CRITICAL MASS AND NEW WAKE PATTERNS FOR FREELY RISING AND FALLING SPHERES

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ABSTRACT

We study the dynamics of spheres rising or falling freely through a fluid at, Re *= 450 and 10,000. Although this problem has been the focus of a number of investigations since it was first considered by Newton (1726), the conditions under which a sphere will vibrate are still not understood clearly. Several studies conclude that the dynamics are determined by the mass ratio (or relative density)* m^**, but while some investigators find that all rising spheres (*$m^* < 1$*) vibrate and all falling spheres (*$m^* > 1$*) descend rectilinearly, others have observed oscillation for both rising and falling cases. For vibrating spheres, it is also unclear what types of trajectories may occur. Both in-plane oscillations and helical motion have been interpreted from experimental results.*

At both values of Re *studied, we find that falling spheres descend rectilinearly. In the case of the rising sphere, we find that there exists a critical value of the mass ratio, below which the sphere undergoes large-amplitude oscillations. Despite the difference in the modes of vortex formation at these two Reynolds numbers, associated in part with the instability of the separated shear layer at higher* Re*, a critical mass exists for both cases. For the higher Reynolds number, we find a critical mass of* $m^*_{crit} = 0.61$*, in good agreement with the result for tethered and elastically mounted spheres at similar* Re *(Govardhan & Williamson, 2005). At Re = 450, performing experiments in glycerin-water mixtures to control the Reynolds number, we find a distinctly lower critical mass,* $m^*_{crit} = 0.36$*.*

For both Reynolds numbers, the motion of the vibrating spheres occurs in a single vertical plane, with no helical trajectories observed. Visualizing the wake of a vibrating sphere at this Reynolds number reveals another interesting phenomenon; rather than two alternately signed vortex loops being shed in a cycle, as might be expected based on studies of the flow past fixed spheres, four vortex rings are formed in each cycle of oscillation.

1. INTRODUCTION

Whether a sphere vibrates as it rises or falls through a fluid is of interest in a wide range of practical applications from sedimentation to atmospheric measurements using weather balloons. Vibration is known to affect drag as well as heat and mass transfer. The earliest observation of vibration of a freely rising or falling sphere is reported by Newton (1726), who writes in the *Principia* that inflated hog bladders "did not always fall straight down, but sometimes flew about and oscillated to and fro while falling. And the times of falling were prolonged and increased by these motions." Such classical observations correspond quite reasonably with recent measurements.

More recently, investigations of freely rising or falling spheres have concluded that the mass ratio of the sphere determines when vibration occurs, with lighter spheres oscillating and heavier ones moving rectilinearly.

The experiments of Preukschat at Re = 1000 – 10,000, and direct numerical simulation by Jenny, Bouchet & Dusek (2004) at Re = 200 – 500 found that falling spheres ($m^* > 1$) have a rectilinear trajectory, while rising spheres ($m^* < 1$) vibrate, suggesting that there may be some special significance to $m^* = 1$, such that there is a clear distinction between rising and falling. On the other hand, MacCready & Jex (1964), Reid (1964) and Veldhuis, Biesheuvel, van Wijngaarden & Lohse (2004) observed both rising and falling spheres undergoing large lateral motions, while a number of studies considering only falling spheres have found vibration. With such major differences between the results of these studies, a key question remains: when does a rising or falling sphere vibrate?

2. EXPERIMENTAL METHODS

Our experiments on freely rising and falling spheres were performed in two vertical tanks, a larger one with dimensions 0.4m x 0.4m x 1.5m, and a smaller one measuring 0.2m x 0.2m x 0.9m. Both solid and hollow spheres were used, with diameters, *D*, ranging from 0.2cm to 3.8cm, deviating from perfect sphericity by no more than 1.5%.

The spheres were held in the tank using a hook inside a hollow launching tube, and were released after the fluid settled. Two ranges of Reynolds number were studied, Re ~ 10,000 in the larger tank, and Re = 450 in the smaller tank, where a constant Reynolds number was achieved using mixtures of glycerin and water to control the viscosity.

To allow flow visualization, sodium fluorescein dye was introduced into the wake of the sphere and illuminated with an argon ion laser.

