Dynamic optimisation of constrained multibody dynamics

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ABSTRACT

This work presents the recently developed combination of two methods, namely the discrete null space method which is suitable for the accurate, robust and efficient time integration of constrained dynamical systems (in particular for multibody dynamics) and a new approach to discrete mechanics and optimal control (DMOC) [1]. The structure preserving time-stepping equations of mechanics subject to holonomic constraints are formulated in terms of the states and controls using a constrained version of the discrete Lagrange-d'Alembert principle. Together with initial and final conditions on the configuration and conjugate momentum, the reduced time-stepping equations (implemented by the discrete null space method) serve as nonlinear equality constraints for the minimisation of a given cost functional. The algorithm yields a sequence of discrete configurations together with a sequence of actuating forces, optimally guiding the system from the initial to the desired final state.

Inspired by space telescopes like e.g. the Hubble telescope, whose change in orientation is induced by external spinning rotors, a multibody system consisting of a main body to which rotors are connected by revolute joints has been analysed. The goal is to determine optimal torques to guide the main body into a prescribed final position, whereby the system starts and ends at rest and the objective function represents the control effort which has to be minimised. Figure 1 shows subsequent configurations of the system. The right hand diagram illustrates the evolution of the kinetic energy and the total angular momentum. Because there are no external torques acting on the system, the total angular momentum remains zero at all times, which is correctly captured by the algorithm. At the same time, an overall rotation occurs during the motion, which may be thought of as a geometric phase.

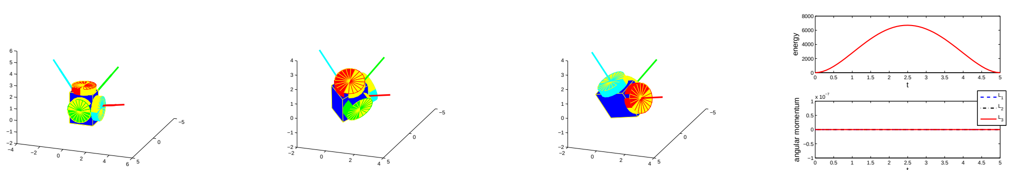

Figure 1: Rigid body with three rotors: configurations at $t = 0, 20, 50$ seconds and evolution of energy and angular momentum ($h = 0.1$).

[1] Leyendecker, S., Ober-Blöbaum, S., Marsden, J.E., and Ortiz, M. (2007). Discrete mechanics and optimal control for constrained multibody dynamics. Proceedings of the 6th International Conference on Multibody Systems, Nonlinear Dynamics, and Control, ASME International Design Engineering Technical Conferences, 4-7 September 2007, Las Vegas, Nevada.