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Study on Band Gaps and Localization Phenomenon in 2D Ordered and Randomly Disordered Phononic Crystals

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Summary

Band gaps and localization phenomenon for both in-plane and anti-plane elastic waves propagating in 2D ordered and disordered phononic crystals are studied in this paper. The localization of wave propagation due to random disorder is discussed by introducing the concept of the localization factor which is calculated by the plane-wave-based transfer-matrix method.

Introduction

Since Kushwaha [1] proposed the concept of "phononic crystal", an artificial periodic elastic/acoustic structure that exhibits so-called "phononic band gaps" [2], a great deal of attention has been focused on this kind of artificial lattice structures [3]. Band gaps involved in phononic crystals have numerous potential engineering applications such as acoustic filters, control of vibration isolation, noise suppression and design of new transducers. So far, many results have been reported on the perfectly ordered phononic crystals or those with defects [4, 5]. However, in practical cases random disorder, usually caused by randomly distributed material defaults or manufacture errors during production process, is very common. This may lead to the localization phenomenon, like the well-known Anderson localization of electron waves in disordered systems [6]. Researches on randomly disordered phononic crystals are very limited [7, 8], especially on two-dimensional (2D) cases. This topic, we believe, is of practical importance not only because the randomly distributed manufacture errors may cause the disorder as we mentioned before, but also because one may expect to tune the band structures of phononic crystals and thus control the propagation behavior of elastic waves by intentionally introducing disorder.

In this paper the band gaps of 2D ordered and randomly disordered phononic crystals are studied by using the plane-wave-based transfer-matrix method [9]. Instead of calculating the transmitted waves, we will use a well-defined localization factor [10] to characterize the band structures and localization phenomenon of the system. As expected, the numerical results show that the localization factor predicts the same band structures as the transmitted coefficients does for both ordered and disordered phononic crystals.

Problem statement and plane-wave-based transfer-matrix method

Consider a 2D solid-solid phononic crystal with lattice constant a as shown in Fig.1a. Rectangle plumbum rods with wide length t_1 are embedded in epoxy host

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material and arranged in square lattice. The filling fraction of the rods is $f = t_1^2/a^2$.

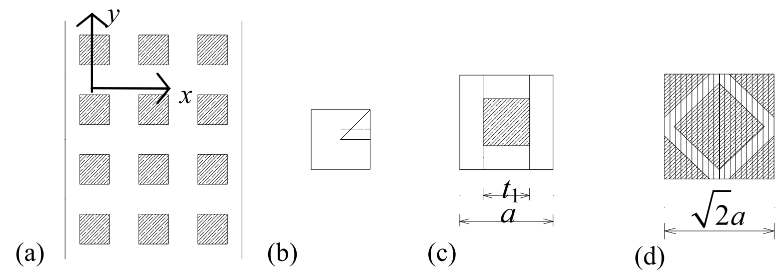

Figure 1: The schematic (a), the first Brillouin zone (b) and the calculated unit cells of the 2D phononic crystal along the high-symmetry lines of the first Brillouin Zone for the ΓX direction (c) and MΓ direction (d).

In this paper the plane-wave-based transfer-matrix method is employed. First, we divide the phononic crystal into a number of thin slices as shown in Fig. 1c and d. Then, in each slice along the y-direction the plane wave expansion method is used. While, along the x-direction the boundary conditions are used to connect the nearest slice. After a simple iteration algorithm, the T-matrix of the unit cell and the whole crystal can be obtained. Finally, the localization factors and the responds of the phononic crystals can be obtained. For details, we refer to Refs. [4] and [9]. It should be noticed that different cutting-directions should be chose to calculate the responds along different high-symmetry lines of the first Brillouin Zone (BZ) (see Fig. 1b), and so do the calculated unit cells. The calculated unit cells along ΓX and MΓ directions are shown in Fig. 1c and 1d, respectively.