3. CRITICAL MASS RATIO FOR FREELY RISING SPHERES

We begin with experiments using spheres with *Re* ~ 10,000. A falling sphere with m^* = 2.84 descends with only small non-periodic transverse motion in figure 1(a). In the case of a buoyant sphere with m^* = 0.75, for which previous studies predict vibration will occur, we find unexpected dynamics: after undergoing an initial transient that is quickly damped out, the sphere rises rectilinearly. This result, shown in figure 1(b), indicates that contrary to previous observations, some rising

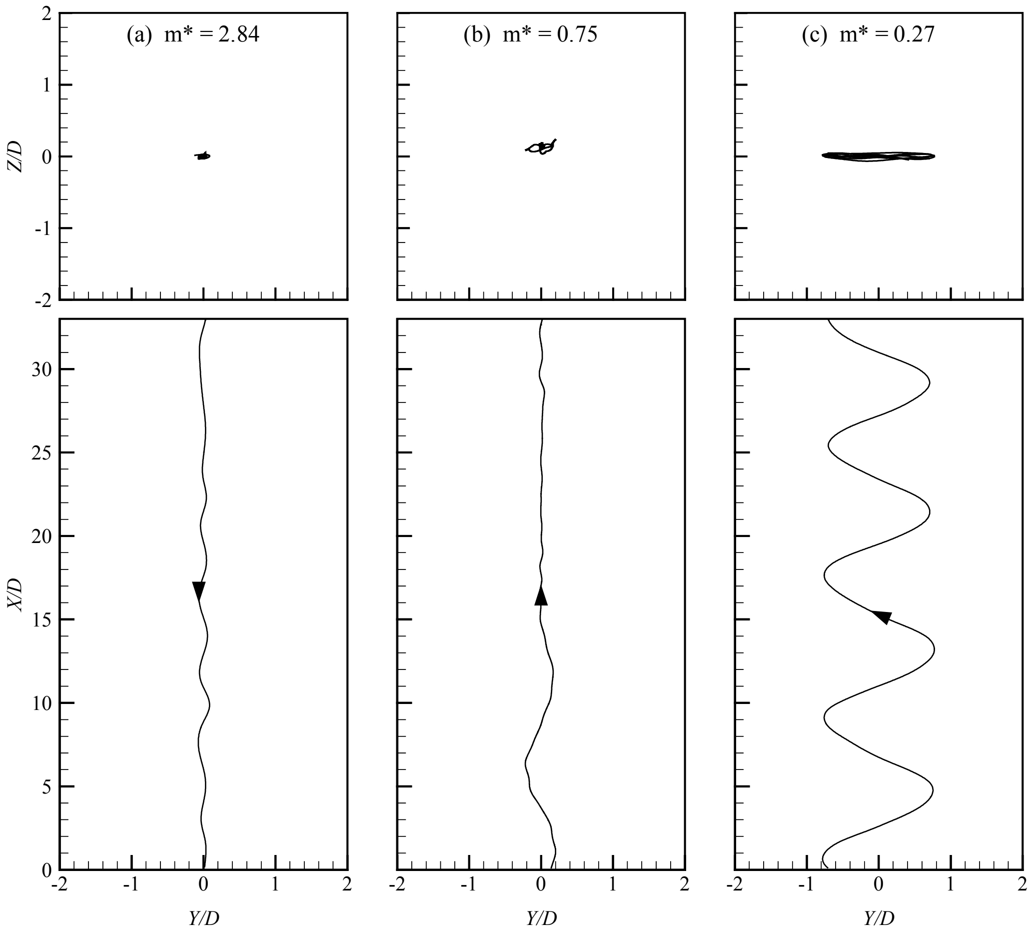

Figure 1: *Trajectories of rising spheres viewed from above (upper row) and from the side (lower row). (a) m* = 2.84. The sphere falls with very small, nonperiodic transverse motion. (b) m* = 0.75. After a brief transient, the sphere rises rectilinearly. (c) m* = 0.27. Very light spheres vibrate in a single plane. Re ~ 10,000.*

