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Contact Problems in Bending of Elastic Plates

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Summary

An initial-boundary value problem for bending of a piecewise homogeneous thermoelastic plate with transverse shear deformation [1] is considered. The unique solvability in distributional spaces is proved by means of a combination of the Laplace transformation and variational methods. This is the first, and essential, step in the construction of boundary element methods for numerical approximations of the solution. The model without thermal effects has been studied in [2]–[6].

Formulation of the Problem

We consider a thin elastic plate of thickness $h_0 = \text{const} > 0$, which occupies a region $\overline{S} \times [-h_0/2, h_0/2]$ in $\mathbb{R}^3$, $S \subset \mathbb{R}^2$. The displacement vector at a point x' at time $t \geq 0$ is $v(x',t) = (v_1(x',t), v_2(x',t), v_3(x',t))^T$, where the superscript T denotes matrix transposition, and the temperature in the plate is $\theta(x',t)$. Let $x' = (x, x_3)$, with $x = (x_1, x_2) \in \overline{S}$. In plate models with transverse shear deformation [2], it is assumed that $v(x',t) = (x_3 u_1(x,t), x_3 u_2(x,t), u_3(x,t))^T$. If thermal effects are taken into account, we also define the "temperature moment" averaged across thickness, by [1]

$$u_4(x,t) = (h^2 h_0)^{-1} \int_{-h_0/2}^{h_0/2} x_3 \theta(x, x_3, t)\, dx_3, \quad h^2 = h_0^2/12.$$

Then $U(x,t) = (u(x,t)^T, u_4(x,t))^T$, where $u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))^T$, satisfies the equation

$$\mathsf{L}U(x,t) = \mathsf{B}_0 \partial_t^2 U(x,t) + \mathsf{B}_1 \partial_t U(x,t) + \mathsf{A}U(x,t) = \mathsf{Q}(x,t), \quad (x,t) \in S \times (0,\infty).$$

Here, $\mathsf{B}_0 = \operatorname{diag}\{\rho h^2, \rho h^2, \rho, 0\}$, $\partial_t = \partial/\partial t$, $\rho > 0$ is the constant density of the material,

$$\mathsf{B}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \eta\partial_1 & \eta\partial_2 & 0 & \varkappa^{-1} \end{pmatrix}, \quad \mathsf{A} = \begin{pmatrix} & & & h^2\gamma\partial_1 \\ & A & & h^2\gamma\partial_2 \\ & & & 0 \\ 0 & 0 & 0 & -\Delta \end{pmatrix},$$

$$A = \begin{pmatrix} -h^2\mu\Delta - h^2(\lambda+\mu)\partial_1^2 + \mu & -h^2(\lambda+\mu)\partial_1\partial_2 & \mu\partial_1 \\ -h^2(\lambda+\mu)\partial_1\partial_2 & -h^2\mu\Delta - h^2(\lambda+\mu)\partial_2^2 + \mu & \mu\partial_2 \\ -\mu\partial_1 & -\mu\partial_2 & -\mu\Delta \end{pmatrix},$$

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$\partial_\alpha = \partial/\partial x_\alpha$, $\alpha = 1,2$, and $\varkappa$, γ, and η are positive constants, which are expressed in terms of the thermal conductivity k, thermal expansion α, specific heat c_e, density ρ, reference temperature θ_0, and the Lamé constants λ and μ by the equalities

$$\varkappa = k/(\rho c_e), \quad \gamma = (3\lambda + 2\mu)\alpha, \quad \eta = (3\lambda + 2\mu)\alpha\theta_0/k.$$

The right-hand side in the above partial differential equation is a combination of the forces and moments acting on the plate and its faces, and $q_4(x,t)$ is a combination of the averaged heat-source density, temperature, and heat flux on the faces.

Without loss of generality [6], we assume that the initial conditions are homogeneous: $U(x,0) = 0$, $\partial_t u(x,0) = 0$, $x \in S$.

