

Application of Kalman Filter and H^∞ Methodologies to Estimate Attitude of a Satellite Control System Simulator

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Summary

Satellite Attitude Control System usually does not have all the states available for feedback; therefore, full state estimation by any sort of filtering methodology becomes essential. A good estimation algorithm must filter out the undesirable noise from the measurement signal. Kalman Filter (KF) technique is a traditional procedure to estimate the states of a linear system that are not always measured directly by the sensors, minimizing the variance of the estimated error. However, the KF is not fully robustness proven in face of unpredictable noise sources and it is not always able to minimize the error under severe conditions. In that case, the H^∞ filter method is an alternative when robustness is at stake. The H^∞ filter is less known and less commonly applied than the Kalman filter for state estimation, but it presents advantages that make it more effective in some situations. This paper presents the application and comparison between the conventional Kalman filter and the H^∞ technique for estimating attitude of a Satellite Attitude Control System Simulator, which has a reaction wheel as actuator and a gyroscope as sensor. Both filters performance are investigated considering noise variation due to uncertainties in the plant and sensors.

Keyword: Estimation, control system, satellite simulator.

Introduction

Space missions involving automatic procedures for guidance and control, require from the satellite Attitude Control System (ACS) reliability and adequate performance. Experimental validation of new equipment and/or control techniques through prototypes is the way to increase confidence of the ACS, mainly when it is in a noisily environment and suffer of parameters variation. In that case robustness need to be experimentally validated in order to improve ACS performance. The Space Mechanics and Control Division (DMC) of INPE is responsible for constructing a Simulation Laboratory to supply the conditions for implementing and testing satellite ACS. The DMC is constructing a 1D simulator, with rotation in the vertical axis. The simulators consist of a disk-shaped platform, supported on a plane air bearing. The platform can accommodate various satellites components; like sensors, actuators, computers and its respective interface and electronic. Depending on the complexity of the satellite mission the ACS design methods can be based on linear or on nonlinear dynamics. Besides, the ACS never can rely completely on the fact that all the states are available for feedback. As a result,

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state estimation by a sort of filtering methodology becomes essential. Kalman filter methodology (Anderson and Moore, 1979) is very well known and it was developed to solve a specific spacecraft navigation problem. Since then, it has been applied in diverse areas. The Kalman filter (Sorenson, 1985) estimates the states of a linear system, often embedded in control systems to obtain an accurate estimation of some states, which are not always measured directly by the sensors. However, KF method has some limitations, when it assumes that the noise properties are known, and minimizes the average estimation error. Actually it is difficult to implement KF properly when one does not know anything about the system noise and when one prefers to minimize the worst-case estimation error. These limitations gave rise to the H^∞ (infinity) filtering approach (Simon, 2006), also known as minimax filtering which minimizes the "worst-case" estimation error. More precisely, the H^∞ filter minimizes the maximum singular value of the transfer function from the noise to the estimation error. While the Kalman filter requires knowledge of the noise statistics of the filtered process, the H^∞ filter requires no such knowledge. Then, the H^∞ filtering problem minimizes the effect of modeling errors on the estimation error and can be designed to be robust to uncertainty in the system model. As one can see both techniques have their pros and cons. Although, the H^∞ filter is less known and less commonly applied than the Kalman filter, it is expected that it have some advantages when dealing with uncertainty.

Steyn and Hashida (1999) have implemented a Kalman filter type during the initial tumbling phase of a satellite to determine the body angular velocity from magnetometers measurements. Souza, Kuga, and Fenili (2004) have used the Kalman filter to estimate elastic parameters of a rigid-flexible satellite in order to improve the controller performance. Clements, Tavares, and Lima (2000) have developed and implemented an extended Kalman filter attitude estimator to be used in a small satellite control system.

Simulator Dynamic Model

In order to apply the Kalman filter and the H^∞ technique for estimating attitude one has to obtain the Simulator Model (Conti and Souza, 2007), which has one reaction wheel as actuator and one gyroscope as sensor with its respective interfaces and one battery, see Fig. 1. Both filters performance are investigated considering the measurements noise source due to the gyro precision, and the process noise source due to uncertainty in the simulator inertia moment.

The simulator model can be represented by a linear system, which is described by the state and output equations, respectively, given by

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k + w_k \quad (1)$$

$$y_k = C\mathbf{x}_k + z_k \quad (2)$$



Figure 1: Simulator with its equipments

where A , B , and C are matrices; k is the time index; x is the state of the system; u is a known input to the system; y is the measured output; and w and z are the process noise and the measurement noise, respectively. The vector x contains all of the information about the present state of the system, but one cannot measure x directly, instead one measures y , which is a function of x and it is corrupted by the noise z . As a result, the filter process uses y to obtain an estimate of x .