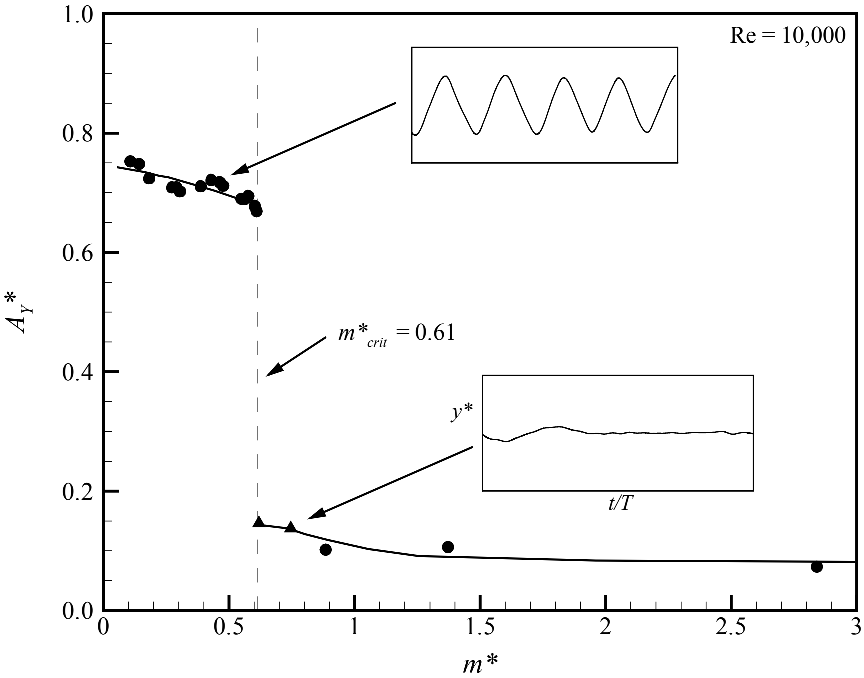

Figure 2: *The critical mass for a rising and falling sphere at Re ~ 10,000 occurs at* $m^*_{crit} = 0.61$*, indicating that some rising spheres do not vibrate. Time histories of* $y^* = y(t)/D$ *are shown for selected* m^**. ●, the sphere quickly reaches a steady state; ▲, transient cases, such as the trajectory shown in figure 1(b).*

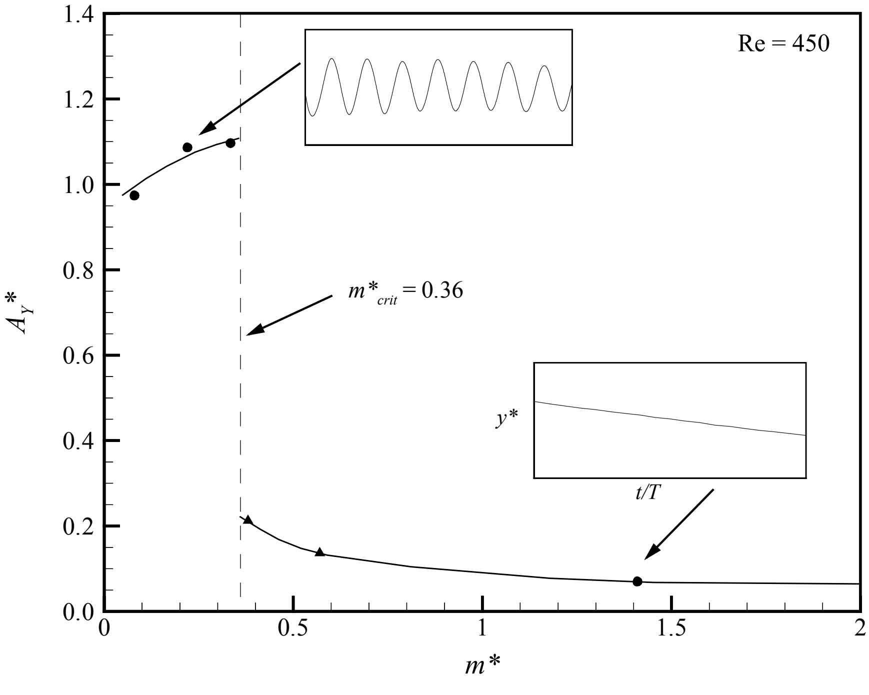

Figure 3: *At Re = 450, the critical mass for a rising and falling sphere is* $m^*_{crit} = 0.36$*, distinctly lower than the higher Reynolds number case, but leaving a wide regime of buoyant mass ratios where vibration exists. ●, steady state; ▲, transient.*