Localization factor

We introduce the concept of the localization factor, which is used to describe the vibration localization phenomenon in nearly periodic engineering structures, to study the present plane wave propagation in randomly disordered phononic crystals. This factor is defined as the smallest positive Lyapunov exponent [10]. And the expression for calculating the localization factor of the system was given by Wolf [11]:

$$\gamma_m = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \ln \left\| \hat{v}_{2R,m}^{(k)} \right\|. \quad (1)$$

Numerical examples and discussion for 2D phononic crystals

Detailed calculation will be performed for the two-situations in this section:

(i) Perfectly periodic phononic crystals;

(ii) Disordered phononic crystals. We take the filling fraction f as the disordered parameter and f is assumed to be a uniformly distributed random variable with the mean value f_0, and the disordered degree, δ. Then f can be expressed as

$$f = f_0[1+\sqrt{3}\delta(2t-1)]. \quad (2)$$

where $t \in (0,1)$ is a standard uniformly distributed random variable.

First, in order to testify the correctness of our method, the localization factors (Fig. 2c), the transmission coefficients (Fig. 2a) and the dispersion curves (Fig. 2b) along the high-symmetry line of the first BZ, i.e., ΓX directions are calculated for the case of the in-plane wave incidence by the plane-wave-based transfer-matrix method [9], the eigenmode match theory method [4] and the plane wave expansion method, respectively.

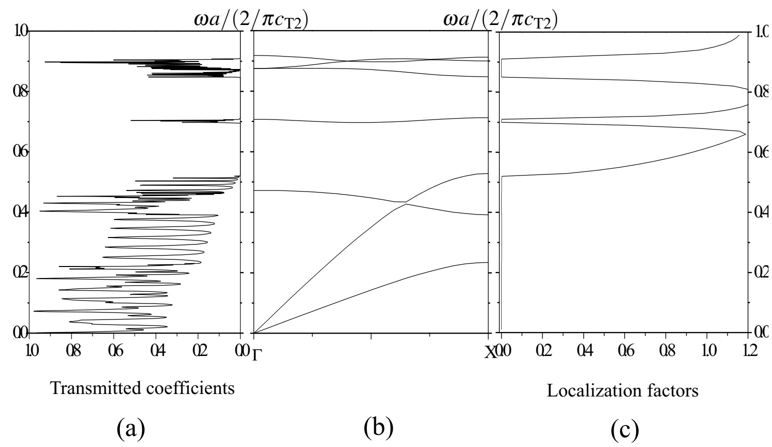

Figure 2: The band gaps along the ΓX direction which are predicted by the transmitted coefficients (a), dispersive curves (b), and the localization factors (c), respectively

According to the definition of the localization factor, if its value is zero, the corresponding frequency intervals are known as passbands; otherwise if its value is positive, the intervals are known as stopbands or band gaps. It can be seen that the band gaps (the frequency intervals (0.52, 0.70), (0.71, 0.845)) predicted by the localization factors have a good agreement with those shown by both the transmission coefficients and the dispersive curves. Similar results are obtained for the XM and MΓ edges and the case of the anti-plane wave incidence. Therefore, it can be concluded that the localization factor is an effective parameter to describe the band gaps of a 2D phononic crystals.

Then the band structures of the randomly disordered 2D phononic crystal with the filling fraction having a disorder degree $\delta = 0.05$ will be considered. Take the

ΓX edge as an example, the band structures characterized by the transmitted coefficients and the localization factors are calculated and shown in Fig. 3a and by the dashed line in Fig. 3b, respectively. The localization phenomenon can be found in the frequency intervals (0.695, 0.715) and (0.845, 0.910) as shown in Fig. 3b. It can be seen that the localization factors and the transmitted coefficients predict the same band structures. This implies that the localization factor is an effective and efficient parameter to describe the band gaps of the 2D randomly disordered phononic crystals.