In what follows, we consider a piecewise homogeneous infinite plate consisting of two homogeneous parts that occupy, respectively, the regions S^+ and S^- interior and exterior to a simple, closed, C^2-curve ∂S. All geometric and physical parameters, external forces, heat sources, displacement vectors, temperature, differential operators, and initial data relating to the plate sections in S^+ and S^- are designated by subscripts $+$ and $-$, as appropriate. Our aim is to solve the problem (TC) that consists in finding vector fields $U_\pm = (u_\pm^T, u_{\pm,4})^T \in C^2(\Sigma^\pm) \cap C^1(\overline{\Sigma}^\pm)$ in $\Sigma^\pm = S^\pm \times (0,\infty)$ which satisfy the system of equations, initial conditions, and transmission (contact) boundary conditions

$$\begin{aligned}
&\mathsf{L}_\pm U_\pm(x,t) = \mathsf{Q}_\pm(x,t), \quad (x,t) \in \Sigma^\pm,\\
&U_\pm(x,0) = 0, \quad \partial_t u_\pm(x,0) = 0, \quad x \in S^\pm,\\
&U_+^+(x,t) - U_-^-(x,t) = F(x,t), \quad (\mathsf{T}_+U_+)^+(x,t) - (\mathsf{T}_-U_-)^-(x,t) = G(x,t),
\end{aligned}$$

where the superscripts $\pm$ denote the limiting values of the corresponding functions as (x,t) tends to Γ from inside $\Sigma^\pm$, respectively,

$$(\mathsf{T}_\pm U_\pm)(x,t) = \begin{pmatrix} (T_\pm u_\pm)(x,t) - h_\pm^2 \gamma_\pm n(x) u_{\pm,4}(x,t) \\ \partial_n u_{\pm,4}(x,t) \end{pmatrix} = \begin{pmatrix} (\mathsf{T}_{\pm,e}U_\pm)(x,t) \\ (\mathsf{T}_{\pm,\theta}U_\pm)(x,t) \end{pmatrix},$$

$T_\pm$ are the boundary moment-stress operators defined by

$$\begin{pmatrix} h_\pm^2\big[(\lambda_\pm + 2\mu_\pm)n_1\partial_1 + \mu_\pm n_2\partial_2\big], & h_\pm^2(\lambda_\pm n_1\partial_2 + \mu_\pm n_2\partial_1) & 0 \\ h_\pm^2(\mu_\pm n_1\partial_2 + \lambda_\pm n_2\partial_1) & h_\pm^2\big[(\lambda_\pm + 2\mu_\pm)n_2\partial_2 + \mu_\pm n_1\partial_1\big] & 0 \\ \mu_\pm n_1 & \mu_\pm n_2 & \mu_\pm\partial_n \end{pmatrix},$$

$n = n(x) = (n_1(x), n_2(x))^T$ is the outward unit normal to ∂S, and $\partial_n = \partial/\partial n$. To keep the notation simple, we have also denoted by $n(x)$ the three-component vector $(n_1(x), n_2(x), 0)^T$. Finally, we assume that $h_+^2\gamma_+\eta_+^{-1} = h_-^2\gamma_-\eta_-^{-1}$, which, since it seems natural to expect that $h_+ = h_-$ and $\theta_{0,+} = \theta_{0,-}$, yields $k_+ = k_-$.

Solvability of the Problem

In what follows, we denote the Laplace transforms of functions by a superposed hat and the transformation parameter by p. The transition to Laplace transforms with respect to t in (TC) leads to problem (TC$_p$) that depends on p and consists in finding $\hat{U}_\pm \in C^2(S^\pm)\cap C^1(\overline{S^\pm})$ which satisfy the equations and boundary conditions

$$\begin{aligned}
\mathsf{L}_{\pm,p}\hat{U}_\pm(x,p) &= p^2\mathsf{B}_{0,\pm}\hat{U}_\pm(x,p) + p\,\mathsf{B}_{1,\pm}\hat{U}_\pm(x,p) + \mathsf{A}_\pm\hat{U}_\pm(x,p)\\
&= \hat{\mathsf{Q}}_\pm(x,p), \quad x\in S^\pm,\\
\hat{U}_+^+(x,p) - \hat{U}_-^-(x,p) &= \hat{F}(x,p), \quad (\mathsf{T}_+\hat{U}_+)^+(x,p) - (\mathsf{T}_-\hat{U}_-)^-(x,p) = \hat{G}(x,p).
\end{aligned}$$

Here and below, we use the superscripts $\pm$ to denote the limiting values of the corresponding functions as x tends to ∂S from inside $S^\pm$, respectively.

