

Piezoelectric Bimorph Response with Imperfect Bonding Conditions

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Summary

The effect of the finite stiffness bonding between the piezoelectric plies of bimorph devices has been investigated. A boundary integral formulation for piezoelectricity, based on a multidomain technique with imperfect interface conditions, has been developed. The imperfect interface conditions between the piezoelectric layers are described in terms of linear relations between the interface tractions, in normal and tangential directions, and the respective discontinuity in displacements. Continuity of the electric potential at the interface is also assumed and an iterative procedure is implemented to avoid interface interference. Numerical analysis has been performed on bimorph configurations with series arrangement and the influence of the adhesive is pointed out for both sensing and actuating functions.

Introduction

Due to the direct or converse piezoelectric effect, piezoelectric materials can be used in the design of many devices working as sensors or actuators [1]. For these reasons piezoelectric materials are a primary concern in the field of Smart-Structure technology [2, 3]. There are basically two strategies adopted to integrate piezoelectric patches into host structures: bonding or embedding. In both cases the interface between the host structure and the piezoelectric material plays a decisive role in terms of strain/stress transfer mechanism. In fact a good interface ensures that effective actuation or sensing is achieved, avoiding the excessive voltages being applied for actuators and inaccurate output results for sensors [4]. Studies including the finite-stiffness character of the interface between the piezoelectric devices and the host structures can be found in literature. Crawley and de Luis [2] assumed that PZT patches are perfectly bonded to the host structure and only shear stresses exist in the adhesive layer. Using similar assumptions, Crawley and Lazarus [5] applied the theory to plates with perfectly bonded PZTs. Luo and Tong [6] modeled the PZT patches and the host beams as Euler-Bernoulli beams and the adhesive as a continuous spring with shear and peel stiffness. Later the same authors developed a laminated beam element for PZT smart beams and a laminated plate element for PZT smart plate including the shear and peel stiffness of the adhesive [7, 8]. In this paper a multidomain boundary integral formulation [9] with imperfect bonding conditions is employed to assess the influence of the bonding layer on the sensing and actuating capabilities of piezoelectric devices. The characterization of the imperfect bonding conditions at the interface is approached by using a spring model

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which involves shear and peel stiffness. Additionally, continuity of electric potential at the interface is assumed. Numerical analyses on piezoelectric bimorph devices are presented for both sensing and actuating functions. The results show that the method proposed is able to point out the impact of the adhesive layer on the static electro-elastic response of this kind of Smart Structures.

Boundary Integral representation and Numerical Model

The formulation is developed for a two-dimensional piezoelectric domain Ω with boundary $\partial\Omega$ lying in the $x_1 x_2$ plane under the hypothesis of generalized plain strain elasticity and in-plane electrostatic. The governing equations of the problem can be found in reference [10]. Considering a particular electroelastic state defined by the generalized displacement field \mathbf{U}_j associated with the concentrated generalized body forces \mathbf{F}_j acting in an infinite domain and applied at the point P_0 , the reciprocity theorem for this particular and the actual electroelastic states leads to the analogous of the Somigliana identity for the electromechanical problem [10]

$$\mathbf{c}^*\mathbf{U}(P_0) + \int_{\partial\Omega} (\mathbf{T}^*\mathbf{U} - \mathbf{U}^*\mathbf{T})d\partial\Omega = \int_{\Omega} \mathbf{U}^*\mathbf{F}d\Omega \quad (1)$$

The boundary integral formulation is numerically implemented by using the boundary element method [11], which provides a linear algebraic resolving system expressed in terms of generalized displacements and tractions nodal values δ and \mathbf{P} , respectively

$$\mathbf{H}\delta + \mathbf{G}\mathbf{P} = \mathbf{0} \quad (2)$$

Eqs. (2), coupled with the electromechanical boundary conditions, provides the solution of the problem for a single domain.

Multidomain BEM and generalized Spring Model at interface

The multidomain boundary element method [10, 11] is based on the division of the original domain into homogeneous subregions so that Eqs. (2) still hold for each single subdomain and the following relation can be written

$$\sum_{j=1}^M \mathbf{H}_{ij}^i \delta_{ij}^i = \sum_{j=1}^M \mathbf{G}_{ij}^i \mathbf{P}_{ij}^i \quad (i = 1, 2, \dots, M) \quad (3)$$

where M is the number of subregions considered, the superscript i indicates the quantities associated with the i -th subdomain. Provided that for $i = j$ the nodes belong to the external boundary of the i -th subdomain, the subscripts ij denote quantities related to the nodes belonging to the interface between the i -th and j -th subdomains (see figure 1). In the Eqs. (3) \mathbf{H}_{ij}^i and \mathbf{G}_{ij}^i are the matrices of influence coefficients pertaining to the quantities δ_{ij}^i and \mathbf{P}_{ij}^i , respectively. To obtain the solution of the problem, the generalized displacement continuity and generalized traction equilibrium conditions along the interfaces between contiguous subdomains

need to be restored. If imperfect bonding is assumed, indicating by $\Delta\delta^{ij}$ the vector of the generalized displacement jumps across the interface between the i -th and j -th subdomains, the compatibility conditions are written as

$$\delta_{ji}^j = \delta_{ij}^i + \Delta\delta^{ij} \quad i = 1, \dots, M-1; \quad j = i+1, \dots, M \quad (4)$$

