Numerical Simulations of Multiphase Flow Problems by an Energy-stable Finite Element Scheme

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Abstract

Multifluid and multiphase flows occur in many scientific and engineering problems. Two key issues in analyzing those flows are to find the position of interfaces separating fluids and to handle the surface tension on the interfaces. Many numerical schemes have been developed and applied to those flow problems, but to the best of our knowledge, there are no numerical schemes whose solutions are proved to converge to the exact one. There are very little discussion even for the stability of schemes.

Recently we have developed a finite element scheme based on energy-stable approximation [1,2]. The scheme is proved to be stable if a quantity corresponding to L^2-norm of the curvature remains bounded in the computation. Since we do not use the maximum norm, the computation proceeds stably while the squared integral value is bounded even if the value becomes very large at a point like a cusp.

By using this scheme, some numerical simulations are performed for rising bubble problems and hourglass problems, where the fluids are governed by the incompressible Navier-Stokes equations and surface tension is exerted on the interface. On the boundary both slip and non-slip boundary conditions are considered. Merging of bubbles are also simulated. Numerical results show the robustness and the applicability of the scheme.

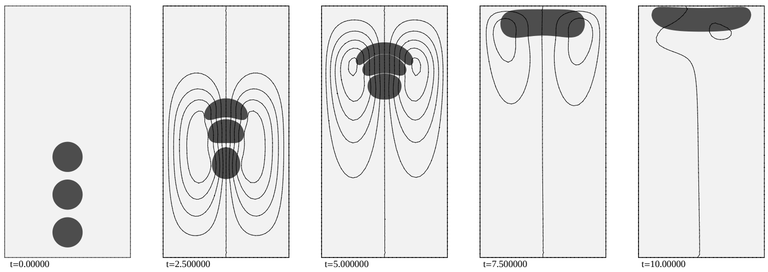

Figure 1: Merging of three bubbles ($\rho_0 = 100.0, \rho_1 = 0.1$, $\rho_2 = 10.0$, $\rho_3 = 19.9$)

Reference

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