For $m\in\mathbb{R}$ and $p\in\mathbb{C}$, we introduce the following functions spaces:

$H_m(\mathbb{R}^2)$: the standard Sobolev space of functions $\hat{v}_4$ on $\mathbb{R}^2$, equipped with norm $\|\hat{v}_4\|_m = \left\{\int_{\mathbb{R}^2}(1+|\xi|^2)^m|\tilde{v}_4(\xi)|^2 d\xi\right\}^{1/2}$, where $\tilde{v}_4$ is the (generalized) Fourier transform of $\hat{v}_4$.

$\mathbf{H}_{m,p}(\mathbb{R}^2)$: the space that coincides with $\left[H_m(\mathbb{R}^2)\right]^3$ as a set but is endowed with the norm $\|\hat{v}\|_{m,p} = \left\{\int_{\mathbb{R}^2}(1+|\xi|^2+|p|^2)^m|\tilde{v}(\xi)|^2 d\xi\right\}^{1/2}$.

$\mathscr{H}_{m,p}(\mathbb{R}^2) = \mathbf{H}_{m,p}(\mathbb{R}^2)\times H_m(\mathbb{R}^2)$, with norm $|||\hat{V}|||_{m,p} = \|\hat{v}\|_{m,p} + \|\hat{v}_4\|_m$, or, equivalently, $\langle\hat{Q}\rangle_{-1,p} = |p|\|\hat{q}\|_{-1,p} + \|\hat{q}_4\|_{-1}$.

$H_m(S^\pm)$, $\mathbf{H}_{m,p}(S^\pm)$: the spaces of the restrictions to $S^\pm$ of all elements $\hat{v}_4\in H_m(\mathbb{R}^2)$ and $\hat{v}\in\mathbf{H}_{m,p}(\mathbb{R}^2)$, respectively, with norms $\|\hat{u}_4\|_{m;S^\pm} = \inf_{\hat{v}_4\in H_m(\mathbb{R}^2):\hat{v}_4|_{S^\pm}=\hat{u}_4}\|\hat{v}_4\|_m$ and $\|\hat{u}\|_{m,p;S^\pm} = \inf_{\hat{v}\in H_{m,p}(\mathbb{R}^2):\hat{v}|_{S^\pm}=\hat{u}}\|\hat{v}\|_{m,p}$.

$\mathscr{H}_{m,p}(S^\pm) = \mathbf{H}_{m,p}(S^\pm)\times H_m(S^\pm)$, with norm $|||\hat{U}|||_{m,p;S^\pm} = \|\hat{u}\|_{m,p;S^\pm} + \|\hat{u}_4\|_{m;S^\pm}$.

If $p=0$, then we write

$$\begin{aligned}
\mathbf{H}_m(\mathbb{R}^2) &= \mathbf{H}_{m,0}(\mathbb{R}^2) = \left[H_m(\mathbb{R}^2)\right]^3,\\
\mathscr{H}_m(\mathbb{R}^2) &= \mathbf{H}_m(\mathbb{R}^2)\times H_m(\mathbb{R}^2) = \left[H_m(\mathbb{R}^2)\right]^4,\\
\mathbf{H}_m(S^\pm) &= \mathbf{H}_{m,0}(S^\pm) = \left[H_m(S^\pm)\right]^3,\\
\mathscr{H}_m(S^\pm) &= \mathbf{H}_m(S^\pm)\times H_m(S^\pm) = \left[H_m(S^\pm)\right]^4.
\end{aligned}$$

$H_{1/2}(\partial S)$, $\mathbf{H}_{1/2,p}(\partial S)$: the spaces of the traces on ∂S of all $\hat{u}_4\in H_1(S^\pm)$ and all $\hat{u}\in\mathbf{H}_{1,p}(S^\pm)$, with norms

$$\begin{aligned}
\|\hat{\varphi}_4\|_{1/2;\partial S} &= \inf_{\hat{u}_4\in H_1(S^+):\hat{u}_4|_{\partial S}=\hat{\varphi}_4}\|\hat{u}_4\|_{1;S^+},\\
\|\hat{\varphi}\|_{1/2,p;\partial S} &= \inf_{\hat{u}\in\mathbf{H}_{1,p}(S^+):\hat{u}|_{\partial S}=\hat{\varphi}}\|\hat{u}\|_{1,p;S^\pm}.
\end{aligned}$$

$\mathscr{H}_{1/2,p}(\partial S) = \mathbf{H}_{1/2,p}(\partial S) \times H_{1/2}(\partial S)$, equipped with norm $|||\hat{F}|||_{1/2,p;\partial S} = \|\hat{\varphi}\|_{1/2,p;\partial S} + \|\hat{\varphi}_4\|_{1/2;\partial S}$.

