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Inverse Estimation of Moisture Diffusivity by Utilizing Temperature Response of a Drying Body

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Summary

This paper deals with the application of inverse concepts in drying. The objective of the paper is estimation of the moisture diffusivity of a drying body by using only temperature measurements. Potato and apple slices have been chosen as representative drying bodies with significant shrinkage effects. A mathematical model of the drying process of shrinking bodies has been applied. The Levenberg-Marquardt method and a hybrid optimization method of minimization of the least-squares norm were used to solve the present parameter estimation problem.

Introduction

An inverse approach to parameter estimation in the last few decades has become widely used in various scientific disciplines. In this paper, application of inverse concepts in drying is analyzed. A mathematical model of the drying process of shrinking bodies has been applied where the moisture content and temperature field in the drying body are expressed by a system of two coupled partial differential equations. The system of equations incorporates several coefficients that are functions of temperature and moisture content and must be determined experimentally. All the coefficients, except for the moisture diffusivity, can be relatively easily determined by experiments. The main problem in the moisture diffusivity determination by classical or inverse methods is the difficulty of moisture content measurements. In this paper the moisture diffusivity is estimated by the temperature response of a drying body. The main idea is to make use of the interrelation between the heat transport and mass (moisture) transport processes within the drying body and from its surface to the surroundings. Then, the moisture diffusivity of the drying body can be estimated on the basis of accurate and easy to perform single thermocouple temperature measurements by using an inverse approach.

A Mathematical Model of Drying

The physical problem involves a single slice of a potato of thickness $2L$ initially at uniform temperature and uniform moisture content (Fig. 1). The surfaces of the drying body are in contact with the drying air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The problem

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is symmetrical relative to the mid-plane of the slice. The thickness of the body changes during the drying from $2L_0$ to $2L_f$.

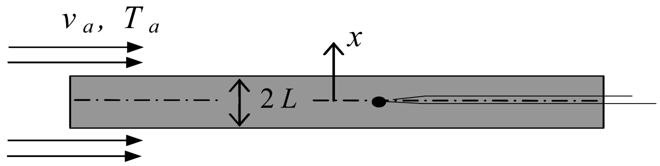

Figure 1: Scheme of the drying experiment

In the case of an infinite flat plate the unsteady temperature, $T(x,t)$, and moisture content, $X(x,t)$, fields in the drying body are expressed by the following system of coupled nonlinear partial differential equations for energy and moisture transport

$$c\rho_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \varepsilon \Delta H \frac{\partial(\rho_s X)}{\partial t} \tag{1}$$

$$\frac{\partial(\rho_s X)}{\partial t} = \frac{\partial}{\partial x}\left(D\rho_s \frac{\partial X}{\partial x}\right) \tag{2}$$

Here, t, x, c, k, ΔH, ε, D, ρ_s are time, normal distance from the mid-plane of the plate, specific heat, thermal conductivity, latent heat of vaporization, ratio of water evaporation rate to the reduction rate of the moisture content, moisture diffusivity, and density of dry solid, respectively.

The shrinkage effect of the drying body was incorporated through the changes of the specific volume of the drying body. There are several models for describing the changes of the specific volume of the body during drying. In this paper, linear relationship between the specific volume, v_s, and the moisture content, X, has been used

$$v_s = \frac{1}{\rho_s} = \frac{V}{m_s} = \frac{1+\beta' X}{\rho_{b0}} \tag{3}$$

where m_s is the mass of the dry material of the drying body, V is the volume of the drying body, ρ_{b0} is the density of a fully dried body and β' is the shrinkage coefficient.

The problem of the moving boundaries due to the changes of the dimensions of the body during the drying was resolved by introducing the dimensionless coordinate

$$\psi = \frac{x}{L(t)} \tag{4}$$

Substituting the above expression for $\rho_s(= 1/v_s)$ and ψ into Eqs. (1) and (2) and rearranging, the resulting system of equations for the temperature and moisture content prediction becomes

$$\frac{\partial T}{\partial t} = \frac{k}{\rho_s c}\frac{1}{L^2}\frac{\partial^2 T}{\partial \psi^2} + \frac{\psi}{L}\frac{dL}{dt}\frac{\partial T}{\partial \psi} + \frac{\varepsilon \Delta H}{c}\frac{\rho_s}{\rho_{b0}}\left(\frac{\partial X}{\partial t} - \frac{\psi}{L}\frac{dL}{dt}\frac{\partial X}{\partial \psi}\right) \tag{5}$$

$$\frac{\partial X}{\partial t} = D\frac{\rho_{b0}}{\rho_s}\frac{1}{L^2}\frac{\partial^2 X}{\partial \psi^2} + \left[\frac{\rho_{b0}}{\rho_s^2}\frac{1}{L^2}\frac{\partial (D\rho_s)}{\partial \psi} + \frac{\psi}{L}\frac{dL}{dt}\right]\frac{\partial X}{\partial \psi} \tag{6}$$

The initial conditions are

$$t = 0: \quad T(\psi, 0) = T_0, \qquad X(\psi, 0) = X_0. \tag{7}$$

The temperature and the moisture content boundary conditions on the surfaces of the drying body in contact with the drying air are

$$-k\frac{1}{L}\left(\frac{\partial T}{\partial \psi}\right)_{\psi=1} + j_q - \Delta H(1-\varepsilon)\, j_m = 0 \tag{8}$$

$$D\rho_s\frac{1}{L}\left(\frac{\partial X}{\partial \psi}\right)_{\psi=1} + j_m = 0 \tag{9}$$

The convective heat flux, $j_q(t)$, and mass flux, $j_m(t)$, on these surfaces are

$$j_q = h\left(T_a - T_{\psi=1}\right) \tag{10}$$

$$j_m = h_D\left(C_{\psi=1} - C_a\right) \tag{11}$$

where h is the heat transfer coefficient, and h_D is the mass transfer coefficient, T_a is the temperature of the drying air, C_a, is the water vapor concentration in the drying air [1], $T_{\psi=1}$ is the temperature on the surfaces of the drying body, and $C_{\psi=1}$ is the water vapor concentration of the air in equilibrium with the surface of the body exposed to convection. The water activity, or the equilibrium relative humidity of the air in contact with the convection surface at temperature $T_{\psi=1}$ and moisture content $X_{\psi=1}$ are calculated from experimental water sorption isotherms.

