\lambda_c}{L_f}\bar{\Phi}\left(f_R\right) \quad (16)$$

and formulation (15) can finally be expressed as:

$$S_{q_nq_n}(f)=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\left|H_n(f)\right|^2\bar{\Phi}_E\left(\frac{fD}{\bar{V}}\right)C_n^2 \quad (17)$$

with the spatial correlation coefficients:

$$C_n^2=2L_f\int_0^{L_f}\left[u(x)^2\phi_n(x)\right]^2dx \quad (18)$$

Equations (17) and (18) will be next used for achieving a consistent time-domain excitation.

3. TIME-DOMAIN FORCE FUNCTIONS

The main and simple idea of the approach developed is to use a set of R uncorrelated point-forces located along the tube, which are generated with spectral properties and amplitudes such that they induce the same modal responses as the original continuous formulation expressed by (17) with (18).

3.1 Equivalent excitation formulation

For a set of random *correlated* point forces, the resulting modal responses read:

$$S_{q_nq_n}(f)=\left|H_n(f)\right|^2\sum_{r=1}^{R}\sum_{s=1}^{R}\phi_n(x_r)\phi_n(x_s)S_f\left(x_r,x_s,f\right) \quad (19)$$

and such excitation would be used if one follows Shinozuka's approach. However, the same result may be obtained much easier assuming that the applied random force set is *uncorrelated*. Then (19) simplifies to:

$$S_{q_nq_n}(f)=\left|H_n(f)\right|^2\sum_{r=1}^{R}\left|\phi_n(x_r)\right|^2\Phi_r\left(x_r,f\right) \quad (20)$$

where $\Phi_r\left(x_r,f\right)$ is the auto-spectrum of the point force applied at location x_r.

We now enforce the condition that modal responses (17) and (20) be the same for all modes of interest. Then:

$$\sum_{r=1}^{R}\left|\phi_n(x_r)\right|^2\Phi_r\left(x_r,f\right)=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\bar{\Phi}_E\left(\frac{fD}{\bar{V}}\right)C_n^2 \quad (21)$$

for modes $n=1,2,...,N$. Based on the previous simplifying assumptions, it is sensible to postulate the same spectral content $\Phi_{exc}\left(f\right)$ for the full set of equivalent point-forces. Furthermore, these should have amplitudes consistent with the local pressure head – in terms of the velocity profile $u(x_r)$ – so that we may write for all point-forces $r=1,...,R$:

$$\Phi_r\left(x_r,f\right)=B_r\,\Phi_{exc}\left(f\right) \quad ; \quad B_r=Au(x_r)^4 \quad (22)$$

where $A>0$ is an unknown coefficient to be computed.

Now, replacing (22) in (21), we obtain:

$$A\Phi_{exc}\left(f\right)\sum_{r=1}^{R}\left[u(x_r)^2\phi_n(x_r)\right]^2=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\bar{\Phi}_E\left(\frac{fD}{\bar{V}}\right)C_n^2 \quad (23)$$

for $n=1,2,...,N$. Identification of the frequency-dependent terms in (23) leads to the spectrum:

$$\Phi_{exc}\left(f\right)=\left(\frac{1}{2}\rho\bar{V}^2D\right)^2\frac{D}{\bar{V}}\bar{\Phi}_E\left(\frac{fD}{\bar{V}}\right) \quad (24)$$

while the amplitude coefficient A must fulfil the condition:

$$A\sum_{r=1}^{R}\left[u(x_r)^2\phi_n(x_r)\right]^2=C_n^2 \qquad (n=1,2,...,N) \quad (25)$$

and we can write equations (25) in matrix form, leading to the least-squares solution:

$$A=\begin{Bmatrix}\sum_{r=1}^{R}\left[u(x_r)^2\phi_1(x_r)\right]^2\\ \sum_{r=1}^{R}\left[u(x_r)^2\phi_2(x_r)\right]^2\\ \vdots\\ \sum_{r=1}^{R}\left[u(x_r)^2\phi_N(x_r)\right]^2\end{Bmatrix}^+\begin{Bmatrix}C_1^2\\ C_2^2\\ \vdots\\ C_N^2\end{Bmatrix} \quad (26)$$

where $\{M\}^+$ is the Moore-Penrose pseudo-inverse of $\{M\}$, defined as:

$$\{M\}^+=\left(\{M\}^T\{M\}\right)^{-1}\{M\}^T \quad (27)$$

Once the parameters of the equivalent force set (22) have been obtained, statistically independent gaussian time-domain realizations $f_{exc}(x_r,t)$ at the R point-forces are generated using the following standard procedure. Each force is the sum of K harmonic components:

$$f_{exc}(x_r,t)=\sum_{k=1}^{K}F_k^{(r)}\sin\left(2\pi f_k t+\theta_k^{(r)}\right) \quad (28)$$

at discrete frequencies $f_k=k\Delta f$, with a zero-centered time-average $\bar{f}_{exc}(x_r)=\langle f_{exc}(x_r,t)\rangle=0$. The component amplitudes $F_k^{(r)}=\left[2\,\Phi_r\left(x_r,f_k\right)\Delta f\right]^{1/2}$ are computed from the force spectra (22) at each frequency f_k, while the component phases $\theta_k^{(r)}$ are random, sampled from a distribution uniform in the range $[0,2\pi]$.

It can be proved that such procedure leads to random gaussian signals which display the target spectral properties – see for instance Shinozuka (1971). The number of components K in (28) should be high enough for an adequate spectral resolution Δf. On the other hand, for a given length T_c of the time-domain computations, we must have $\Delta f\le 1/T_c$ in order to avoid repetitions of the pseudo-random forces within T_c.

In practice, (28) being a rather lazy numerical procedure, computations are speeded replacing this sum by a fast equivalent inverse FFT routine.

3.2 Convergence of the results

As an illustration, Figures 1 and 2 demonstrate the convergence of the obtained modal excitations, for two given velocity profiles, shown in (a) – the first one being uniform and the other localized. These excitations are applied to $N = 50$ modeshapes $\phi_n(x) = \sin(n\pi x / L)$ of a tube with $L = 1.5$ m. Also displayed, in (b), are typical turbulence-simulating equivalent force sets (using 20 forces), their amplitudes being given by the coefficients $B_r = A u(x_r)^4$ which scale the excitation spectrum $\Phi_{exc}(f)$ of the point-forces. Finally, in (c), the global quadratic error δ of the modal excitations, computed from (29), as the number $R = 1 \sim 100$ of random point-forces is increased.

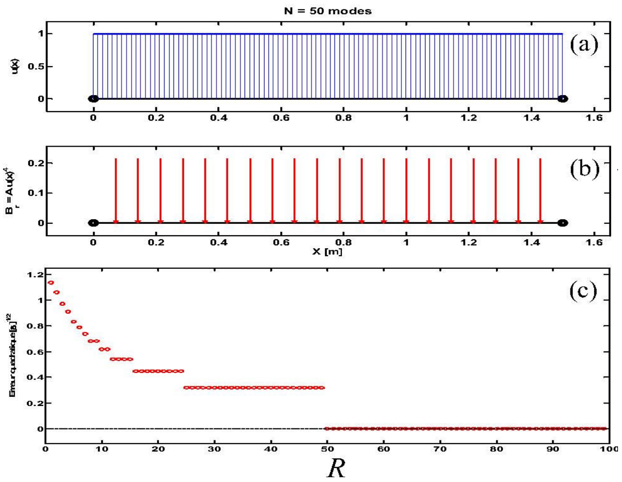

Figure 1: Uniform velocity profile – Convergence of the modal excitations as the number of equivalent excitation point-forces increases.