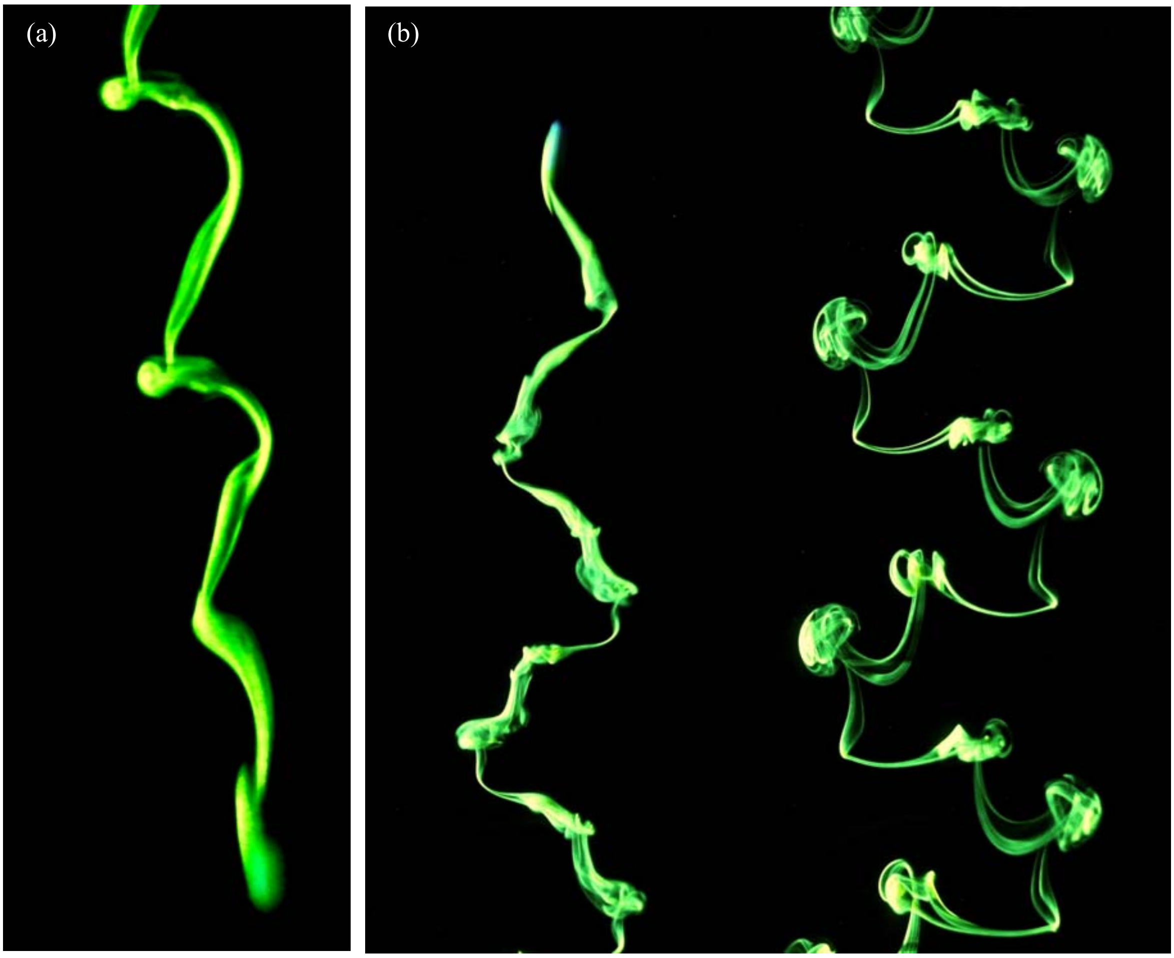

Figure 4: *(a) A single-sided chain of vortex loops in the wake of a falling sphere in rectilinear motion, $m^* = 1.41$. (b) In the wake of a very light rising sphere, $m^* = 0.08$, four distinct vortex structures are created in each cycle of oscillation, twice as many as have previously been observed in flows past static or elastically mounted spheres. $Re = 450$.*

spheres do not vibrate. As a result of conducting an extensive set of such experiments, it appears there is nothing inherently special about $m^* = 1$. With a sufficient reduction of the mass ratio, however, buoyant spheres begin to undergo large-amplitude oscillations, as shown in figure 1(c) for $m^* = 0.27$. Although it is not evident from the figure, if the mean rising velocity is subtracted, there also exists streamwise vibration with an amplitude $A_X^* = 0.14$. The top view of the trajectory shows that although the sphere is free to move in three-dimensions, the oscillation is confined to a single plane, the orientation of which is determined by the direction of the initial velocity.

