EFFECTS OF INTERMEDIATE SUPPORTS AND A FLOW MIXER ON THE EIGENVALUE OF A MULTI-SPAN CYLINDRICAL ROD SUBJECTED TO AXIAL AND PERIODIC MIXED FLOWS

Kang-hee Lee
Innovative Nuclear Fuel Technology Development, KAERI, Daejon, Korea
Hyung-kyu Kim
Innovative Nuclear Fuel Technology Development, KAERI, Daejon, Korea
Chang-whan Shin
Innovative Nuclear Fuel Technology Development, KAERI, Daejon, Korea
Heung-seok Kang
Department of Mechanical Engineering, Ecole Polytechnique de Montreal, Montreal, Canada

ABSTRACT

Long and slender body with or without flexible supports under severe operating condition can be unstabilized even by the small cross flow. Turbulent flow mixer, which actually increases thermal-hydraulic performance of the nuclear fuel by boosting turbulence, disturbs the flow field around the fuel rod and affects dynamic behavior of the nuclear fuel rods. Few studies on this problem can be found in the literature because these effects depend on the specific natures of the support and the design of the system. This work shows how the dynamics of a multi-span fuel rod can be affected by the turbulent flow, which is discretely activated by a flow mixer, and the looseness of intermediate supports. By solving a state-space form of the eigenvalue equation for a multi-span fuel rod system, the critical velocity at which a fuel rod becomes unstable was established. Based on the simulation results, we evaluated how stability of a multi-spanned nuclear fuel rod with mixing vanes can be affected by the coolant flow in an operating reactor core.

1. INTRODUCTION

The flow velocity in most engineering systems is usually far below that of the critical velocity at which an oscillating body in an axial flow becomes unstable. However a long and slender body, such as a fuel rod, with or without supports under a severe operating condition can be unstabilized even by a small cross flow. From this parallel respect, a turbulent flow mixer (called the mixing vane, see Figure 2) attached to a downstream of the spacer grid disturbs the flow field around a fuel rod and affects the dynamic behavior of these nuclear fuel rods; it actually increases the thermal-hydraulic performances of the nuclear fuel by boosting the turbulence. Few studies on this problem can be found in the literature because these effects depend on the specific natures of the support and the design of the system and are difficult to be modeled quantitatively. Thus, it is necessary to know how this mixing effect due to the flow mixer can change the dynamic characteristics of the nuclear fuel rod.

Mathematical modeling of the hydrodynamic force acting on a body due to the flow mixer can be derived by generalizing representative design features and simplifying the flow downstream of the flow mixer while confining assumptions as follow: 1) a specific design of the flow mixer can be characterized by the experimental model test and resultant empirical parameters, 2) the flow down stream can be assumed as a combination of the swirl flow and its dissipation. And we assume the swirl flow as the unique, biggest component in the flow downstream of the mixer and concentrate on its effect as a result of the flow mixer of the conventional nuclear fuel rod.

Nuclear fuel rod is a multi-span, flexible cylindrical rod which is supported by spacer grids which are spaced along the rod length. Spacer grids provide a frictional grip with their spring elements so as not to allow a fuel rod to move arbitrary in any direction. The stability for a flow-induced vibration of a fully supported fuel rod is not a concern because its critical velocity is far from the operating flow velocity in the reactor core (Kang, 2003). But, as the residence period inside reactor increases, the gripping force can be lost partially or entirely due to the thermal and irradiation-induced degradation. This looseness of support issues a stability problem and a nonlinear dynamics.

On analyzing the dynamics of a flexible cylinder submerged in a flow, Paidoussis (1973, 1976, 1998,

2004) had accomplished remarkable works in this field of a flow-induced vibration. Au-Yang (2001) has proposed a semi-empirical methodology applicable to the analysis of the behaviors of a fuel rod. Because the exciting forces by a coolant flow are severe turbulence as well as the functions of the environmental parameters in a reactor core, theoretical analysis can be confined to a simple problem. Based on Paidoussis's works, Chen and Wambsganss (1972) studied a beam with more general, arbitrary boundary conditions. Based on the potential flow theory, Chen (1975) and his colleagues (Chung and Chen, 1977; Yeh and Chen, 1978) also computed the hydrodynamic mass of a group of cylinders and a coupled vibration of a fuel bundle was discussed.

