TRANSITION OF FLOW-INDUCED BRIDGE GIRDER OSCILLATIONS FROM RESONANCE TO CRITICAL AND POST-CRITICAL BEHAVIOUR

Rüdiger Höffer & Stefan Lachmann
Ruhr-Universität Bochum, Faculty of Civil Engineering, Building Aerodynamics Laboratory, Bochum, Germany

Stanislav Pospíšil
Academy of Science of the Czech Republic, Institute of Theoretical and Applied Mechanics, v.v.i. Prague, Czech Republic

ABSTRACT

The fluid-structure interaction of turbulent wind is investigated with regards to the transition from pre-critical to post-critical response state. In the frequency-domain, aeroelastic forces due to aerodynamic stiffness and damping are commonly linked to linear terms of displacements and velocities, with so-called flutter derivatives as connecting parameters. Validity of the linearized model and its non-linear enhancement is discussed. Different interpretations of the self-excited forces, used with different experimental techniques for determination of flutter derivatives, are reviewed. The aeroelastic behaviour is illustrated by data from wind tunnel tests of a bridge girder and on a bluff bridge-like cross-section. The article examines the adaptability of the non-linear model under conditions of drastical amplitude growth. The principal mechanical modes of heave and torsion are decoupled. The coupling of the complete aeroelastic system is caused by the flow-structure interaction forces.

1. INTRODUCTION

Slender line-like structures, such as long cable-supported bridges and light footbridges have typically low eigen-frequencies and low damping, and are therefore sensitive to wind loading. Once such a structure starts to vibrate, a complex interaction between the moving boundary and the airflow takes place, which may either effectively attenuate or reinforce the driving force of the wind. Because of the absence of closed forms of analytical solutions for flow-induced pressures and forces on complicated shapes of box- or beam/plate or truss girders as in Figure 1, fluid-structure interaction in case of bridges is normally assessed experimentally, in wind tunnel investigations with a sectional model. The results obtained are then projected to full-scale conditions, using mathematical models of various levels of complexity, e.g. by considering a pair of most important vertical and torsional modes, or carrying out a multi-mode aeroelastic analysis, see Scanlan (1992).

Figure 1: *Section model of the bridge girder.*

From the theoretical point of view, the fluid-structure-interaction leads to the origin of non-conservative forces contributing the to stiffness matrix and in the same time to the origin of forces influencing the damping matrix in linear and non-linear way. On the linear level two parallel, time and frequency ways can be formulated and investigated, see e.g. Caracoglia (2003). The frequency domain analysis is based on a well known approach. In the time domain formulation of self-excited forces on a bridge deck, indicial functions are adopted, see e.g. Costa (2007). The majority of the models have however either obvious or hidden linear character being based on various types of convolution formulations or other superposition related principles. This is the reason why the analyses have revealed relatively considerable diversity of conclusions in the basic terminology as well as in results based on experimental studies. Therefore it is important, regarding the reliability of the system, to analyse also the post critical system behavior, which cannot be represented using the linear approach only. Non-linear processes in the post critical regime are decisive from the point of view of a possible restabilization of the system due to non-linear forces, while linear configuration would lead to infinitely rising response due to positive real part of at least one eigen-value of the system matrix.

2. AEROELASTIC EQUATIONS

Fluid-structure interaction of turbulent flow and oscillating sectional bridge models of width B (see Figure 1) with two degrees of freedom z (heave) and θ (torsion) is often represented as a coupled system of inertial forces $\mathbf{F}_I(t)$, viscous damping forces $\mathbf{F}_C(t)$ and elastic stiffness forces $\mathbf{F}_K(t)$ of the structural part, buffeting forces $\mathbf{F}_B$ due to wind gusts including resonance and aeroelastic forces $\mathbf{F}_{ae} = [L_{ae} \quad M_{ae}]^T$ due to flow-structure interaction as shown in following equation:

$$\begin{aligned} \mathbf{F}_I + \mathbf{F}_C + \mathbf{F}_K &= \\ \mathbf{M}\cdot\ddot{\mathbf{u}}^T + \mathbf{C}\cdot\dot{\mathbf{u}}^T + \mathbf{K}\cdot\mathbf{u}^T &= \\ \mathbf{F}_B(t) + [L_{ae}(\mathbf{u},\dot{\mathbf{u}},t), M_{ae}(\mathbf{u},\dot{\mathbf{u}},t)]^T & \quad (1) \end{aligned}$$

where $\mathbf{u} = (z, \theta)$ is the vector of generalized displacements. In a mixed frequency and time-domain representation, the aeroelastic forces in vertical directions L_{ae} and the aeroelastic moment M_{ae} due to aerodynamic stiffness and aerodynamic damping contributions are commonly linked to linear terms of girder displacements and velocities via so-called flutter derivatives. The flutter derivatives are non-dimensional functions of reduced wind speed $u_{red} = U/fB$, or reduced frequency, $k = 2\pi fB/U = 2\pi/u_{red}$.

The aeroelastic strip forces after Scanlan (1992) for a two-dimensional representation are listed in the following equations:

$$L_{ae} = qB\left[\frac{kH_1^*}{U}\dot{z} + \frac{kH_2^*B}{U}\dot{\theta} + k^2H_3^*\theta + \frac{k^2H_4^*}{B}z\right] \quad (2)$$

$$M_{ae} = qB^2\left[\frac{kA_1^*}{U}\dot{z} + \frac{kA_2^*B}{U}\dot{\theta} + k^2A_3^*\theta + \frac{k^2A_4^*}{B}z\right] \quad (3)$$

Here, q is the mean stagnation pressure in bridge height. According to this formula, the relationship between structural deformations, z and θ, and velocities, $\dot{z}$ and $\dot{\theta}$, in the two dimensional case, and the associated aeroelastic lift L_{ae} and the overturning moment M_{ae} per unit length can be formulated in terms of the flutter derivatives H_i^* or A_i^*, $i = 1, 2\ldots 4$.

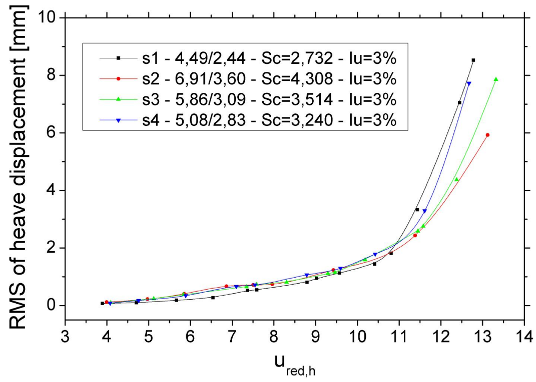

Figure 2: *Standard deviation of the heave amplitudes over the reduced velocities* $u_{red,h} = u/(Bf_h)$

Such solution, however, is linear and can be used to investigate the transition and especially post-critical state only with certain limitations. It is also mathematically problematic, as it works as the combined time/frequency system. One of the possibilities to avoid this, is to consider the lifting force and the moment as a certain functions of generalized displacement z, rotation θ, and their derivatives. For the linear parts expressions as for example in equation (2) can be used. The non-linear parts can be assumed in a form of a third degree polynomial. This represents a natural extension of the linear approach multiplying that by an Eulerian homogeneous function of the second degree. This formulation results from the experience with experimental investigations in a wind channel performed in post-critical regimes. With the adoption of the approach theoretically analyzed in Náprstek et al (2008) the the formulas for lift and moment can be written as:

$$\begin{aligned} L_{ae} = qBm\,(1 - \beta_{zz}z^2 - \beta_{z\theta}\theta^2)\,\cdot \\ (b_{zz}\dot{z} + b_{z\theta}\dot{\theta} + c_{zz}z + c_{z\theta}\theta) \quad (4) \end{aligned}$$