The simulator model is obtained considering that its dynamic is based on a double integrator given by

$$\ddot{\theta} = \frac{N}{I} \quad (3)$$

where $\ddot{\theta}$ is the simulator acceleration around the vertical axis, N is the total torque applied and I the moment of inertia around the simulator vertical axis to be controlled.

In order to put Eq. (3) in states space form one integrates it twice, obtaining

$$\dot{\theta}_{k+1} = \dot{\theta}_k + \frac{N}{I}t + \dot{\bar{\theta}}_k \quad (4)$$

$$\theta_{k+1} = \theta_k + \frac{1}{2} \frac{N}{I}t^2 + \bar{\theta}_k \quad (5)$$

where $\dot{\bar{\theta}}_k$ and $\bar{\theta}_k$ are the velocity and the position noises.

Considering a state vector x given by

$$x_k = \begin{bmatrix} \theta_k \\ \dot{\theta}_k \end{bmatrix} \quad (6)$$

and changing the variables in the way

$$x_1 = \theta \Rightarrow \dot{x}_1 = \dot{\theta} \quad (7)$$

$$x_2 = \dot{\theta} \Rightarrow \dot{x}_1 = \ddot{\theta} \quad (8)$$

The simulator dynamics in state space form is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{N}{I} \quad y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (9)$$

Comparing Eq. (9) with Eqs (1) and (2) it is easy to identify the matrices A, B and C. One observes that the input N is a torque generated by a reaction wheel and the output y is the angular velocity measured by the gyro. Therefore, the state that is not measured and should be estimated is the simulator angular position. As a result, both the Kalman filter and the H^∞ filtering algorithms, besides estimating the angular velocity, will estimate also the angular position, once the angular position is fundamental for implementing control strategies based on traditional PID or optimal LQR and LQR/LTR controllers.

In order to put the system in discrete state space form with sampling period of Δt , one needs to find the transition matrix given by

$$\Phi(s) = (sI - A)^{-1} \quad (10)$$

For a state space system given by Eq. (9), one has

$$x(t) = \Phi(t)x(0) + \int \Phi(t - \tau)Bu(\tau)d\tau \quad (11)$$

which after some manipulation becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix} \frac{N}{I} + w_k \quad (12)$$

$$y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + z_k \quad (13)$$

where z_k is the measurement noise and w_k is the process noise.

Kalman Filter Algorithm

The Kalman filter methodology is very well known, but the main aspects of it are highlighted here in order to compare with the H^∞ technique. In the KF methodology, it is assumed that, the expected value of the estimate should be equal to the expected value of the state. The algorithm minimizes the expected value of the square of the estimation error, which means that on average, the algorithm gives the smallest possible estimation error. One also assumes that the average value of the process noise w and the measurement noise z are zero and that there is no correlation between them.

The process noise covariance matrix S_w and the measurement noise covariance matrix S_z are defined as

$$S_w = E(w_k w_k^T) \quad (14)$$

$$S_z = E(z_k z_k^T) \quad (15)$$

where $(\cdot)^T$ represent matrix transposition and $E(\cdot)$ the expected value.

There are many equivalent ways to express the Kalman filter algorithm equations (Simon, 2007), the formulation adopted here is given by

$$K_k = AP_k C^T (CP_k C^T + S_z)^{-1} \quad (16)$$

$$\hat{x}_{k+1} = (A\hat{x}_k + Bu_k) + K_k(y_{k+1} - C\hat{x}_k) \quad (17)$$

$$P_{k+1} = AP_k A^T + S_w - AP_k C^T S_z^{-1} CP_k A^T \quad (18)$$

where $(\cdot)^{-1}$ indicates matrix inversion. The K matrix is called the Kalman gain, and the P matrix is called the estimation error covariance.

In the state estimate Eq. (17) the first term is the state estimate at time $(k+1)$ that is just A times the state estimate at time k , plus B times the known input at time k . The second term is called the correction term and it represents the amount to correct the propagated state estimate due to our measurement.

Inspection of the K gain equation shows that if the noise covariance matrix S_z is large, K will be small and one will not give much credibility to the measurement y when computing the estimation of x . On the other hand, if S_z is small, K will be large and one will give a lot of credibility to the measurement when computing the estimation of x .