This work shows how the dynamics of a multi-span fuel rod can be affected by the turbulent flow, which is discretely activated by a flow mixer, and the looseness of intermediate supports. By solving a generalized eigenvalue equation of a multi-span fuel rod system, the critical velocity at which the fuel rod becomes unstable was established. Based on the simulation results, we evaluated how stability of a multi-spanned nuclear fuel rod with mixing vanes can be affected by the coolant flow in an operating reactor core.

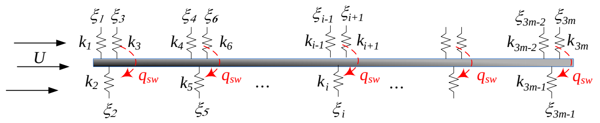


Figure 1: Multi-spaned, elastically supported beam subjected to axial and periodic mixed flows.

2. BASIC EQUATION AND SWIRL MODELING

2.1 Basic Equation

By assuming a small lateral motion and uniform incompressible flow, the nondimensional linear differential equation of motion applicable to a vibration analysis of a nuclear fuel rod can be founded in Paidoussis (2004), which was derived for a flexible beam in a bundle and a confined axial flow(see Figure 1). For a simplicity, the viscoelastic property of the beam material, gravity, axial tension and external pressure are ignored here such that

$$\eta'''' + \ddot{\eta} + u^2\eta'' + 2u\chi^{1/2}\dot{\eta}' + a_1\varepsilon u(\chi^{1/2}\dot{\eta} + u\eta') + a_2\varepsilon\chi^{1/2}\dot{\eta} + f_{sw} + \sum_{j=1}^{3m}\bar{k}_j\,\eta\,\delta(\xi-\xi_i) = 0. \qquad (1)$$

where (′) and (.) indicate the derivative with respective to dimensionless variables of a coordinate variable ξ (=x/L) and a time τ (=$[EI/(m+M)]^{1/2}t/L^2$), respectively. The various symbols in Eq. (1) are: η (=v/L) is a dimensionless lateral displacement of the beam, u (=$[M/EI]^{1/2}UL$) is a dimensionless flow velocity , χ (=$[M/(M+m)]^{1/2}$) is a mass ratio of added or virtual mass M , ε (=L/D) is a ratio of length to diameter, U is a uniform axial flow velocity, EI is a bending stiffness of the beam, a_i (=$2C_i/\pi$) are coefficients related to the viscosity in the normal direction to the beam length, f_{sw} is a dimensionless hydrodynamic force due to the swirl flow downstream of the mixer, discussed in the next section. The last term represents the elastic supports force ($\bar{k}_i$ being a dimensionless spring constant) by the discretely located spacer grids; δ is the Dirac delta function which is activated at a supports location ξ_i. Inviscid and viscous hydrodynamic forces of the third to the sixth terms in Eq. (1) were derived with a simple mathematical form by a slender body approximation (Lighthill, 1960) and Taylor's unconfined flow relationship (Taylor, 1952).

It is not easy to solve equation (1) analytically. So, a discretized model should be introduced to compute the eigenvalues approximately. By applying a variational formulation, a finite element approximation(Vendhan and Bhattachryya, 1997) and several mathematical manipulations, the matrix form of a state-space equation for the eigenvalue problem can be derived as

$$\{\dot{D}\} = [A]\{D\}$$

where

$$\{D\} = [\eta, \dot{\eta}]^T, [A] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \qquad (2)$$

Elements of each matrix of [M], [C] and [K] in equation (2) are written as,

$$M_{ij} = c_{1,ij}\int_{-1}^{1} N_j'' N_i \, d\xi$$

$$C_{ij} = \int_{-1}^{1} [c_{2,ij}N_j' + c_{3,ij}N_j] N_i \, d\xi$$

$$\begin{aligned} K_{ij} = &\int_{-1}^{1} [c_{4,ij}N_j''N_i'' + c_{5,ij}N_j'N_i' + c_{6,ij}N_jN_i']d\xi \\ &+ \bar{k}_i N_j(\xi_i)N_i(\xi_i) + \int_{*} \bar{k}_{sw} N_j N_i \, d\xi \end{aligned} \qquad (3)$$

The symbols used in Eq. (2) and (3) are as follows: D is a state variable defined by v and $\dot{v}$, [0] and [I] are zero and identity matrix, respectively, N is a cubic polynomial shape function as a function of the non-dimensional distance in a typical finite element, $c_{i(=1\sim3)}$ are coefficients related to the mass, the damping from coriolis and viscous forces), $c_{i(=4\sim6)}$ are coefficients related to a stiffness from a centrifugal, two viscous hydrodynamic forces, $\bar{k}_i$ is a dimensionless spring constants of the discrete grids supports, $\bar{k}_{sw}$ is a dimensionless stiffness of the elastic foundation due to the swirl effects. The symbol (*) indicates elemental integration over affected region by the flow mixer; $x < 40D_h$, where x is a distance from the mixer and D_h is the hydraulic diameter.