$H_{-1/2}(\partial S)$, $\mathbf{H}_{-1/2,p}(\partial S)$, $\mathscr{H}_{-1/2,p}(\partial S)$: the duals to the spaces $H_{1/2}(\partial S)$, $\mathbf{H}_{1/2,p}(\partial S)$, $\mathscr{H}_{1/2,p}(\partial S)$ with respect to the dualities generated by the inner products $(\cdot,\cdot)_{0;S^\pm}$ in $L^2(\partial S)$, $\left[L^2(\partial S)\right]^3$, $\left[L^2(\partial S)\right]^4$, equipped with norms $\|\hat{g}_4\|_{-1/2;\partial S}$, $\|\hat{g}\|_{-1/2,p;\partial S}$, $|||\hat{G}|||_{-1/2,p;\partial S} = \|\hat{g}\|_{-1/2,p;\partial S} + \|\hat{g}_4\|_{-1/2;\partial S}$ or, equivalently, $\langle \hat{G} \rangle_{-1/2,p;\partial S} = |p| \|\hat{g}\|_{-1/2,p;\partial S} + \|\hat{g}_4\|_{-1/2;\partial S}$.

The continuous (uniformly with respect to $p \in \mathbb{C}$) trace operators from $H_1(S^\pm)$ to $H_{1/2}(\partial S)$, from $\mathbf{H}_{1,p}(S^\pm)$ to $\mathbf{H}_{1/2,p}(\partial S)$, and from $\mathscr{H}_{1,p}(S^\pm)$ to $\mathscr{H}_{1/2,p}(\partial S)$ are denoted by the same symbols $\gamma^\pm$.

We denote by c all positive constants in estimates, which do not depend on the functions in those estimates or on $p \in \mathbb{C}_\kappa$, where $\mathbb{C}_\kappa = \{p = \sigma + i\zeta \in \mathbb{C} : \sigma > \kappa\}$, but may depend on κ.

We say that $\hat{U} = \{\hat{U}_+, \hat{U}_-\}$, $\hat{U}_\pm \in \mathscr{H}_{1,p}(S^\pm)$, is a variational (weak) solution of (TC$_p$) if $\gamma^+\hat{U}_+ - \gamma^-\hat{U}_- = \hat{F}$ and

$$\Upsilon_{+,p}(\hat{U}_+, \hat{W}_+) + \Upsilon_{-,p}(\hat{U}_-, \hat{W}_-) = (\hat{\mathrm{Q}}, \hat{W})_0 + (\hat{G}, \hat{W})_{0;\partial S} \quad \forall \hat{W} \in \mathscr{H}_{1,p}(\mathbb{R}^2),$$

