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Mechanics of Inflatable Fabric Beams

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Summary

In this paper we present a summary of the behaviour of inflatable fabric beams. Analytical studies on inflatable fabric beams are presented and inflatable fabric beam finite elements are described. The stiffness matrixes take into account the inflation pressure. Shakedown analysis is used to calculate limit loads of inflatable fabric beams. Results on the dynamic behaviour of inflatable beams are finally displayed.

Introduction

The aim of the paper is to display the mechanics of inflatable fabric beams, and in particular results on the deflections for a given bending load, collapse loads, and dynamic behaviour of beams. Two kinds of beams are studied: panels and tubes.

The first section is concerned with deflections under static loading. The first papers [1], [2], dealing with beam theory applied to inflatable structures, suppose an Euler-Bernoulli behaviour, which doesn't let appear explicitly the pressure in the formulae giving the deflections. In our opinion, the inflation pressure must appear in the stiffness of the inflated beam. A new theory has been recently built following the hypothesis that equilibrium equations must be written in the deformed state and that Timoshenko's assumptions must be used to describe the kinematics of the beam. Analytical formulae have been established for simply supported or cantilever beams and take into account the internal pressure. They can be found in [3] for inflated panels and in [4] for inflated tubes.

The second section of the paper is devoted to construct inflatable beam finite elements able to give accurate values of the displacement field for hyperstatic beams and also for structures made of inflatable beams. For inflatable panels, the compliance matrix of a cantilever-inflated panel is the sum of the yarn and beam compliances [3]. The stiffness matrix depends on the inflation pressure and is simply obtained by the usual theory of the equilibrium FEM [5]. For tubes problems the effects of large rotations and shear deformation must be added, but the final result [6] is similar to the previous ones. Results given by the tube finite elements are compared to results given by 3D membrane finite elements and to experiments.

In the third section, we show that one can make an analogy between plastic hinges and pneumatic hinges which arise in inflatable beams. Limit bending momentum is given and collapse loads of inflatable beams are calculated by means of the usual theorems of limit analysis.

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In the last section of the paper, we deal with the first results on the dynamic behaviour of inflatable tubes. For isostatic configurations, analytical formulae may be derived, and show that frequencies depend lightly on the inflation pressure. A continuous element is described and comparisons between theoretical and experimental results are displayed.

Analytical deflections of inflatable beams

In this section, we consider, as the first example, the linearized problem of an inflated cantilever beam under bending. The beam is made of a cylindrical membrane; its reference length is ℓ_o, its reference radius R_o and its reference thickness h_o. The beam is built-in at end $X = 0$, subjected to an internal pressure p and a transverse force $F\mathbf{y}$ at end $X = \ell_o$. P is the global load due to the pressure applied at the ends of the beam; kGS_0 is the shear coefficient. Deflection and rotation are denoted by V and θ. From the equilibrium equation and the boundary conditions, one readily gets [6]:

$$V(x) = \frac{F}{(E+P/S_o)I_o}\left(\frac{\ell_o^2 x}{2} - \frac{x^3}{6}\right) + \frac{Fx}{P+kGS_o} \tag{1}$$

$$\theta(x) = \frac{F}{(E+P/S_o)I_o}\left(\ell_o x - \frac{x^2}{2}\right) \tag{2}$$

The solution is linear with respect to force F, yet non linear with respect to the pressure. First, the pressure appears in the denominators of the right-hand sides of (1) and (2). Second, the reference dimensions ℓ_o, S_o, and I_o themselves depend on the pressure; however, further numerical results show that this dependence may not be too strong.

If the internal pressure is zero, these relations give the well-known results for the Timoshenko beam model. However, contrary to a classical beam, here the inflatable beam is made of a membrane, so the pressure cannot be equal to zero for the beam not to collapse. The influence of the internal pressure on the beam response is clearly shown in the previous relations: the inflation amounts to strengthen the Young modulus and the shear modulus. In particular, when p tends to infinity, so do the equivalent material properties and the deflection and the rotation are identically zero.

Finite element for static analysis of inflatable beams

A tube finite element is now developed. This time the tube is modeled as a straight beam of length ℓ, radius R, section area S, and second moment of area I. It is assumed that the beam undergoes axial stretch and bending in the plane, the bending stiffness is characterized by EI and the shear effect by a factor denoted kGS. As in 3D analysis, use is made again of the virtual power principle, which is written in terms of Lagrangian variables. However, there are two differences here:

- The choice of the Timoshenko kinematics to describe the real and virtual displacement fields,
- And the linearization of the discretized equations around a prestressed state, which corresponds to the preliminary inflation of the beam.

The resulting stiffness matrix contains the material and the geometric matrices, including the presence of the internal pressure. In the sequel, the computations are carried out using a 2-node element, with two degrees of freedom at each node (deflection v and rotation θ) interpolated by a cubic shape function.

$$[K] = \frac{\left(EI_0 + \frac{PI_0}{S_0}\right)}{\ell_0^3(1+\phi_p)} \begin{bmatrix} 12 & 6\ell_0 & -12 & 6\ell_0 \\ & \ell_0^2(4+\phi_p) & -6\ell_0 & \ell_0^2(2-\phi_p) \\ & & 12 & -6\ell_0 \\ sym & & & \ell_0^2(4+\phi_p) \end{bmatrix} \tag{3}$$

where ϕ_P has been introduced for convenience:

$$\phi_P = \frac{12\left(EI_0 + \frac{PI_0}{S_0}\right)}{(P + kGS_0)\ell_0^2} \tag{4}$$

Shakedown analysis

Shakedown analysis of plastic beams is well known. When plasticity appears on the inner or outer fibers of a beam, the load, which gives the beginning of plasticity, seems to the wrinkling load of an inflated beam. One can define M_0, the maximum elastic momentum (equivalent to the momentum which gives the wrinkle of an inflatable beam). When the load is increased, a plastic zone grows until a plastic hinge appears and the section of the beam is entirely plastified. In the case of inflatable beams, we have the same situation: the stress falls in the fabric until the collapse load. The collapse loads of inflatable beams are therefore given by the same formulae that those of plastic beams provided that the total or plastic bending momentum M_1 is well defined for inflatable beams. This limit momentum has been defined for inflatable panels in [3].