$$\begin{aligned} M_{ae} = qB^2J/2\,(1 - \beta_{\theta z}z^2 - \beta_{\theta\theta}\theta^2)\,\cdot \\ (-b_{zz}\dot{z} + b_{\theta\theta}\dot{\theta} - c_{\theta z}z + c_{\theta\theta}\theta) \quad (5) \end{aligned}$$

in which m is the mass and J is the mass moment of inertia of the cross-section. The coefficients β, b and c, the counterparts of flutter derivatives, shall be determined experimentally. Because higher degrees are hardly to be identified experimentally and problematic from a physical point of view, only the first and third degree forms are meaningful in the equations (4) and (5). The second degrees, on the other hand, can be avoided due to cross-section symmetry. The whole system (1) then represents a deterministic synthesis of generalized Van der Pol and Duffing types of non-linear equations. Briefly summarized; from

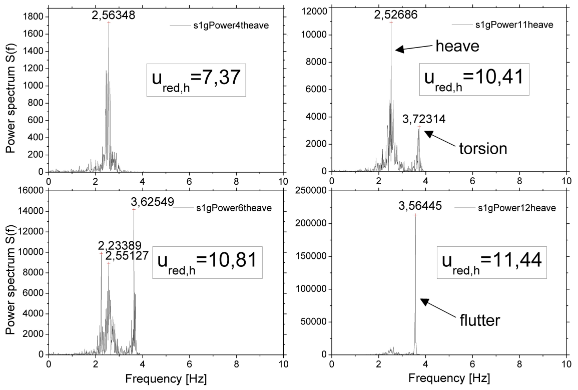

Figure 3: *Contribution of the full aeroelastic system to the deformation energy of the heave mode in terms of the distribution of variances.*

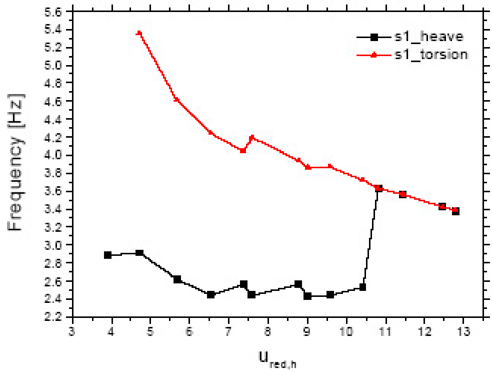

Figure 4: *Heave and pitch frequency as the function of the reduced velocities* $u_{red,h} = u/(Bf_h)$

the physical point of view the above model separates aeroelastic effects into two groups: (i) a linear part represented by non-conservative and gyroscopic forces producing non-symmetric character of the operator and therefore being responsible for the stability loss; (ii) a third degree part characterizing the system behavior in the post-critical state.

3. CRITICAL STATE

Critical velocities identified by tests in turbulent flow can in the majority of the cases be accepted as a reference value for the bridge behaviour in full scale. A turbulent flow generated in a boundary layer wind tunnel is used to represent wind in the neutral atmosphere. The ambient oscillations of a section model of a bridge girder are then examined increasing the wind speed stepwise, until violent responses develop. Figure 1 shows a section model constructed to test the aeroelastic performance of the mid span of the new Rhine crossing near Wesel. The section model is equipped with all aerodynamically relevant details, including railings with screens and balustrades and a central barrier. The model is mounted on a set of springs which are tuned to produce an appropriate relation of the modal stiffnesses. The eigen-frequencies of the two modes as well as other important parameters are scaled with regard to the test wind speeds and the scaling of geometry and mass. As in many applications, the deformability in horizontal directions is restrained.