The boundary conditions on the mid-plane of the drying slice are

$$\left(\frac{\partial T}{\partial \psi}\right)_{\psi=0} = 0, \quad \left(\frac{\partial X}{\partial \psi}\right)_{\psi=0} = 0. \tag{12}$$

Problem defined by Eqs. (5-12) is referred to as a direct problem when initial and boundary conditions as well as all the parameters appearing in the formulation are known. The objective of the direct problem is to determine the temperature and moisture content fields in the drying body.

In order to approximate the solution of Eqs. (5) and (6), an explicit numerical procedure has been used.

Inverse Approach

For the inverse problem of interest here, the moisture diffusivity of a drying body and the boundary conditions parameters are regarded as unknown parameters.

The estimation methodology used is based on the minimization of the ordinary least square norm

$$E(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^T [\mathbf{Y} - \mathbf{T}(\mathbf{P})]. \tag{13}$$

Here, $\mathbf{Y}^T = [Y_1, Y_2, \ldots, Y_{imax}]$ is the vector of measured temperatures, $\mathbf{T}^T = [T_1(\mathbf{P}), T_2(\mathbf{P}), \ldots T_{imax}(\mathbf{P})]$ is the vector of estimated temperatures at time $t_i (i=1,2,\ldots,imax)$, $\mathbf{P}^T = [P_1, P_2, \ldots P_N]$ is the vector of unknown parameters, imax is the total number of measurements, and N is the total number of unknown parameters ($imax \geq N$).

A hybrid optimization algorithm OPTRAN [2] and the Levenberg-Marquardt method have been utilized for the minimization of $E(\mathbf{P})$ representing the solution of the present parameter estimation problem.

The Levenberg-Marquardt method is a stable and straightforward gradient search minimization algorithm that has been applied to a variety of inverse problems. It belongs to a general class of damped least squares methods.

An alternative to the Levenberg-Marquardt algorithm, especially when searching for a global optimum of a function with possible multiple minima, is the hybrid optimization program OPTRAN. OPTRAN incorporates six of the most popular optimization algorithms: the Davidon-Fletcher-Powell gradient search, sequential quadratic programming algorithm, Pshenichny-Danilin quasi-Newtonian algorithm, a modified Nelder-Mead simplex algorithm, a genetic algorithm, and a differential evolution algorithm. Each algorithm provides a unique approach to optimization with varying degrees of convergence, reliability and robustness at different stages during the iterative optimization procedure. The hybrid optimizer OPTRAN includes a set of rules and switching criteria to automatically switch back and forth among the different algorithms as the iterative process proceeds in order to avoid local minima and accelerate convergence towards a global minimum.

Results and Discussion

Real experiments have been conducted to investigate the applicability of the method. Thin flat samples of potato and apple were dried in an experimental convective dryer [1]. For estimation of the unknown parameters, the transient readings of a single temperature sensor located in the mid-plane of the slice exposed to drying have been used.

The experiments were designed by using the so-called D-optimum criterion. From the relative sensitivity coefficients analysis, the drying air velocity, drying air temperature and drying body dimension were defined. From this analysis it was concluded that in the convective drying experiment it is possible, based on a single thermocouple temperature response, to estimate simultaneously the moisture diffusivity, the convection heat and mass transfer coefficients, and the relative humidity of the drying air. All other quantities appearing in the direct problem formulation were taken from published data of other authors. Determinant of the information matrix with normalized elements has been calculated in order to define drying time. The duration of the drying experiment corresponding to the maximum determinant value was used.

In Figures 2 the estimated moisture diffusivities are compared with the results published by other authors [3].

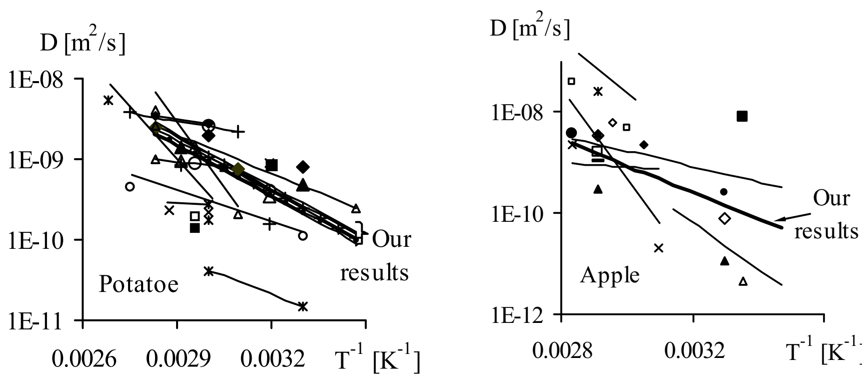

Figure 2: Moisture diffusivity of potatoes and apples

Conclusions

It can be concluded that in the convective drying experiments of bodies with significant shrinkage effects it is possible, based on a single thermocouple temperature response, to estimate the moisture diffusivity. Estimated moisture diffusivities compare well with the values obtained by other authors who utilized different methods.

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