According to the Interface Spring Model [9] and referring to local coordinate systems centered at each node of the interface boundary belonging to the domain i (see figure 1), the normal and tangential components of the displacement jump and the electric potential jump across the interface can be written as

$$\begin{aligned} \delta_N^j &= \delta_N^i + \Delta\delta_N^{ij} \quad \text{with } \Delta\delta_N^{ij} = k_N \mathbf{P}_N^i \\ \delta_T^j &= \delta_T^i + \Delta\delta_T^{ij} \quad \text{with } \Delta\delta_T^{ij} = k_T \mathbf{P}_T^i \\ \varphi^j &= \varphi^i + \Delta\varphi^{ij} \quad \text{with } \Delta\varphi^{ij} = k_e \mathbf{D}_N^i \end{aligned} \quad (5)$$

where k_N and k_T denote the interface compliance coefficients along normal and tangential directions, while k_e is the electric fictitious compliance coefficient to be set to zero in order to restore the continuity of the electric potential between two contiguous subregions.

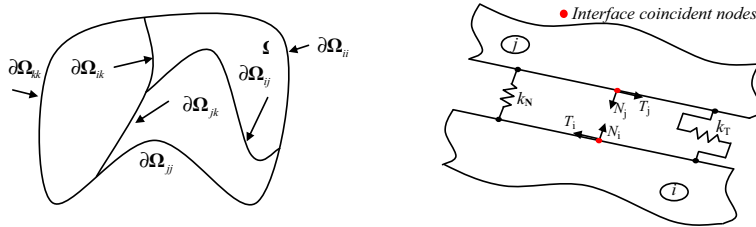


Figure 1: Multidomain configuration and Spring Model at interface.

Consequently in the global coordinate system one has [9]

$$\delta_{ji}^j = \delta_{ij}^i + \mathbf{K}_{ij} \mathbf{P}_{ij}^i \quad i = 1, \dots, M-1; \quad j = i+1, \dots, M \quad (6)$$

where \mathbf{K}_{ij} is a matrix containing the compliance interface constants and the transformation matrix from local to global reference at the considered interface node. It is worth nothing that the modeling of the displacement jump at the interface does not require auxiliary interface elements since the compliance constants, characterizing the elastic behavior of the adhesive between two different layers, enter the assembling of matrices \mathbf{H} and \mathbf{G} . The system of Eqs. (3) and the interface conditions provides a set of relationships, which, together with the external boundary conditions on the external boundaries, allows to obtain the electromechanical response of each subdomain. An iterative procedure need to be used to avoid overlap

between interface coincident nodes. The algorithm employed in the present work detects the contact conditions by checking the sign of the normal component of the mechanical displacement jump at the interface. In the case of detected contact the compliance constant k_N is set to zero.

Numerical applications and discussion

The first configuration analysed consists of a piezoelectric series bimorph in which the piezoelectric layers are used as sensors in closed circuit. The length of the bimorph is $L=25\text{mm}$ and a slender ratio $L/h=10$ is considered. The boundary conditions and the material properties are taken from [12].

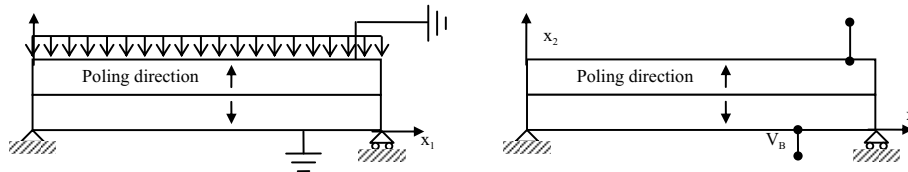


Figure 2: Series bimorph configuration as sensor and actuator.

Figure 3 show the through-the-thickness normalised vertical displacement and electric potential distributions at $x_1 = L/2$ in the case of perfect bonded interface. the results are compared with those obtained by the finite element analysis performed for the full 3D model [12].