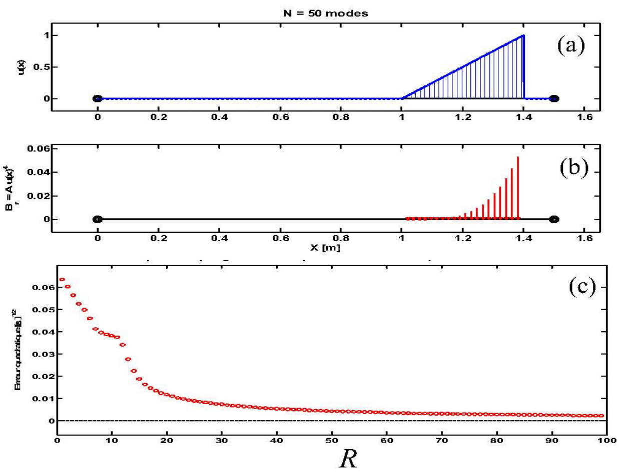

Figure 2: Localized velocity profile – Convergence of the modal excitations as the number of equivalent excitation point-forces increases.

$$\delta = \sum_{n=1}^{N} \left[\left(\sum_{r=1}^{R} \left[u(x_r)^2 \phi_n(x_r) \right]^2 \right) A - C_n^2 \right]^2 \quad (29)$$

These computations show that, for the *uniform* velocity profile, the global modelling error δ of the equivalent excitations becomes nil whenever $R \geq N$. For the *localized* excitation, there is always some residual error in the modal excitations, which nevertheless becomes negligible beyond some value R_δ (typically $R_\delta < N$). Other computations have fully confirmed these trends. Furthermore, as expected, time-domain numerical simulations of the system turbulent responses proved to be insensitive (in the statistical sense) to the number R of point-forces used, provided $R \geq N$ or $R \geq R_\delta$. This demonstrates the robustness of the method.

3.3 Modal forces due to turbulence excitations

It proves enlightening to examine how the modal excitations behave, when computed from the previously described equivalent turbulent model. More specifically, we will look at the correlation between the modal forces, when excitations are due to both uniform and localized velocity profiles. The modal forces $f_n(t)$ are computed from the equivalent discrete excitation model $f_{exc}(x_r, t)$ using the modal projection equation (2):

$$f_n(t) = \sum_{r=1}^{R} \phi_n(x_r) u(x_r)^2 f_{exc}(x_r, t) \qquad (n = 1, 2, ..., N) \quad (30)$$

Given any two modes i and j, the N^2 correlation coefficients between modal forces $f_i(t)$ and $f_j(t)$ are given by:

$$\mathcal{R}_{ij} = \frac{Cov\left[f_i(t), f_j(t)\right]}{\sqrt{Cov\left[f_i(t), f_i(t)\right] Cov\left[f_j(t), f_j(t)\right]}} \qquad \left(-1 \leq \mathcal{R}_{ij} \leq 1\right) \quad (31)$$

with:

$$Cov\left[f_i(t), f_j(t)\right] = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[f_i(t) - \bar{f}_i\right]\left[f_j(t) - \bar{f}_j\right] dt \quad (32)$$

Where the average values of modal forces are $\bar{f}_i = \bar{f}_j = 0 \left(\forall i, j\right)$. Obviously, $\mathcal{R}_{ii} = 1 \left(\forall i\right)$ and $\mathcal{R}_{ji} = \mathcal{R}_{ij}$. We will now address two pertinent cases:

1) *Uniform velocity profile*: As $u(x_r) \equiv 1 \left(\forall r\right)$, one may deduce from (31) and (32):

$$\mathcal{R}_{ij} = \frac{I_{ij}}{\sqrt{I_{ii} I_{jj}}} \quad ; \quad I_{ij} = \begin{Bmatrix} \phi_i(x_1) \\ \vdots \\ \phi_i(x_R) \end{Bmatrix}^T \begin{Bmatrix} \phi_j(x_1) \\ \vdots \\ \phi_j(x_R) \end{Bmatrix} \quad (33)$$

which, accounting for the orthogonality of the tube modeshapes of a axially uniform tube, implies:

$$\mathcal{R}_{ij} = 0 \quad (i \neq j) \quad ; \quad \mathcal{R}_{ij} = 1 \quad (i = j) \qquad (34)$$

meaning that, for uniform flows, all turbulence-induced modal forces are uncorrelated.

