Particular Solutions of Chebyshev Polynomials for Polyharmonic and Poly-Helmholtz Equations

C.C. Tsai¹

Summary

In this paper we develop analytical particular solutions for the polyharmonic and the products of Helmholtz-type partial differential operators with Chebyshev polynomials at right-hand side. Our solutions can be written explicitly in terms of either monomial or Chebyshev bases. By using these formulas, we can obtain the approximate particular solution when the right-hand side has been represented by a truncated series of Chebyshev polynomials. These formulas are further implemented to solve inhomogeneous partial differential equations (PDEs) in which the homogeneous solutions are complementarily solved by the method of fundamental solutions (MFS). Numerical experiments, which include fourth order PDEs and three-dimensional cases, are carried out. Due to the exponential convergence of the Chebyshev interpolation and the MFS, our numerical results are extremely accurate.

keywords: Particular solution, Chebyshev polynomials, polyharmonic equation, product of Helmholtz equation, boundary element method, method of fundamental solutions, Trefftz method, radial basis function

¹Department of Information Technology, Toko University, Chia-Yi County, 61363, Taiwan, E-mail: tsaichiacheng@ntu.edu.tw