where

$$\begin{aligned}
&\Upsilon_{\pm,p}(\hat{U}_\pm, \hat{W}_\pm) = a_\pm(\hat{u}_\pm, \hat{w}_\pm) + (\nabla \hat{u}_{\pm,4}, \nabla \hat{w}_{\pm,4})_{0;S^\pm} + p^2 (B_{0,\pm}^{1/2} \hat{u}_\pm, B_{0,\pm}^{1/2} \hat{w}_\pm)_{0;S^\pm} \\
&\quad + \varkappa_\pm^{-1} p (\hat{u}_{\pm,4}, \hat{w}_{\pm,4})_{0;S^\pm} - h_\pm^2 \gamma_\pm (\hat{u}_{\pm,4}, \operatorname{div} \hat{w}_\pm)_{0;S^\pm} + \eta_\pm p (\operatorname{div} \hat{u}_\pm, \hat{w}_{\pm,4})_{0;S^\pm}, \\
&a_\pm(\hat{u}_\pm, \hat{w}_\pm) = 2 \int_{S^\pm} E_\pm(\hat{u}_\pm, \hat{w}_\pm)\, dx, \\
&2E_\pm(\hat{u}_\pm, \hat{w}_\pm) = h_\pm^2 E_{\pm,0}(\hat{u}_\pm, \hat{w}_\pm) + h_\pm^2 \mu_\pm (\partial_2 \hat{u}_{\pm,1} + \partial_1 \hat{u}_{\pm,2})(\partial_2 \overline{\hat{w}_{\pm,1}} + \partial_1 \overline{\hat{w}_{\pm,2}}) \\
&\quad + \mu_\pm [(\hat{u}_{\pm,1} + \partial_1 \hat{u}_{\pm,3})(\overline{\hat{w}_{\pm,1}} + \partial_1 \overline{\hat{w}_{\pm,3}}) + (\hat{u}_{\pm,2} + \partial_2 \hat{u}_{\pm,3})(\overline{\hat{w}_{\pm,2}} + \partial_2 \overline{\hat{w}_{\pm,3}})], \\
&E_{\pm,0}(\hat{u}_\pm, \hat{w}_\pm) = (\lambda_\pm + 2\mu_\pm)[\partial_1 \hat{u}_{\pm,1} \partial_1 \overline{\hat{w}_{\pm,1}} + \partial_2 \hat{u}_{\pm,2} \partial_2 \overline{\hat{w}_{\pm,2}}] \\
&\quad + \lambda_\pm [\partial_1 \hat{u}_{\pm,1} \partial_2 \overline{\hat{w}_{\pm,2}} + \partial_2 \hat{u}_{\pm,2} \partial_1 \overline{\hat{w}_{\pm,1}}], \\
&B_{0,\pm} = \operatorname{diag}\{\rho_\pm h_\pm^2, \rho_\pm h_\pm^2, \rho_\pm\}.
\end{aligned}$$

Theorem 1 *For any given functions $\hat{\mathrm{Q}} = (\hat{q}^T, \hat{q}_4)^T \in \mathscr{H}_{-1,p}(\mathbb{R}^2)$, $\hat{F} \in \mathscr{H}_{1/2,p}(\partial S)$, and $\hat{G} \in \mathscr{H}_{-1/2,p}(\partial S)$, $p \in \mathbb{C}_\kappa$, $\kappa > 0$, problem* (TC$_p$) *has a unique solution of components $\hat{U}_\pm(x,p) \in \mathscr{H}_{1,p}(S^\pm)$, which satisfy the estimate*

$$|||\hat{U}_+|||_{1,p;S^+} + |||\hat{U}_-|||_{1,p;S^-} \le c\big(\langle \hat{\mathrm{Q}} \rangle_{-1,p} + |p|\, |||\hat{F}|||_{1/2,p;\partial S} + \langle \hat{G} \rangle_{-1/2,p;\partial S}\big).$$

We introduce a few more function spaces for $\kappa > 0$ and $k \in \mathbb{R}$.

$\mathbf{H}^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2)$, $\mathbf{H}^{\mathscr{L}}_{1,k,\kappa}(S^\pm)$, $\mathbf{H}^{\mathscr{L}}_{\pm 1/2,k,\kappa}(\partial S)$**:** the spaces of three-component vector functions $\hat{q}(x,p)$, $\hat{u}(x,p)$, $\hat{e}(x,p)$ that define holomorphic mappings $\hat{q} : \mathbb{C}_\kappa \mapsto \mathbf{H}_{-1}(\mathbb{R}^2)$, $\hat{u} : \mathbb{C}_\kappa \mapsto \mathbf{H}_1(S^\pm)$, $\hat{e} : \mathbb{C}_\kappa \mapsto \mathbf{H}_{\pm 1/2}(\partial S)$ and have norms defined by

$$\|\hat{q}\|^2_{-1,k,\kappa} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{q}(x,p)\|^2_{-1,p}\, d\tau < \infty,$$
$$\|\hat{u}\|^2_{1,k,\kappa;S^\pm} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{u}(x,p)\|^2_{1,p;S^\pm}\, d\tau < \infty,$$
$$\|\hat{e}\|^2_{\pm 1/2,k,\kappa;\partial S} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{e}(x,p)\|^2_{\pm 1/2,p;\partial S}\, d\tau < \infty.$$

