Numerical Revisit to a Food Web of Four Species

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Abstract

Numerical simulations are made for studying a food web of four species including a bottom prey X, two competing predators, Y and Z, on X, and a super-predator W only on Y. Such model had been studied by [1] on the existence of a chaotic attractor, on which all species coexist, by using a geometric method of singular perturbations. It was shown later in [2] that the chaotic attractor was formed via period-doubling cascades from a Hopf bifurcation point. A one-dimensional bifurcation diagram using the relative reproduction rate of Z as the bifurcation parameter was computed to show period-doubling cascades leading to chaos.

In this work, the simulations are performed with the aid of a numerical method for locating the boundaries of existence of equilibria and the local bifurcation boundaries of equilibria in the above-mentioned system on the parameter space. The numerical method for locating bifurcation boundaries developed in this work does not need the computation of the normal form in contrast to the typical work presented in [3]. Two-parameter bifurcation diagrams of the system are constructed using this numerical method. Preliminary results have found bifurcation curves on the parameter planes such as transcritical, saddle-node, Hopf bifurcations, etc., of equilibria. The bifurcation curves divide the two-dimensional parameter planes into several regions on which asymptotic behaviors will be discussed. Coexisting attractors and the existence of chaotic attractors are found in some of the regions. The bifurcation diagrams also contain codimension-two points. The phase portraits in the vicinity of each codimension-two point will be shown to fully understand the bifurcation diagrams. Three-parameter diagrams will be constructed for the search of higher codimension points.

References

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