Snap-back tests are conducted in still air to identify frequencies and damping in terms of linearized approximations. The damping of such a model is typically amplitude dependent, but at small amplitudes a certain viscous range can usually be established. In turbulent flow, the model is performing stochastic oscillations when the flow speed and the associated excitation force are sufficiently high. As the response amplitudes are small at low reduced velocities in the non-critical range, see Figure 2, their squares in the first brackets in equation (4) and (5) will not dominate the response but remaining small unless larger amplitudes are generated. The structure of the terms in the second brackets is comparable to the traditional linearized approach. The coefficients b_{ij}, c_{ij}, $i, j = z, \theta$, of this part of the model will be frequency dependent and can be practically expressed through flutter derivatives and be identified e.g. from free vibration tests on sectional models. Figure 2 shows the development of dominant frequencies in the scanned time histories of heave and torsion for test series with subsequently increased reduced velocity u_{red}. In turbulent flow, the model is performing stochastic oscillations with possible resonant amplifications if the Scruton number (Sc) of the model is limited to a suitable low level. It is found that the coupling of the aeroelastic system becomes visible at further increasing reduced velocities through the frequency signal of a new mode of the coupled aeroelastic system which is visible also in the heave spectral density. The new mode develops from the torsional degree of freedom as it can be understood from the decaying eigenfrequency of torsion shown in Figure 4. Violent responses occur when the flow speed in the wind tunnel is near to, or identical to the critical velocity. Pure heave is damped out, and periodic oscillations are performed in the coupled mode shape consisting of both heave and torsional component. The validity of the equations (2) and (3) is limited to this range. The amplitudes grow drastically for $u_{red} > u_{red,crit}$ as it is observed in experiments, e.g. Náprstek et al (2008), so that the squares of

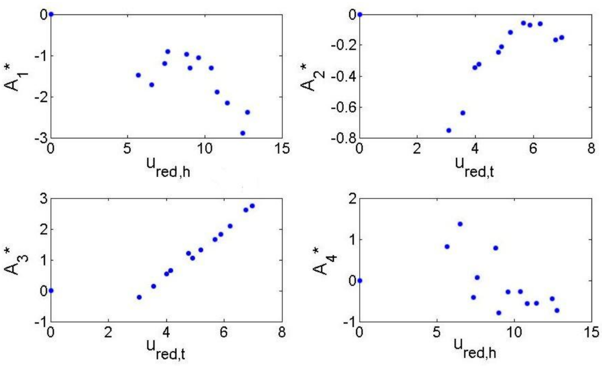

Figure 5: *Estimated girder A^* derivatives.*

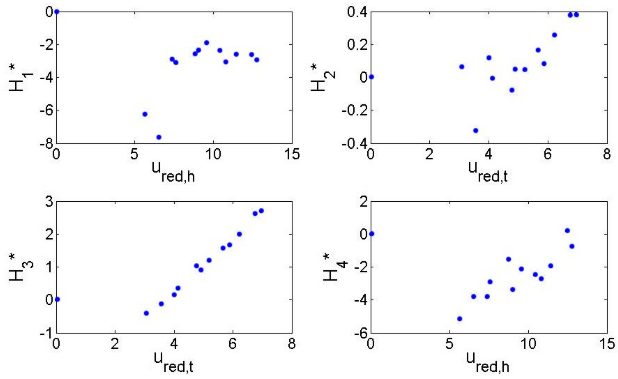

Figure 6: *Estimated girder H^* derivatives.*

the amplitudes and the coefficients β_{ij}, $i, j = z, \theta$ can gain importance.