$H^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2)$, $H^{\mathscr{L}}_{1,k,\kappa}(S^\pm)$, $H^{\mathscr{L}}_{\pm 1/2,k,\kappa}(\partial S)$**:** the spaces of all $\hat{v}_4(x,p)$, $\hat{u}_4(x,p)$, $\hat{e}_4(x,p)$ that define holomorphic mappings $\hat{q}_4 : \mathbb{C}_\kappa \mapsto H_{-1}(\mathbb{R}^2)$, $\hat{u}_4 : \mathbb{C}_\kappa \mapsto H_1(S^\pm)$, $\hat{e}_4 : \mathbb{C}_\kappa \mapsto H_{\pm 1/2}(\partial S)$ and have norms defined by

$$\|\hat{q}_4\|^2_{-1,k,\kappa} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{q}_4(x,p)\|^2_{-1}\, d\tau < \infty,$$
$$\|\hat{u}_4\|^2_{1,k,\kappa;S^\pm} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{u}_4(x,p)\|^2_{1;S^\pm}\, d\tau < \infty,$$
$$\|\hat{e}_4\|^2_{\pm 1/2,k,\kappa;\partial S} = \sup_{\sigma>\kappa} \int_{-\infty}^{\infty} (1+|p|^2)^k \|\hat{e}(x,p)\|^2_{\pm 1/2;\partial S}\, d\tau < \infty.$$

$\mathscr{H}^{\mathscr{L}}_{-1,k,l,\kappa}(\mathbb{R}^2) = \mathbf{H}^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2) \times H^{\mathscr{L}}_{1,l,\kappa}(\mathbb{R}^2)$, $\mathscr{H}^{\mathscr{L}}_{1,k,l,\kappa}(S^\pm) = \mathbf{H}^{\mathscr{L}}_{1,k,\kappa}(S^\pm) \times H^{\mathscr{L}}_{1,l,\kappa}(S^\pm)$, $\mathscr{H}^{\mathscr{L}}_{\pm 1/2,k,l,\kappa}(\partial S) = \mathbf{H}^{\mathscr{L}}_{\pm 1/2,k,\kappa}(\partial S) \times H^{\mathscr{L}}_{\pm 1/2,l,\kappa}(\partial S)$ of $\hat{V} = \{\hat{v}, \hat{v}_4\}$, $\hat{U} = \{\hat{u}, \hat{u}_4\}$, $\hat{\mathscr{E}} = \{\hat{e}, \hat{e}_4\}$ with norms $|||\hat{V}|||_{-1,k,l,\kappa} = \|\hat{v}\|_{-1,k,\kappa} + \|\hat{v}_4\|_{-1,l,\kappa}$, $|||\hat{U}|||_{1,k,l,\kappa;S^\pm} = \|\hat{u}\|_{1,k,\kappa;S^\pm} + \|\hat{u}_4\|_{1,l,\kappa;S^\pm}$, $|||\hat{\mathscr{E}}|||_{1,k,l,\kappa;\partial S} = \|\hat{e}\|_{1,k,\kappa;\partial S} + \|\hat{e}_4\|_{1,l,\kappa;\partial S}$.

Theorem 2 *Let $\kappa > 0$ and $l \in \mathbb{R}$. If $\hat{\mathrm{Q}} \in \mathscr{H}^{\mathscr{L}}_{-1,l+1,l,\kappa}(\mathbb{R}^2)$, $\hat{F} \in \mathscr{H}^{\mathscr{L}}_{1/2,l+1,l+1,\kappa}(\partial S)$, $\hat{G} \in \mathscr{H}^{\mathscr{L}}_{-1/2,l+1,l,\kappa}(\partial S)$, then the components $\hat{U}_\pm(x,p)$ of the (weak) solution $\hat{U}(x,p)$ of problem* (TC$_p$) *belong to $\mathscr{H}^{\mathscr{L}}_{1,l,l,\kappa}(S^\pm)$ and*

$$|||\hat{U}|||_{1,l,l,\kappa;S^+} + |||\hat{U}|||_{1,l,l,\kappa;S^-} \le c\big\{|||\hat{\mathrm{Q}}|||_{-1,l+1,l,\kappa} + |||\hat{F}|||_{1/2,l+1,l+1,\kappa;\partial S} + |||\hat{G}|||_{-1/2,l+1,l,\kappa;\partial S}\big\}.$$