2) *Localized velocity profile*: Analysis now produces the following result:

$$\mathcal{R}_{ij} = \frac{J_{ij}}{\sqrt{J_{ii} J_{jj}}} \quad ; \quad J_{ij} = \begin{Bmatrix} \phi_i(x_1) \\ \vdots \\ \phi_i(x_R) \end{Bmatrix}^T \left[U^4\right] \begin{Bmatrix} \phi_j(x_1) \\ \vdots \\ \phi_j(x_R) \end{Bmatrix} \qquad (35)$$

where:

$$\left[U^4\right] = \begin{bmatrix} u(x_1)^4 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u(x_r)^4 \end{bmatrix} \qquad (36)$$

One may notice that, in general, the tube modeshapes will not be orthogonal with respect to matrix $\left[U^4\right]$, so that $J_{ij} \neq 0 \; (\forall i, j)$. Therefore, from (35) and (36):

$$-1 \leq \mathcal{R}_{ij} \leq 1 \quad \Leftrightarrow \quad \left|\mathcal{R}_{ij}\right| \leq 1 \qquad (37)$$

and we conclude that localized turbulence excitations lead to partially correlated modal forces. In the extreme case of a single random point excitation, all modal forces are fully correlated.

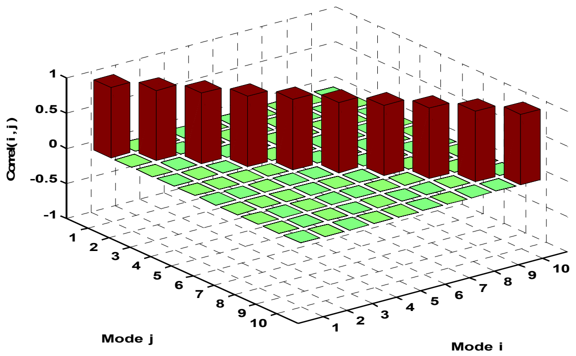

Figure 3: Uniform velocity profile – Correlation matrix between turbulence-generated modal forces.

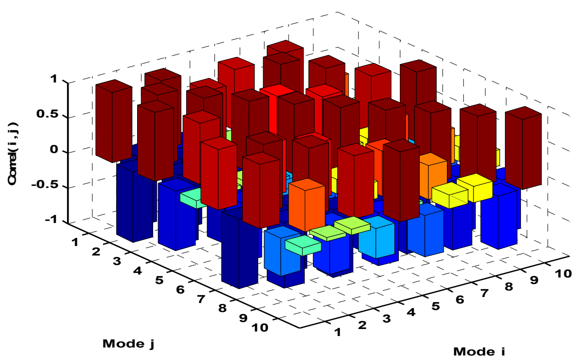

Figure 4: Localized velocity profile – Correlation matrix between turbulence-generated modal forces.

Figures 3 and 4 illustrate the previous results, respectively for the uniform and the localized flows. These results, which stem from the theoretical conclusions (35) and (37), where fully confirmed when processing the time-domain modal forces through equation (31).

4. CONCLUSIONS

We proposed a simple and consistent method for the time-domain simulation of turbulence excitations, which accounts for the space-correlation of the fluctuations and is well suited for highly non-uniform flow fields. Using this approach we demonstrated that the turbulence-induced modal forces are uncorrelated for uniform flows but become partially correlated for localized flows.

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