4. PROSPECTIVE OF EXPERIMENTAL POSSIBILITIES

Many different and also related methodologies for the identification of aeroelastic systems and the respective authors are compiled in Table 1. Further contributions are also known e.g. by Diana, Hjorth-Hansen, Matsumoto and others. The advantages and disadvantages of analysing ambient data are beside others that the natural coupling between vertical and torsional motion and the influence of turbulence are preserved. The methods are applicable to wind tunnel data and full scale tests. The effect of noise is present but 70% of the values of a derivative meet an interval of 8% (smooth) and 19% (turbulent). A further disadvantage is that effects of size of motion and of the mean angle of attack of the flow are difficult to be separated. In the Figures 5 and 6 various calculated derivatives of the cross section are shown. The time series are measured from a 2DOF system oscillating in a flow with a turbulence intensity of $I_u = 3\%$. The heave and torsion frequencies in still air are $2.44\,Hz$ and $4.49\,Hz$ respectively. The aerodynamic derivatives are determined by author SL applying the Covariance Block Hankel Matrix method (CBHM) after Brownjohn & Bogunovic-Jakobsen (2001).

Clear results are found e.g. for the derivatives H_3^* and A_3^*. Some identified derivatives have a large scattering although the results are calculated from repeated experiments. This can be an indication that the linear model of the aeroelastic subsystem has deficiencies. Nonlinear modelling promises an improvement in such cases. A step forward can be the use of the forced motion method where the motion of the structure is controlled and can generate associated aeroelastic forces. An example of such control mechanism is shown in Figure 7. The evaluation of flow-induced forces is not only straightforward within a linear approach as here aeroelastic forces related to mass and stiffness are linked to a cosinusoidal term and damping related components are linked to a sinusoidal term. Advantages are expected also in case of the adaption of a non-linear approach with squared components in terms of the amplitude as the scattering of the amplitudes evaluated from respective experiments remain very small, and the important components of the bias are known (see Figure 4) or can be controlled. A similar mechanism is constructed at present at the Building Aerodynamics Laboratory at Bochum and shall be put into action for further investigations. An another example is shown on the Figures 10-11. It is related to the large amplitude response of the bridge-like structure with rectangular cross-section and aspect ratio 1:5 using a newly developed testing rig (Figure 9). By the system of mechanically indepen-

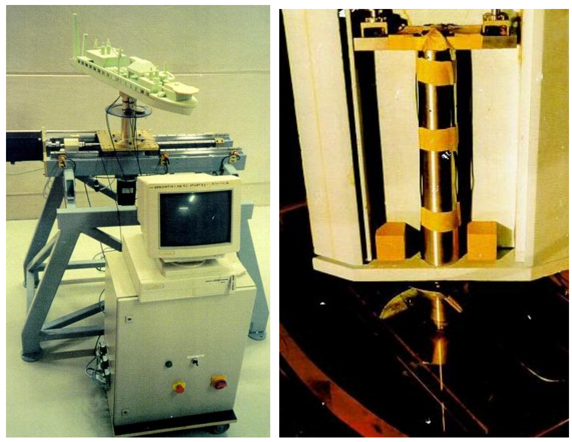

Figure 7: *Forced motion mechanism as used by author RH at the Danish Maritime Institute, Lyngby, Denmark (Force AG), and model of the girder fixed at the forced motion mechanism.*

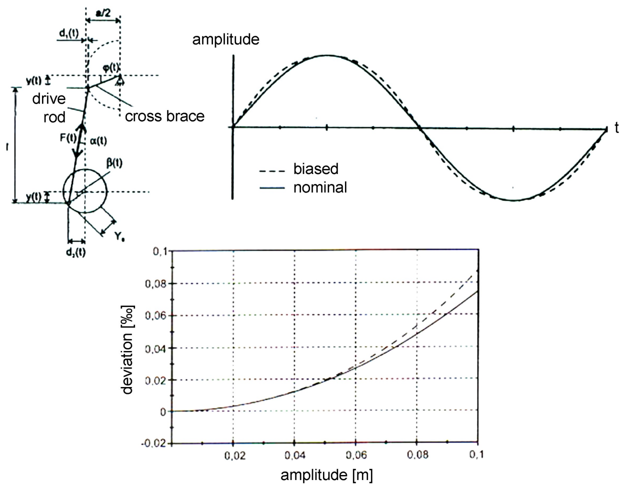

Figure 8: *Known bias due to deviation of the amplitude from the projected run of the curve. (Taken from Hortmanns (1996))*