Let $\kappa > 0$ and $k, l \in \mathbb{R}$, and let $\mathbb{R}^3_+ = \mathbb{R}^2 \times (0,\infty)$. By

$$\mathbf{H}^{\mathscr{L}^{-1}}_{-1,k,\kappa}(\mathbb{R}^3_+), \quad H^{\mathscr{L}^{-1}}_{-1,l,\kappa}(\mathbb{R}^3_+), \quad \mathscr{H}^{\mathscr{L}^{-1}}_{-1,k,l,\kappa}(\mathbb{R}^3_+) = \mathbf{H}^{\mathscr{L}^{-1}}_{-1,k,\kappa}(\mathbb{R}^3_+) \times H^{\mathscr{L}^{-1}}_{-1,l,\kappa}(\mathbb{R}^3_+),$$
$$\mathbf{H}^{\mathscr{L}^{-1}}_{1,k,\kappa}(\Sigma^\pm), \quad H^{\mathscr{L}^{-1}}_{1,l,\kappa}(\Sigma^\pm), \quad \mathscr{H}^{\mathscr{L}^{-1}}_{1,k,l,\kappa}(\Sigma^\pm) = \mathbf{H}^{\mathscr{L}^{-1}}_{1,k,\kappa}(\Sigma^\pm) \times H^{\mathscr{L}^{-1}}_{1,l,\kappa}(\Sigma^\pm),$$
$$\mathbf{H}^{\mathscr{L}^{-1}}_{\pm 1/2,k,\kappa}(\Gamma), \quad H^{\mathscr{L}^{-1}}_{\pm 1/2,l,\kappa}(\Gamma), \quad \mathscr{H}^{\mathscr{L}^{-1}}_{1,k,l,\kappa}(\Gamma) = \mathbf{H}^{\mathscr{L}^{-1}}_{1,k,\kappa}(\Gamma) \times H^{\mathscr{L}^{-1}}_{1,l,\kappa}(\Gamma)$$

we denote the spaces consisting of the inverse Laplace transforms of the elements of

$$\mathbf{H}^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2),\quad H^{\mathscr{L}}_{-1,l,\kappa}(\mathbb{R}^2),\quad \mathscr{H}^{\mathscr{L}}_{-1,k,l,\kappa}(\mathbb{R}^2)=\mathbf{H}^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2)\times H^{\mathscr{L}}_{-1,l,\kappa}(\mathbb{R}^2),$$
$$\mathbf{H}^{\mathscr{L}}_{1,k,\kappa}(S^\pm),\quad H^{\mathscr{L}}_{1,l,\kappa}(S^\pm),\quad \mathscr{H}^{\mathscr{L}}_{1,k,l,\kappa}(S^\pm)=\mathbf{H}^{\mathscr{L}}_{1,k,\kappa}(S^\pm)\times H^{\mathscr{L}}_{1,l,\kappa}(S^\pm),$$
$$\mathbf{H}^{\mathscr{L}}_{\pm1/2,k,\kappa}(\partial S),\quad H^{\mathscr{L}}_{\pm1/2,l,\kappa}(\partial S),\quad \mathscr{H}^{\mathscr{L}}_{1,k,l,\kappa}(\partial S)=\mathbf{H}^{\mathscr{L}}_{1,k,\kappa}(\partial S)\times H^{\mathscr{L}}_{1,l,\kappa}(\partial S),$$

with norms

$$\|v\|_{-1,k,\kappa}=\|\hat v\|_{-1,k,\kappa},\quad \|v_4\|_{-1,l,\kappa}=\|\hat v_4\|_{-1,l,\kappa},\quad |||V|||_{-1,k,l,\kappa}=|||\hat V|||_{-1,k,l,\kappa},$$
$$\|u\|_{-1,k,\kappa;\Sigma^\pm}=\|\hat u\|_{-1,k,\kappa;S^\pm},\quad \|u_4\|_{1,l,\kappa;\Sigma^\pm}=\|\hat u_4\|_{1,l,\kappa;S^\pm},$$
$$|||U|||_{1,k,l,\kappa;\Sigma^\pm}=|||\hat U|||_{1,k,l,\kappa;S^\pm},$$
$$\|e\|_{\pm1/2,l,\kappa;\Gamma}=\|\hat e\|_{\pm1/2,l,\kappa;\partial S},\quad \|e_4\|_{\pm1/2,l,\kappa;\Gamma}=\|\hat e_4\|_{\pm1/2,l,\kappa;\partial S},$$
$$|||\mathscr{E}|||_{\pm1/2,k,l,\kappa;\Gamma}=|||\hat{\mathscr{E}}|||_{\pm1/2,k,l,\kappa;\partial S}.$$