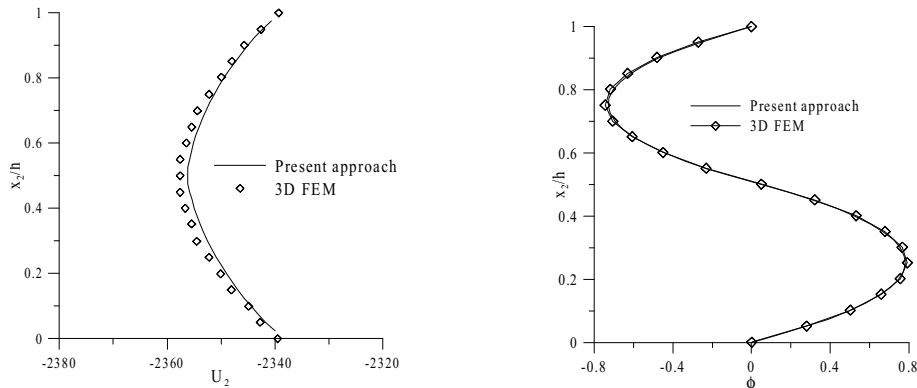


Figure 3: Through-the-thickness Vertical Displacement, Electric Potential.

Figure 4 shows the vertical displacement distribution obtained for both perfect and imperfect bonding conditions. A 32% increment of the deflection in the case of imperfect bonding, due to the softening of the bimorph, can be observed. This leads to a change of the through-the-thickness electric potential distribution as shown in figure 4. The modelling of the adhesive layer is then of significant importance in order to correctly interpret the output of a bimorph used with sensing function. To point out the effect of the adhesive layer on the actuating performances of a

piezoelectric bimorph with series arrangement, an electric potential is applied to the top and bottom faces of the plate ($V_B = -50V$ at $x_2 = 0$ and $V_U = 50V$ at $x_2 = h$) as shown in figure 2. Figure 5 shows the through-the-thickness normalised vertical and longitudinal displacement. The softening of the bimorph actuator due to the adhesive layer can be pointed out by observing a 17% reduction of the through-the-thickness deflection. As a consequence, the bonding layer affects the effectiveness of a bimorph with actuating function since, in order to obtain a fixed deflection, more voltages is needed.

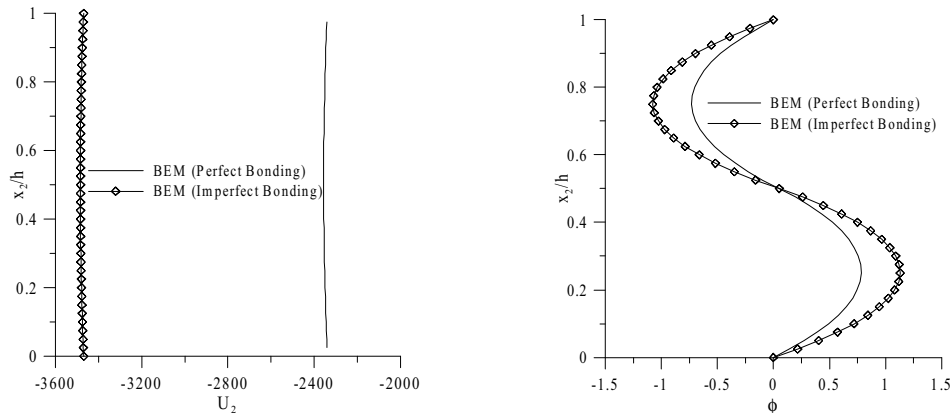


Figure 4: Vertical Displacement BEM, Electric Potential BEM (perfect/imperfect bonding).

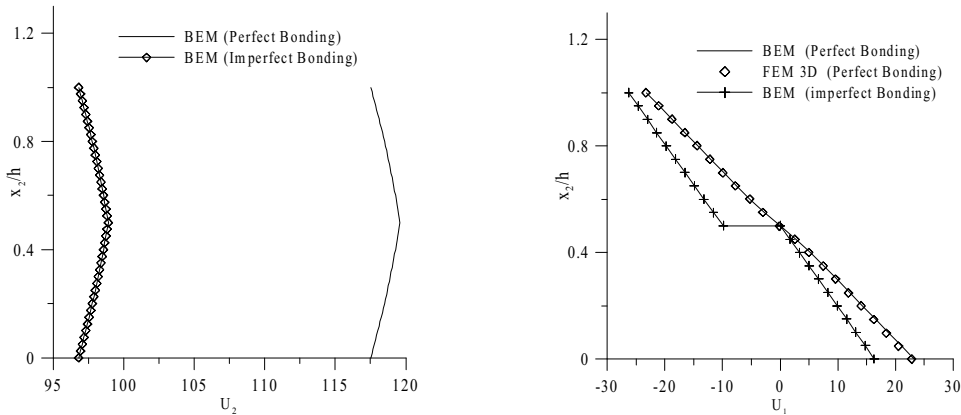


Figure 5: Through-the-thickness Vertical Displacement, Longitudinal Displacement.

Conclusion

The effect of the adhesive layer on the piezoelectric bimorph response has been investigated by implementing a multidomain boundary integral formulation

for piezoelectricity with imperfect bonding conditions. From the analyses performed on piezoelectric bimorph with series arrangement a noticeable variation of the electromechanical response for both sensing and actuating functions has been pointed out. In particular, the presence of the adhesive layer leads to a reduction of the structure stiffness and consequently to a decay of the bimorph effectiveness.

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