Dynamic stability of the flexible beam according to the flow velocity is determined by the sign of the real part of the eigenvalue; However, only the first loss of a stability can be predicted from a linear equation. If the real part of an eigenvalue is negative, the beam is stable (in linear manner). Otherwise, the beam is unstable; if an imaginary part of the eigenvalue is a non-zero, a divergence, static instability or buckling through loss of the restoring system stiffness, will happen in the rod, if an imaginary part of the eigenvalue is zero, a flutter will appear.

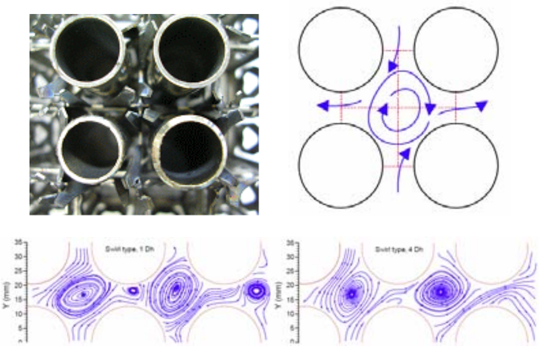


Figure 2: *Mixing vane, flow distribution in a subchannel downstream of the mixing vane.*

2.2 Swirl modeling

Mixing vane or flow mixer attached to the downstream of a spacer grid generates a strong turbulence in a subchannel and promotes a flow mixing among subchannels. Various types of mixing vanes have been developed to produce swirling flows in subchannels as well as a crossflow between subchannels. The shapes of the mixing vane have been improved to strengthen turbulence and a cross flow mixing (Chang, 2006; Yang et al, 1998; Rehme, 1987). Figure 2 shows a typical flow velocity distribution in a subchannel downstream of a vane with a specific bent angle. The swirl flow exponentially decays as the distance from the vane increases.

Based on the assumption of a incompressible, laminar flow and a plane vibration, Langthjem and Nakamura(2006) showed that the swirl flow generated in an annular flow in a narrow passage in-between acts as an elastic foundation (continuously and distributed spring supports) with a negative stiffness; Its magnitude is proportional to square of the mean circumferential flow rate. By using a dimensionless stiffness coefficient ($\bar{k}_{sw}$), a swirling flow (q_{sw}) and some mathematical manipulations, the mathematical form of a hydrodynamic force due to a swirl flow can be written as

$$f_{sw} = \bar{k}_{sw}\eta = \theta(\rho, D_h, h)q^2{}_{sw}\eta \qquad (4)$$

where coefficient θ is a function of the fluid density, the rod diameter and the inter-gap distance between neighboring rods. Their functional relationship can be found in Langthjem (1999, 2006). However the flow through the fuel elements is turbulent, the induced swirling flow boosts a three-dimensional motion of a rod. These turbulence and three-dimensional vibrations will complicate an analysis considerably. To obtain simple, analytical results, the present study uses the assumptions of a laminar flow and plane vibrations.

The swirling flow in equation (4) can be defined by the swirl mixing factor (S_M) which is the ratio between the inlet uniform axial flow and the swirling flow integrated along the centerline of a subchannel (In, 2000; Kim, 1996). Swirl mixing factor decays exponentially to nearly zero as the distance x increases from the vane; $S_M = S_0 e^{-\beta x/d}$, where β is a swirling decay rate, d is the hydraulic diameter. Swirl flow can be uniquely defined by the S_0 and β. Swirl mixing factor (S_M) is defined as:

$$S_M = \int_0^R r^2 V_{lat} dr / \{R\int_0^R rU^2 dr\} \qquad (5)$$

where U is a uniform axial flow velocity, V_{lat} is a lateral flow velocity and R is a radius of the circular section or a hydraulic radius of the subchannel test section. Decay rate of S_M depends primarily on the angle of the vane root and the shape of the outer confinement channel.

3. SIMULATION AND RESULTS

To investigate the effect of 1) the swirl flow generated discretely by the flow mixer and 2) severely supported boundary condition due to the support looseness on the dynamic stability of the fuel rod, free vibration analysis for the fully supported fuel rod was performed. The two cases of swirl-mixing parameter were considered from the experimental model test as discussed in (In, 2000).

