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Compact integration factor methods in high spatial dimensions

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Summary

The dominant cost for integration factor (IF) or exponential time differencing (ETD) methods is the repeated vector-matrix multiplications involving exponentials of discretization matrices of differential operators. Although the discretization matrices usually are sparse, their exponentials are not, unless the discretization matrices are diagonal. For example, a two-dimensional system of $N \times N$ spatial points, the exponential matrix is of a size of $N^2 \times N^2$ based on direct representations. The vector-matrix multiplication is of $O(N^4)$, and the storage of such matrix is usually prohibitive even for a moderate size N. In this paper, we introduce a compact representation of the discretized differential operators for the IF and ETD methods in both two and three dimensions. In this approach, the storage and CPU cost are magnitudely reduced for both IF and ETD methods. For the case of two-dimensional systems, the required storage and CPU cost are reduced to $O(N^2)$ and $O(N^3)$, respectively. The improvement on three-dimensional systems is even more significant. We analyze and apply this technique to a class of semi-implicit integration factor method recently developed for stiff reaction-diffusion equations. Direct simulations on test equations along with applications to a morphogen system in two dimensions and an intra-cellular signaling system in three dimensions demonstrate an excellent efficiency of the new approach.