

Numerical Study of Dynamics of Point Vortex Configurations

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Summary

We study the dynamics of planar vortices. We follow the analysis of the integrable 3-vortex problem initiated by Synge (Can. J. Math. 1, 1949) and supplemented by Tavantzis & Ting (Phys. Fluids 31, 1988), the perturbation analysis of the 3-vortex problems by Blackmore et al. (J. Math. Phys. 48, 2007), and recent studies of clusters of vortices by Ting et al. presented in ICIAM'07. By a cluster of J vortices, we mean that the distances between these vortices are much smaller than their distances to vortices not in the cluster, so the leading order dynamics in a cluster is independent of the vortices outside the cluster and is integrable when J is less or equal to 3. The cluster is binary or ternary, respectively, when J equals 2 or 3. Here we report on numerical studies of cluster interactions. For a binary interacting with a third vortex, we show that there are three possibilities: weak interaction, scattering of the binary and the interchange of a vortex in the binary with the third vortex. These behaviors were predicted by Synge and Tavantzis and correspond to the three types known in celestial mechanics for the interaction of a binary star with a third body. Since the vortex strengths can be positive or negative and the 3-vortex problem is integrable, in contrast to the 3-body problem in celestial mechanics, the interactions of vortices should generate new types of interactions not found in classical mechanics. For example, the motion of a vortex in a corner region can be considered as the interaction of two binaries of opposite orientation and initially far apart. Their interaction produces exchange of vortices to form two new widely separated binaries. Similar interactions of N binaries would be expected from the simulations of a single vortex moving in a wedge shaped region with an angle of $180/N$ degrees. Simulations of interaction of a cluster of three vortices with another cluster of one, two or three vortices should generate new types of interactions.

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