Since no ambiguity occurs, the norms on $\mathbf{H}^{\mathscr{L}}_{-1,k,\kappa}(\mathbb{R}^2)$, $H^{\mathscr{L}}_{-1,l,\kappa}(\mathbb{R}^2)$, $\mathscr{H}^{\mathscr{L}}_{-1,k,l,\kappa}(\mathbb{R}^2)$, and on $\mathbf{H}^{\mathscr{L}^{-1}}_{-1,k,\kappa}(\mathbb{R}^3_+)$, $H^{\mathscr{L}^{-1}}_{-1,l,\kappa}(\mathbb{R}^3_+)$, $\mathscr{H}^{\mathscr{L}^{-1}}_{-1,k,l,\kappa}(\mathbb{R}^3_+)$, are denoted by the same symbols. We also extend the use of the symbols $\gamma^\pm$ to the trace operators from $\Sigma^\pm$ to Γ. By $\gamma_0^\pm$ we denote the trace operators from $\Sigma^\pm$ to $S^\pm\times\{0\}$.

We say that $U(x,t)=\{U_+(x,t),U_-(x,t)\}$, $U_\pm\in\mathscr{H}^{\mathscr{L}^{-1}}_{1,0,0,\kappa}(\Sigma^\pm)$, is a weak solution of (TC) if $\gamma_0^\pm u_\pm=0$, $\gamma^+U_+-\gamma^-U_-=F(x,t)$, $(x,t)\in\Gamma$, and for all $W=\{W_+,W_-\}\in C_0^\infty(\overline{\mathbb{R}}^3_+)$,

$$\Upsilon_+(U_+,W_+)+\Upsilon_-(U_-,W_-)=\int_0^\infty\big\{(\mathsf{Q},W)_0+(G,W)_{0;\partial S}\big\}\,dt,$$

where

$$\Upsilon_\pm(U_\pm,W_\pm)=\int_0^\infty\big\{a_\pm(u_\pm,w_\pm)+(\nabla u_{\pm,4},\nabla w_{\pm,4})_{0;S^\pm}-(B^{1/2}_{0,\pm}\partial_t u_\pm,B^{1/2}_{0,\pm}\partial_t w_\pm)_{0;S^\pm}$$
$$-\varkappa_\pm^{-1}(u_{\pm,4},\partial_t w_{\pm,4})_{0;S^\pm}-h_\pm^2\gamma_\pm(u_{\pm,4},\operatorname{div}w_\pm)_{0;S^\pm}-\eta_\pm(\operatorname{div}u_\pm,\partial_t w_{\pm,4})_{0;S^\pm}\big\}\,dt.$$

Theorem 3 *Let $U(x,t)=\mathscr{L}^{-1}\hat U(x,p)$ be the inverse Laplace transform of the weak solution $\hat U(x,p)$ of problem* (TC$_p$)*. If $\mathsf{Q}\in\mathscr{H}^{\mathscr{L}^{-1}}_{-1,l+1,l,\kappa}(\mathbb{R}^3_+)$, $F\in\mathscr{H}^{\mathscr{L}^{-1}}_{1/2,l+1,l+1,\kappa}(\Gamma)$, and $G\in\mathscr{H}^{\mathscr{L}^{-1}}_{-1/2,l+1,l,\kappa}(\Gamma)$, where $\kappa>0$ and $l\in\mathbb{R}$, then $U_\pm\in\mathscr{H}^{\mathscr{L}^{-1}}_{1,l,l,\kappa}(\Sigma^\pm)$ and*

$$|||U_+|||_{1,l,l,\kappa;\Sigma^+}+|||U_-|||_{1,l,l,\kappa;\Sigma^-}$$
$$\le c\big\{|||\mathsf{Q}|||_{-1,l+1,l,\kappa}+|||F|||_{1/2,l+1,l+1,\kappa;\Gamma}+|||G|||_{-1/2,l+1,l,\kappa;\Gamma}\big\}.$$

If, in addition, $l\ge0$, then U is a weak solution of problem (TC).

It can be further shown that the solution of (TC) supplied by this theorem is unique.

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