In the case of inflatable tubes, Comer & Levy [1] have supposed that this bending momentum is obtained when the stress is nil in the entire tube, except at a point of the generative of the tube. Our experiments have shown that collapse appears when only half of the tube section is submitted to zero normal stress. This experimental assumption will be traduced in the following manner: we will suppose that the collapse load is obtained when the stress distribution has the shape defined below.

We will therefore suppose that the bending momentum is given by:

$$M_1 = \frac{p\pi R^3}{4} \tag{5}$$

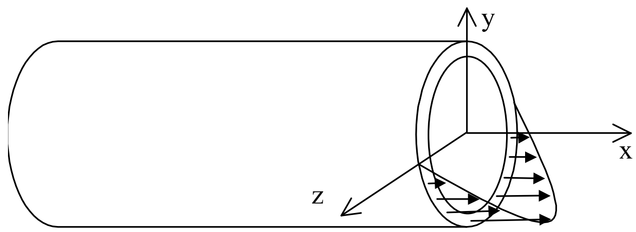

Figure 1: Stress distribution in a tube at its collapse load

Experiments have been done on inflated panels and tubes for three kinds of boundary conditions (simply supported at the two ends, simply supported at one end and clamped at the other end, and finally clamped at the two ends) and for various values of internal pressure. Results are given figure 2.

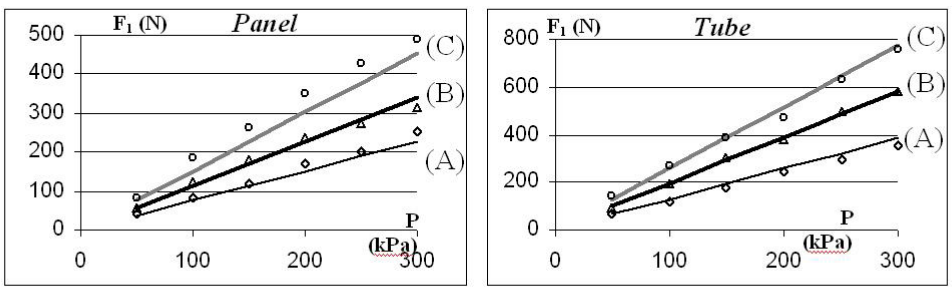

Figure 2: Comparison between theoretical and experimental results

Dynamics of inflatable beams

By introducing inertia effects in the linearized beam equations [7], one gets:

$$\rho_0 S_0 \ddot{V} - (P+KGS)V_{,x}^2 + (P+KGS)\theta_{,x} = 0 \tag{6}$$

$$-\left(EI + P\frac{I}{S}\right)\theta_{,x}^2 - (P+KGS)(V_{,x} - \theta) = 0 \tag{7}$$

We have chosen to deal the dynamic behaviour of beams by means of the continuous element method, which has the main advantage to lead to an exact solution. We will construct a dynamic stiffness matrix depending on the circular frequency ω, which connects the vector of the generalized displacements (transverse displacement V, cross-sectional rotationθ) to the vector of the generalized loads (shear load T, bending momentum M). The expression of the dynamic matrix can be found in [7], and the natural frequencies of vibration are given by :

$$\omega_n = \sqrt{\frac{n^4\pi^4}{\frac{\rho_0 S_0 l^4}{EI+\frac{PI}{S}} + \frac{\rho_0 S_0 n^2 \pi^2 l^2}{P+KGS}}} \tag{8}$$

Experiments have been done to detect the natural frequencies of an inflatable simply supported beam. They are displayed in next Table:

Table 1: Natural frequencies – experiment

Pressure (kPa)	**50**	**75**	**100**	**125**
f1 (Hz)	18.90	19.41	19.96	20.63
f2 (Hz)	57.53	59.55	60.71	63.00
f3 (Hz)	109.50	112.72	114.68	119.00

The exact solutions calculated by equation (8) are displayed below:

Table 2: Natural frequencies – exact solutions

Pressure (kPa)	**50**	**75**	**100**	**125**
f1 (Hz)	16.68	17.12	17.41	17.70
f2 (Hz)	58.60	60.37	61.70	63.00
f3 (Hz)	112.22	116.01	119.09	122.12

One can see that the inflation pressure lightly increases the values of the frequencies. A good agreement between theory and experiment is once again found.

Conclusion

Inflatable fabric prototype beams are studied. Deflections are given by analytical formulae for isostatic configurations for panels and for tubes. The inflation pressure appears in the solution. Inflatable fabric beam finite elements are constructed. The stiffness matrixes take into account the inflation pressure. We take advantage of an analogy between plasticity and inflatable behaviour to use shakedown analysis in order to calculate limit loads of inflatable fabric beams provided the limit momentum is well defined. First results on the dynamic behaviour of an inflated beam are finally shown. Comparisons between experimental and numerical results prove the accuracy of shakedown and finite element analysis for solving problems of inflatable fabric beams.

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