The H^∞ Algorithm

As mentioned before, the Kalman Filter theory has serious limitations when one does not know much about the system noise or when one wishes to minimize the worst-case estimation error rather than the variance of the estimation error. In that case one option available is the H^∞ filter, which does not make any assumptions about the noise, and minimizes the worst-case estimation error (Simon, 2007).

The H^∞ filter theory deals with a linear dynamic system as defined before and it intends to solve the following problem

$$\min_{\hat{x}} \max_{w,v} J \quad (19)$$

where J is some measure of how good our estimator is. The noise terms w and v can be seen as adversaries that try to worsen our estimate. Given the worst possible values of w and v , it is desirable to find a state estimate that will minimize the worst possible effect that w and vz imposes on our estimation error. This is the problem

that the H^∞ filter tries to solve. This is why the H^∞ filter is sometimes called the minimax filter; once it tries to minimize the maximum estimation error.

The function J can be defined by

$$J = \frac{\text{ave} \|x_k - \hat{x}_k\|_Q}{\text{ave} \|w_k\|_w + \text{ave} \|v_k\|_v} \quad (20)$$

where the averages are taken over all time samples k . In other words, one wants to find a state estimate that minimizes J , so that the estimation error is as minimal as possible.

Equation (20) is the statement of the H^∞ filtering problem, which has the task of finding a state estimate that makes J small even when the noise matrices Q , W , and V used in the weighted norms are large. Therefore, the designer has the trade-offs in choosing these matrices. For example, if the w noise will be smaller than the v noise, one should make the W matrix smaller than the V matrix. Similarly, if one is more concerned about estimation accuracy in specific elements of the state vector x , one should define the Q matrix accordingly.

One observes that the function J is bit complicate to be represented mathematically. However, one can solve a related problem, considering that the function to be minimized is given by

$$J < 1/\gamma \quad (21)$$

where γ is some constant number chosen by the designers. So one can find a state estimate with the maximum value of J regardless of the values of the noise terms w and v .

The H^∞ filter equations that minimize Eq. (21) are given by

$$\mathbf{L}_k = (\mathbf{I} - \mathbf{Q}\mathbf{P}_k + \mathbf{C}^T \mathbf{V}^{-1} \mathbf{C}\mathbf{P}_k)^{-1} \quad (22)$$

$$\mathbf{K}_k = \mathbf{A}\mathbf{P}_k \mathbf{L}_k \mathbf{C}^T \mathbf{V}^{-1} \quad (23)$$

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{B}\mathbf{u}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_k) \quad (24)$$

$$\mathbf{P}_{k+1} = \mathbf{A}\mathbf{P}_k \mathbf{L}_k \mathbf{A}^T + \mathbf{W} \quad (25)$$

They have the same form as the Kalman filter equations, but the details are different, where \mathbf{K}_k is the H^∞ gain matrix and \mathbf{I} is the identity matrix. The initial state estimate x_0 and the initial value P_0 should be initialized aiming at acceptable filter performance. This means that no matter the noise terms w and v , the ratio of the estimation error to the noise will always be less than $1/\gamma$.

The mathematical derivation of the H^∞ equations is valid only if γ is chosen such that all of the eigenvalues of the P matrix have magnitudes less than one. If

γ is too large, a solution to the H^∞ filtering problem does not exist. That is, one cannot find an estimator that will make the estimation error arbitrarily small. More details of the H^∞ filter theory can be found in Simon (2006).

Simulation Results

One assumes that the angular velocity is measured with nominal noise of 0.05rad/s (one standard deviation). The commanded torque is a constant of 1Nm and the process nominal noise of 0.01Nm (one standard deviation). The angular position is measured 10 times per second (sample time $\Delta T = 0.1$ s). In the simulation one will evaluate the Kalman filter performance keeping the process noise constant and varying the measured noise and vice-versa. As for the and H^∞ filtering performance one uses in Eq.(20) the same measurement and process noise as before and the state error equal to 0.0001. In the control law design is important to vary the measured noise to represent gyro errors and process noise to represents the uncertainty in the simulator inertia moments so that one obtains a robust control law.

Figure 2 shows Kalman filter (red) and H^∞ filter (blue) performance for a measured noise of 0.05rad/s and the process noise of 0.01Nm. In the first graph one observes that the noisy looking curves of the angular position error for the Kalman filter is bigger than H^∞ filter. In the second graft one observes that the angular velocity error for the Kalman filter is also bigger than the H^∞ filter. Although, for both filters the error is still inside 0.15 rad/s (3 sigma), for the Kalman filter the angular position is a little worse than the H^∞ filter.