One has a higher decay rate and a little lower initial swirl-mixing factor (S_0) than those of the other as shown in Figure 3. For the typical vane angle (30°), the decay rates for two swirl models are 0.03 and 0.06, respectively. The higher decay rate is due to not only the decrease of the lateral flow distribution by the surrounding rods but also the cross flow among the neighboring subchannels.

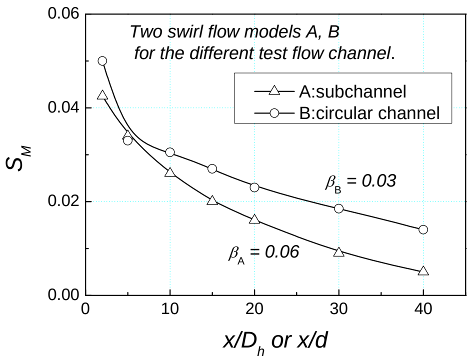


Figure 3: *Two swirl flow models characterized by β and S_0 (Vane angle 30°).*

Owing to a relaxation by thermal and neutron irradiation effect in a reactor core, the spring force of supporting the fuel rod can be lost even at the early stage of a fuel life. It results in an intermediate gap at the supporting locations along fuel rod length. The possibility of a support looseness due to an in-core thermal and neutron irradiation is much lower at the bottom or top grid than that at the middle grids because of the power distribution along the fuel rod length and the material resistance of the bottom grid to an irradiation (Kim, 2004; Park, 2007). So, two cases of a supporting condition are considered: one has a spring support at both ends (like a simply supported beam) and the other has one spring support at the bottom grid (like a cantilever beam). These two extreme supporting conditions are based on the fact that a different material for the top/bottom grids is used to reduce spring force degradation.

3.1 The swirl effect on the stability of fully supported rod

Figure 4 shows the Argand diagram for the eigenvalue for the fully supported rod according to an increase of the dimensionless flow velocity up to 28.7(120 m/s). It shows the effect of a swirl flow on the stability (critical velocity) of a fully supported fuel rod. The boundary condition of a fuel rod is actually free-free. But series of elastic support make the system nearly conservative. So, divergence instability occurred in the case 1 and case 2. It is interesting that flutter type instability can occur in nearly conservative system due to the intermediate swirl effect as case 3. This may be caused by the loss of balance between a restoring elastic support force and a devastating hydrodynamic force due to the flow mixer. The leaned circle in the dotted line in Figure 4 before the instability is probably resulted from the mode crossing or exchange.

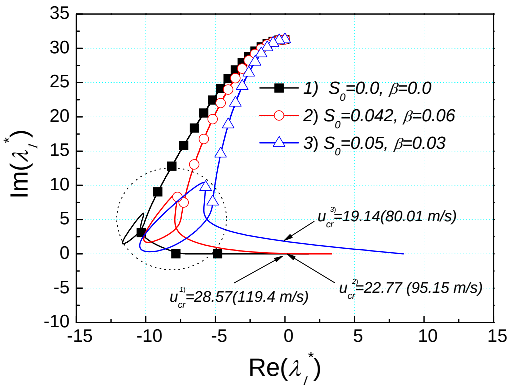
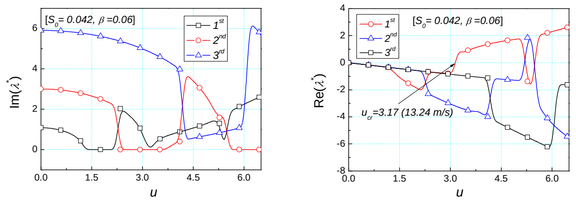


Figure 4: Argand diagram of the first eigenvalue for the fully supported fuel rod; 1) the swirl effect is neglected, 2) S_0=0.042, β=0.06 (result from the subchannel test) and 3) S_0=0.05, β=0.03 (result from circular channel test).

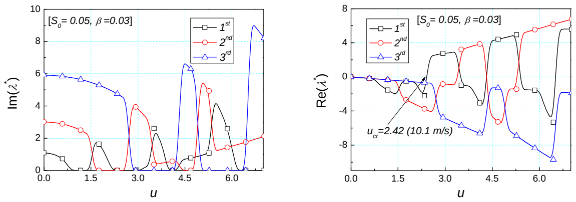
By comparing the critical velocity for the cases of 1), 2) and 3), the consideration of the swirl flow clearly reduces the critical flow velocity of the system by 19.14 (80.01 m/s) (about 33 % reduction); but, it is still far from the operating flow velocity. So, there is no possibility of instability in the case of the fully supported rod under the core flow condition (Park, 2007). As the flow velocity rises, the imaginary part (natural frequencies of the rod) of eigenvalues decreases gradually and the rate of the frequency decrease becomes higher, then drops in a high flow region eventually. This means that the reduction of the system stiffness according to the flow increase has a different rate for the range of the flow. The real part (system damping) of the eigenvalue linearly increases according to the flow velocity until a sudden change appears at around the onset of the instability.