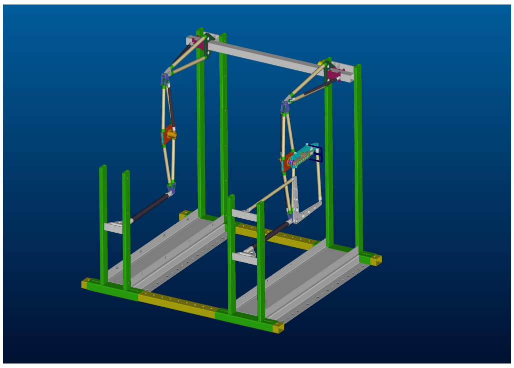

Figure 9: *New test rig used by authors for identification of non-linear aeroelastic coefficients.*

dent adjustable torsional springs, the frequencies of the girder can be tuned to theoretically arbitrary values. The regular pressure oscillations have been measured. The red (dash-dotted) lines are the pressures on the top surface of the beam. The blue (dashed) line represents the pressures on the bottom face of the body. The positive pressures are pointing out of the body. The Figure 10 represents the time shot, when the excessive amplitude growth started during the transition through the critical (bifurcation) state. Especially rotation, a driving DOF, exceeded the angle, where the linear assumptions are unsatisfactory.

The pressures measured at the model surface has been used for the identification of flutter derivatives according to the procedure in Ricciardelli (2002), where the coefficients are established from the response of the model excited by the wind and which uses the linear assumption as in equations (2) and (3). Figure 11 represents

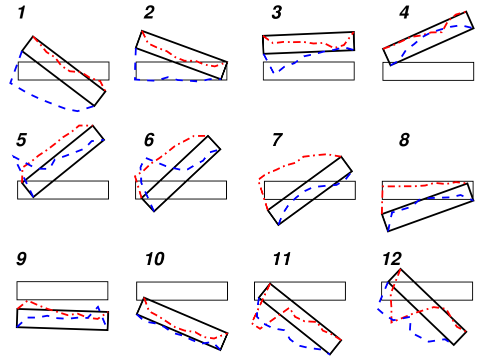

Figure 10: *Snapshots of large amplitudes oscillation the beam during one period.*

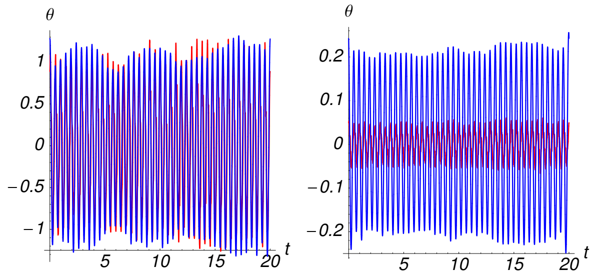

Figure 11: *Heave (left) and pitch (right) response of the bridge model in post-critical range.*

the results of comparison of the left-hand side with the right-hand side of the equation (1). In the case of heave, the experimentally determined (red line) forces perfectly matches the left-hand side of the equation (blue line), whereas in the case of rotational movement, the difference due to using just linear identification procedure is apparent.

5. CONCLUSIONS

Based on the frequency analysis of the time series of the monitored degrees of freedom heave and flutter the phenomena of aeroelastical coupling with increasing reduced velocities can be described as a successive development of a new mode consisting from coupled torsional and heave motions with phases of decreasing band width from the previously (aeroelastically) uncoupled torsional and heave modes. A sequence of power spectra, each evaluated at increasing levels of the reduced velocity shows the successive growth of the energy contribution of the new mode to both, the power spectra of the heave signals and of the signals of the rotation. At the critical velocity the original heave mode is strongly damped and not reflecting into the power spectrum of flutter oscillations. Such behaviour must be included into