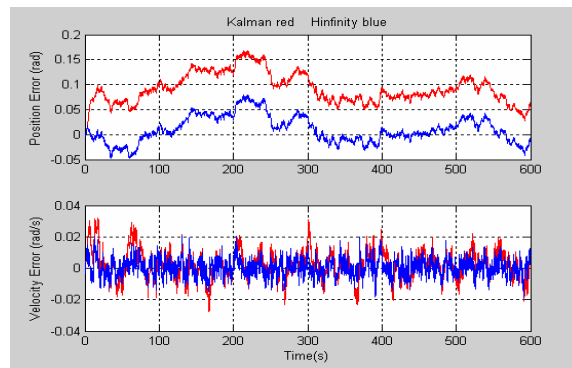


Figure 2: Kalman and H^∞ filters errors for Nominal noise

Figure 3 shows the Kalman filter and H^∞ filter performance for a measurement noise of 0.5rad/s and the process noise of 0.01Nm. In the first graph one observes that in the first 200 seconds the angular position error for the Kalman filter is much more degraded than H^∞ filter. In the second graft one observes that the angular velocity error for the Kalman filter is also degraded as for the H^∞ filter. Now, both filter error still inside 1.5 rad/s (3 sigma).

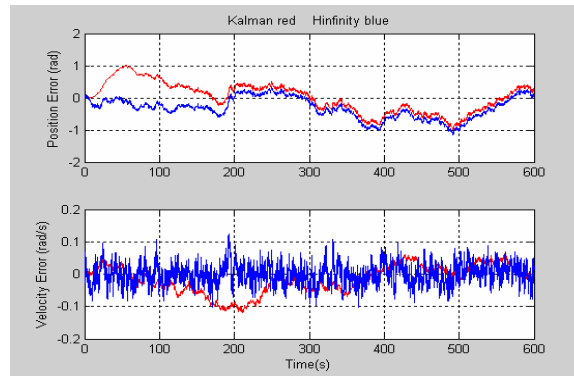


Figure 3: Kalman and H^∞ filters errors for variation in the measurement noise

Figure 4 shows the Kalman filter and H^∞ filter performance for a measurement noise of 0.05rad/s and the process noise of 0.1Nm . In the first graph one observes that in the first 50 seconds the angular position error for the Kalman filter is similar to the H^∞ filter. However, after that, the Kalman filter angular position error increases. In the second graph one observes that the angular velocity error for the Kalman filter is similar to the H^∞ filter. For both filters the errors is still inside 0.15 rad/s (3 sigma).

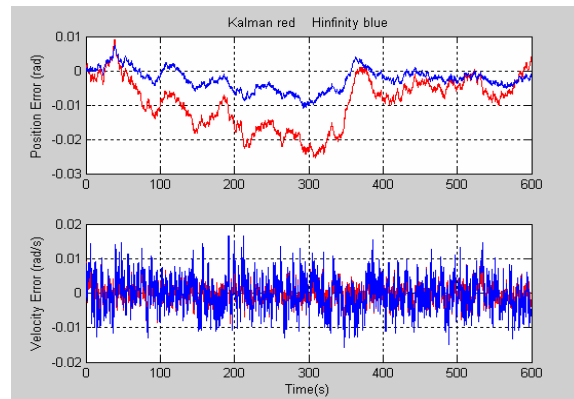


Figure 4: Kalman and H^∞ filters errors for variation in the process noise

Conclusions

This paper presents the application of the Kalman filter and the H^∞ filter techniques for estimating attitude of a Satellite Attitude Control System Simulator, which has a reaction wheel as actuator and a gyro as sensor. Both filters performance are investigated considering measurement and process noise variation, which represents uncertainties in sensor and in the plant. The simulator equations of motions were obtained for a platform with rotation around the vertical axis. Once

the simulator has only angular velocity sensor and it is planned to implement PD, PID, LQR and LQG control law type, both filters algorithm can be used to estimate also the angular position. From simulations one observes that the Kalman filter performance is worse than the H^∞ filter when the measurements noise and process noise increased separately. This means that the Kalman filter may not comply with the attitude control specification when there is great uncertainty in the sensor and plant. As a result, the H^∞ filter algorithm is a good candidate to be implanted in the simulator control system.

Acknowledgement

The author wish to thank CNPq who has support that research by the grant no 306534/2006-1.

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