3.2 Extreme support condition due to an intermediate support looseness

Figure 5 illustrates the imaginary and the real part of the lowest three eigenvalues of the elastically supported fuel rod at both ends according to the nondimensional flow velocity. This indicates the characteristics of the natural frequency and a damping of the fuel rod according to the two swirl mixing parameters and the flow velocity. As the flow velocity increases, the frequency of the fuel rod decreases gradually to zero. Then, both frequency and damping of the rod are changed suddenly while a mode crossing or exchange occurs at a certain flow velocity. The critical velocities for the two swirl mixing parameters are 3.17 (13.24 m/s) and 2.42 (10.1 m/s), respectively. Those are considerably reduced from the 5.64 (23.57 m/s) of the swirl effect not being considered.



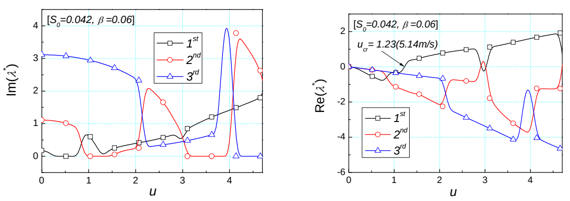
(a) (b)



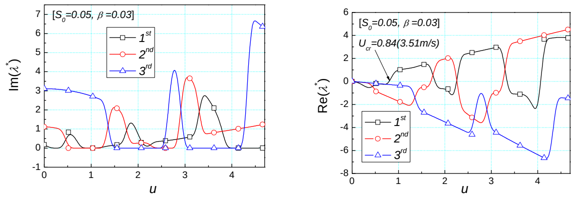
(c) (d)

Figure 5: Imaginary and real value of the lowest three eigenvalues for the both ends spring supported rod; (a), (b): S_0=0.042,β=0.06 from the subchannel test, (c), (d):S_0=0.05,β=0.03 from the circular channel test.

Fig. 6 indicates the results of the other case (nearly cantilever type boundary condition) of the extreme supporting condition. It also shows the lowest three eigenvalues of the fuel rod according to the two swirl mixing parameters and the flow velocity. The natural frequency and damping, with the flow velocity, change in a similar manner to the other extreme support case, but the critical velocity is considerably lower than that of the previous boundary condition. The critical velocities for the two swirl mixing parameters are 1.23 (5.14 m/s) and 0.84 (3.51 m/s), respectively. Those are considerably reduced compared to 4.07 (17.03 m/s) of the swirl effect not being considered.



(a) (b)



(c) (d)

Figure 6: Imaginary and real value of the lowest three eigenvalues for the bottom-end supported rod;(a), (b): S_0=0.042, β=0.06 from the subchannel test, (c), (d):S_0=0.05,β=0.03 from the circular channel test.

4. CONCLUSION AND SUMMARY

In this study, the effect of a swirl flow generated by a flow mixer on the dynamics of a fuel rod was investigated through a free vibration analysis of a fully supported fuel rod. By modeling the swirl flow as an elastic foundation with a negative stiffness, the stability of a severely supported fuel rod with support looseness was also evaluated. Findings from the analysis are summarized as:

1) The critical velocity of the fully supported fuel rod is 20.3 ~ 33 % lower (80.01 m/s) than that of the case when ignoring the swirl effect. But the critical velocity is still far from the operating flow velocity 0.96 ~ 1.44 (4~6 m/s); There is no possibility of instability in the case of the fully supported rod under the core flow condition.

2) Series and intermediate swirl effect can alter type of instability, i.e. divergence or flutter, for the nearly conservative system of elastically supported ends.

3) Because the swirl flow generated by the flow mixer lowers the critical velocity of the axial flow, it is necessary to consider in a dynamic analysis of a fuel rod to compensate for underestimates of the critical velocity.

4) For the two severely supported fuel rods, the critical velocity becomes close to, even lower than the operating flow velocity. It is more necessary to consider the effect of a flow mixer in the case of a severely supported fuel rod due to support looseness.

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