Measuring the oscillation amplitude of spheres at many different mass ratios, plotted in figure 2, we find a critical value of the mass ratio below which a sphere will vibrate, $m^*_{crit} = 0.61$. Consequently, there is a broad range of mass ratios over which buoyant spheres will rise without vibration. We attribute this difference with previous studies to the sensitivity of the sphere dynamics to experimental conditions. In particular, spheres heavier than the critical mass were very sensitive to small disturbances in the fluid that could induce transient motions. To minimize these disturbances and ensure that the fluid was truly quiescent, a settling time of at least two hours between experiments was required. It is also noteworthy that the value of the critical mass found for rising spheres agrees well with the estimate of the critical mass made by Govardhan & Williamson (2005) for elastically mounted and tethered spheres, where they find $m^*_{crit} \approx 0.6$.

A similar set of experiments was performed for spheres at $Re = 450$. At this Reynolds number, we find an even wider range of mass ratios where rising spheres do not vibrate, corresponding to a critical mass $m^*_{crit} = 0.36$, shown in figure 3. Like the higher Reynolds number case, vibrating spheres

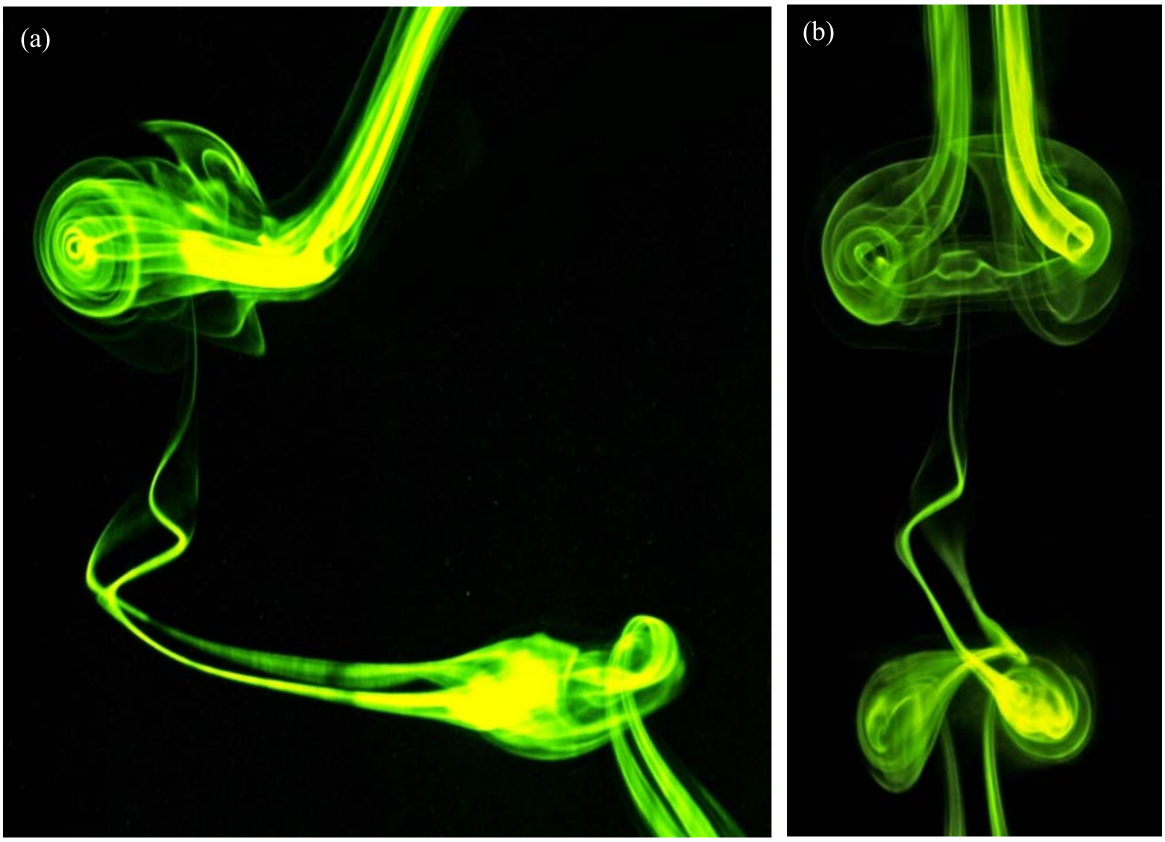

Figure 5: *Detailed view of the wake of a vibrating sphere. (a) A half-cycle of oscillation results in the creation of a stronger, primary structure (upper left) and a weaker, secondary structure (lower right). In between these structures, the trailing vortex pair crosses over (lower left). Wake viewed normal to the plane of oscillation. (b) Simultaneous view of the wake viewed parallel to the plane of oscillation.*