Authors	Year	Name
Klöppel & Thiele	1967	Formbeiwerte
Scanlan	1970	Scanlan methods. later important extensions
Shinozuka	1982	ARMA method
Yamada & Ichikawa	1992	Extended Kalman Filter
Poulsen	1992	Control theory & system identification
Zasso et al., Ljung	1996	Transfer function analysis, PEM
Jakobsen et al.	1995, 2001, 2003	CBHM[1], CPS
Sarkar, Scanlan et al.	1992, 1994, 2004, 2005	MITD, RFA
Bartoli, Righi et al.	2004, 2006	CSIM
Gu et al.	2001, 2006	Unified leastsquares approach

Table 1: *Procedures for the identification of aeroelastic systems. CBHM [1] method is used here.*

the representation through flutter coefficients.

The paper examines the adaptability of the non-linear model under conditions of drastical amplitude amplitude growth. The main idea is to adapt quasi-stationary β-coefficients as they become active at high wind velocities where often a quasi-stationary structure of models of fluid-structure interaction is adopted through experiments. The experimental method of free vibration tests cannot deliver suitable data as in regular the model oscillations grow too large in the post-critical range and can harm the test equipment. A second, principal drawback of the free vibration method is that the measured amplitudes of heave and torsion are aeroelastically coupled by strong stochastic nature what requires a stochastic identification procedure with associated difficulties of accuracy. It seems more suited to implement a forced-motion test, Höffer (2007). Here, heave and torsion amplitudes are defined for each harmonical oscillation in a single frequency. Also coupled motions are possible.

The more advanced "non-linear" identification procedures must be employed in order to identify the aeroelastic coefficients introduced in the proposed non-linear model. The realized experiments on the newly developed experimental facilities and test rigs will be focusing on that.

6. ACKNOWLEDGEMENT

The support of Grant No. 103/07/J060, Grant DFT 476 TSE 113/52/0-1 and the institutional support AV 07020710524 (ITAM) is acknowledged.

7. REFERENCES

Brownjohn, J.M.W., Bogunovic Jakobsen, J., 2001, Strategies for aeroelastic parameter identification from bridge deck free vibration data. *Journal of Wind Engineering and Industrial Aerodynamics* **89**: 1113-1136.

Caracoglia L., Jones, N. P., 2003, Time domain vs. frequency domain characterization of aeroelastic forces for bridge sections. *Journal of Wind Engineering and Industrial Aerodynamics*, **91**: 1135-1150.

Costa C., 2007, Aerodynamic admittance functions and buffeting forces for bridges via indicial functions, *Journal of Fluids and Structures*, **23**: 413-428.

Hortmanns, M., 1996, *Zur identifikatin und Berücksichtigung nichtlinearer aeroelastischer Effekte.* Schriftenreihe Stahlbau, RWTH Aachen, Heft 34

Höffer, R., Bogunovic Jakobsen, J., Lachmann, S., 2007, Accuracy and equivalency of modelling of aeroelastic bridge girder response in time and frequency domain. In *Proc. 12th Int. Conference on Wind Engineering* (ed Cheung) 215-223, Cairns, Australia, July.

Náprstek, J., Pospíšil S., Fischer C., 2008, Post-critical limit cycles and non-stationary response types of an aeroelastic system. In *Flow-Induced Vibrations FIV 2008* (eds I. Zolotarev & J. Horacek), Prague, June 30 – July 3.

Ricciardelli, F., de Grenet, E., 2002, Evaluation of bridge flutter derivatives from wind excited vibration of section models. In *Proc. EURODYN 2002* (eds G. I. Schuëller & H. Grundmann), Munich, 2002, 589-592

Scanlan, R.H., 1992, Wind dynamics of long-span bridges. In *Aerodynamics of Large Bridges* (ed A. Larsen) 47-57. Balkema,