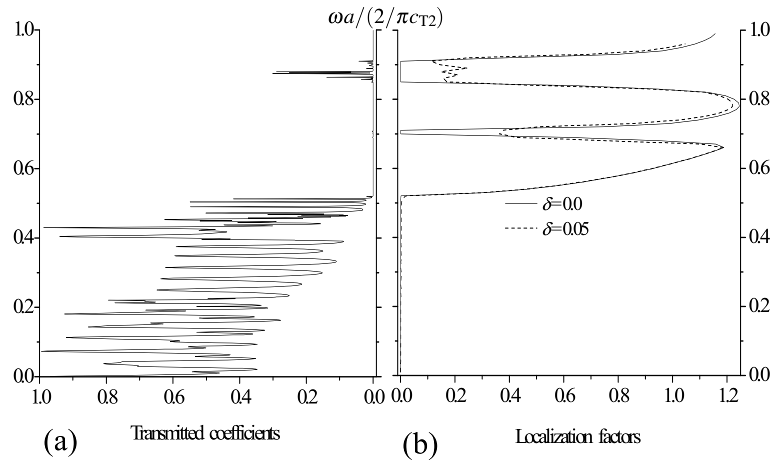

Figure 3: The transmitted coefficients (a) and the localization factors (the dashed line in (b)) of the disordered phononic crystal with the disordered degree is 0.05. The solid line in (b) is the band structures along ΓX direction of the perfect one (δ=0).

As the last example, we illustrate the localization factors varying with the normalized frequency $\omega a/(2/\pi c_{T2})$ and k_y in the grey-scale maps for ordered (Fig.4a) and disordered phononic crystals with $\delta = 0.05$ (Fig.4b) in the case of oblique propagation of the anti-plane wave. For ordered system (Fig.4a), it can be seen that the frequency interval (0.9, 1.0) changes to a band gap and a band gap appears in the frequency interval (0.5, 0.6) and then disappears (which is different in Fig. 4b) with the increase of k_y. Comparison between the two figures shows that the localization phenomenon appears in the disordered system (Fig. 4b) and the values of the localization factors in Fig. 4b are generally smaller than those shown in Fig. 4a. It should be noticed that the solid line in Fig. 4 describes the band structure for the inner line of the first BZ which parallel to the ΓX direction (such as the dashed line in Fig. 1b). Particularly notice that a band gap appears first as the frequency increases from zero for $k_y > 0$, which is due to the total reflection of waves at the interfaces.

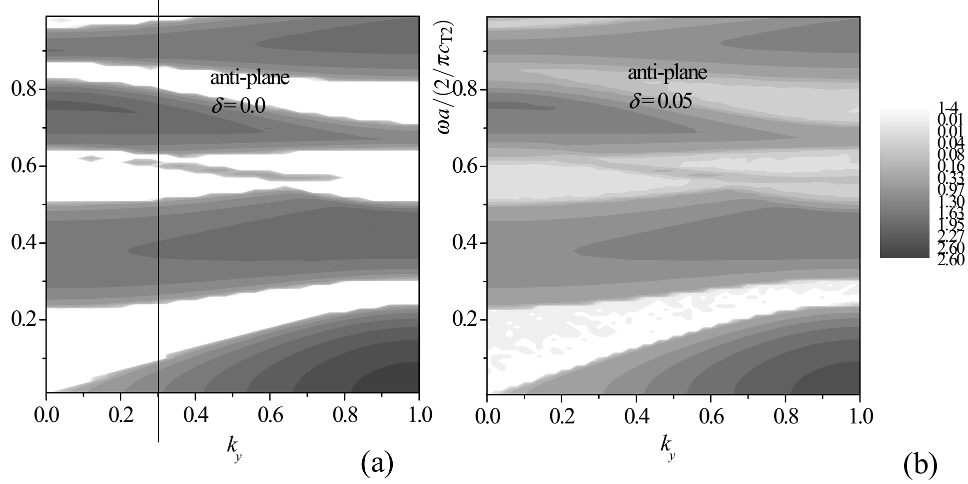

Figure 4: Localization factors and band structures vary with the normalized frequency and normalized wave number along the *y*-axis for the anti-plane wave propagating in crystals. (a) is for the ordered phononic crystal and (b) is for the disordered one with the disordered degree is 0.05.

Conclusion Remarks

The concept of the localization factor is introduced to describe the band structures and localization behaviors of 2D perfect, randomly disordered phononic crystals. Comparison with other methods such as the dispersive relation and transmission coefficients, etc. shows

(i) The localization factor is an accepted and effective parameter in characterizing the band gaps and localization behaviors of 2D perfect and randomly disordered phononic crystals. Band structures of the phononic crystals can be tuned by introducing random disorder to the them.

(ii) The localization factor can character the band structures not only along the high-symmetry edges but also the inner of the first Brillouin Zone of the 2D phononic crystals.

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