

Model Adaptivity and Green's Functions

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Summary

The reliability of computer predictions has become known as the subject of Verification and Validation (V&V). The verification process addresses the quality of the numerical treatment of the model while the validation process addresses the quality of the model.

The central issue for both aspects is the Green's function of the governing equation. If we define Green's function as the inverse operator which connects the input with the output then both issues obviously lead to the same question: How accurate is the approximation of the Green's function and does the model approximate the correct Green's function? The numerical error as well as the modelling error strongly depend on the nature of the Green's functions, their rate of decay, their domain of influence and their shifts (Green's function are pseudo differential operators).

Things become additionally complicated in that structures are made of composite materials so that homogenization techniques tend to blur the picture and multiscale modelling complicates the resolution of the (numerical) Green's functions.

We know that the error in a (linear) FEanalysis is attributable to the error $G_0 - G_h$ in the Green's functions and to gain accuracy we must better the projections G . Adaptive methods based on this duality technique have been applied very successfully in recent years.

This technique is also the key to the assessment of the modelling error. Unlike standard error analysis where $u - u_h$ is the FEerror we must now be concerned, additionally, with a modelling error $u - u_0$ where u_0 is the exact solution of a simplified problem (after some homogenization techniques are applied).

The issue are therefore techniques which allow to adaptively refine a model so that the sensitivity and reliability of the model can be 'guaranteed' with regard to a certain output value.

The a posteriori error estimates required can be developed in the same framework as the now 'classical' duality approach of Rannacher [1]. We shall outline a general technique for comparing coarse and fine models and to produce error estimates for computing modelling errors, [2]

References

1. Bangerth W, Rannacher R. (2003) Adaptive Finite Element Methods for Differential Equations. Birkhäuser Verlag Basel Boston Berlin.

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2. Hartmann F, Katz C (2007) Structural Analysis with Finite Elements Springer-Verlag Berlin Heidelberg New York.