would undergo periodic, large amplitude oscillation in a single plane, and spheres slightly heavier than the critical mass showed transient small-amplitude behavior. The heavier spheres moved rectilinearly, but rather than being vertical, their trajectories were slightly oblique. Although the Reynolds number of the sphere is similar to that of spiraling bubbles, no evidence of spiral trajectories was found.

4. VORTEX DYNAMICS IN THE WAKE OF FREELY RISING AND FALLING SPHERES

Several studies have examined the wakes of fixed spheres, however much less is known about the vortex dynamics behind unrestrained spheres, where the vortex dynamics can interact with the body motion. Using laser-induced fluorescence, we find that in the case of rectilinear trajectories, the sphere sheds a single-sided chain of vortex loops (figure 4a), resembling the wake of a fixed sphere observed by Sakamoto & Haniu (1990) at similar Reynolds numbers. Since the wake is single-sided, there is a mean transverse force that causes the trajectory to deviate from the vertical. From the angle of the trajectory, this force is found to be $C_Y = 0.04$. Such an asymmetric wake pattern would not be expected from the oscillating sphere, whose trajectory is periodic. One might expect to have a double-sided chain instead, as has been found for vibrating tethered spheres by Govardhan & Williamson (2005). However, the actual pattern, shown in figure 4(b) is unlike any wake mode found previously for either fixed or vibrating spheres, in which there are four distinct vortex structures formed in each cycle of oscillation.

Due to the small size ($D \approx 0.2$ cm) and high velocities ($U \approx 20$ cm/s)of the freely rising spheres, it is difficult to precisely identify these structures and see how they are formed. To provide a better understanding of these phenomena, we performed further experiments in a computer-controlled towing tank, in which trajectories measured from the freely rising spheres were matched in a towing tank. This allowed for the use of much larger spheres ($D \approx 3.81$ cm) that could be towed at a lower velocity ($U \approx 1$ cm/s), while maintaining the same Reynolds number. With these experiments, we were able to achieve superior spatial and temporal resolution for the visualizations.

Images obtained from these towing tank experiments are shown in figure 5. In each half cycle, a large primary structure is shed. This structure appears to develop into a vortex ring (the upper structure in figure 5 a,b), while the counter-rotating vortex pair subsequently develops into a second, weaker structure (figure 5 a,b, lower). Immediately preceding the formation of this secondary structure, the two counter-rotating streamwise vortices in the pair are found to cross over one another, providing a mechanism for the change in sign of the streamwise vortex pair as the body moves from one half cycle to the next. Evidence for the vortical structure is provided by extensive PIV measurements, which will be presented at the conference.

5. CONCLUSIONS

While several previous studies have found that the boundary between the vibrating and rectilinear regimes for freely rising and falling spheres occurs at $m^* = 1$, our experiments indicate that this mass ratio has no special significance. Instead, we find values of the critical mass, $m^*_{crit} = 0.38$ at $Re = 450$ and $m^*_{crit} = 0.61$ at $Re = 10{,}000$. Consequently, there exists a wide range of mass ratios where rising spheres can ascend without vibration. Since the spheres were observed to be very sensitive to small disturbances in the surrounding fluid, these results are highly dependent on careful control over the experimental conditions: the presence of significant background disturbances in a facility would lead to vibration where it should otherwise not occur, and hence to incorrect deductions.

We performed studies on the wake of freely rising and falling spheres at $Re = 450$, and discovered a new mode of vortex formation, in which a vibrating sphere sheds four vortex structures per cycle of oscillation, twice as many as have been observed in the wakes of tethered and fixed spheres. Experiments in a towing tank, in which rising sphere trajectories are precisely replicated, show that this wake comprises a primary structure, which originates as a vortex loop shed from the sphere, and subsequently develops into what appears to be a vortex ring, and also a secondary structure, which evolves from the trailing streamwise vortex pair. Further detailed PIV measurements will show the precise vorticity within the primary and secondary structures over a range of Reynolds numbers.

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