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Unsupervised Support Vector Machine Based Principal Component Analysis for Structural Health Monitoring

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Summary

Structural Health Monitoring (SHM) is concerned with identifying damage based on measurements obtained from structures being monitored. For the civil structures exposed to time-varying environmental and operational conditions, it is inevitable that environmental and operational variability produces an adverse effect on the dynamic behaviors of the structures. Since the signals are measured under the influence of these varying conditions, normalizing the data to distinguish the effects of damage from those caused by the environmental and operational variations is important in order to achieve successful structural health monitoring goals. In this paper, kernel principal component analysis (kernel PCA) using unsupervised support vector machine is developed and incorporated with a time prediction model for data normalization by characterizing the relationship between the extracted features and an unmeasured environmental parameter. This method performs a nonlinear principal component analysis by using kernel functions in high-dimensional feature spaces without involving computationally expensive nonlinear optimization. The advantages of the proposed method are demonstrated using a numerical example with comparison results obtained by applying an autoassociative neural network.

keywords: Kernel Principal Component Analysis, Novelty Detection, Damage Diagnosis, Environmental and Operational Variations, Support Vector Machine

Introduction

The goal of Structural Health Monitoring (SHM) is to provide reliable information regarding damage states including its presence, location and severity. Damage identification is the first step and, if damage is detected, the subsequent damage assessments are performed in order to investigate the location and severity. The basic premise for this diagnosis is that damage alters the dynamic characteristics of the structures, when the damage occurs. However, in reality the environmental and operation conditions, such as temperature, often cause an adverse effect on the dynamic behavior of the structures. Therefore, for the civil structures exposed to time-varying environmental and operational conditions, normalizing the data to distinguish the effects of damage from those caused by the environmental and operational variations is important in order to achieve successful SHM goals.

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In this paper, kernel principal component analysis (kernel PCA), i.e., unsupervised support vector machine based PCA, incorporated with a novelty detection method is introduced for SHM applications. This proposed method characterizes the relationship between the extracted features and unmeasured environmental parameters so that the damage states affected by environmental and operational conditions can be effectively predicted. The main procedures of the proposed method are (1) to extract the damage-sensitive features from the measurements during the undamaged normal condition, (2) to compute principal components by the proposed kernel PCA, and (3) to predict damage states by novelty indices calculated with the subsequent data from the possibly damaged structure. Here, the novelty index is defined to show the degree of divergence between the data obtained from an undamaged state and those from possibly damaged states. The adopted method has advantages that no measurements of environmental and operation parameters are required and no physics-based models are necessary for damage indication.

In the next section, kernel PCA is introduced as a way of nonlinear PCA, and then a novelty index is defined. An illustrative example of a computer hard disk shows a damage diagnosis procedure and results with the conclusions followed at the end.

Kernel Principal Component Analysis

Principal Component Analysis (PCA) is an orthogonal transformation of the coordinate system in which the given data can be described as new variables [1,2]. This method utilizes the eigenvector decomposition of the data's covariance matrix, and new variables, called principal components, are obtained by projecting the data onto eigenvectors. The new variable projected onto the eigenvector corresponding to the largest eigenvalue is called the first principal component, and the subsequent principal components can be computed similarly by projecting the data onto the subsequent eigenvectors. PCA is a well-known method for reducing dimensionality of the data, since a small number of principal components are sufficient to account for most of the structure in the data [1].

Linear PCA can be generalized to nonlinear PCA (NLPCA) to reveal nonlinear correlations inherent in data. One such method is an auto-associative neural network (AANN) [3-5]. This network consists of three hidden layers such as the mapping, the bottleneck, and the de-mapping layers, and it is trained to reproduce input values as outputs, i.e., in an auto-associative mode. The hidden unit activations perform NLPCA via the bottleneck layer whose dimension is smaller than those of input. AANN was applied for damage diagnosis of a computer hard disk considering temperature as an operational variation [6]. However, AANN needs to solve a complex nonlinear optimization problem for estimating unknown parameters with the possibility of getting trapped in local minima as well as the well-known over-fitting problem. On the contrary, the proposed kernel PCA method involves solving only a simple eigenvalue problem. In this section, a mathematical background of kernel PCA is described which could be translated as a generalization from linear PCA [7].

Linear Principal Component Analysis (linear PCA)

Let $F_N = \{\underline{x}_j \in R^{m\times 1},\ j = 1,\ldots,N\}$ denote a set of N number of centered m-dimensional feature vector extracted from measurements. Then, linear PCA is performed by an eigenvector decomposition of the covariance matrix $\mathbf{C}_1$:

$$\mathbf{C}_1 = \frac{1}{N}\sum_{j=1}^{N} \underline{x}_j \underline{x}_j^T \tag{1}$$

which leads to the following eigenvalue problem:

$$\lambda^* \underline{w} = \mathbf{C}_1 \underline{w} \tag{2}$$

where λ^* and $\underline{w}$ are eigenvalues and corresponding eigenvetors, respectively. Hereafter, underlines and boldfaces are used to denote vectors and matrices, respectively. From Eq. (2), m number of eigenvalues and corresponding eigenvectors can be obtained and the k^{th} principal components for feature vector $\underline{x}_j$ are computed as a dot product between $\underline{x}_j$ and the corresponding k^{th} eigenvector $\underline{w}^k$:

$$(PC(\underline{x}_j))^k = \underline{x}_j \cdot \underline{w}^k \tag{3}$$

where $(PC(\underline{x}_j))^k$ represents the k^{th} principal component for feature vector $\underline{x}_j$.

Kernel Principal Component Analysis (Kernel PCA)

Kernel PCA is a generalization of linear PCA where the original m-dimensional features are transformed to a higher, possibly infinite, dimension space via nonlinear mapping function, $\underline{\phi}(\cdot)$. Here, $\underline{\phi}(x_j)$ denotes the transformed feature of $\underline{x}_j$ satisfying $\Sigma_j \underline{\phi}(\underline{x}_j)$=$\underline{0}$; this constraint is called centering and an unconstrained situation is considered shortly. For this centered and transformed feature, $\underline{\phi}(\underline{x}_j)$, the covariance matrix, $\mathbf{C}_2$, can be constructed in a similar manner as $\mathbf{C}_1$ in Eq. (1):

$$\mathbf{C}_2 = \frac{1}{N}\sum_{j=1}^{N} \underline{\phi}\left(\underline{x}_j\right) \underline{\phi}\left(\underline{x}_j\right)^T \tag{4}$$

Then, Eq. (4) leads to the same eigenvalue problem as before.

$$\lambda \underline{v} = \mathbf{C}_2 \underline{v} \tag{5}$$

where λ and $\underline{v}$ are eigenvalues and corresponding eigenvetors, respectively.

Similar to linear PCA, $\underline{v}$ can be expressed as a span of $\underline{\phi}(\underline{x}_i)$, $i = 1,\ldots,N$, such as:

$$\underline{v} = \sum_{i=1}^{N} \alpha_i \underline{\phi}\left(\underline{x}_i\right) \tag{6}$$

where α_i is an unknown coefficient. Combining Eq. (5) and Eq. (6) leads to another eigenvalue problem [9]:

$$N\lambda\underline{\alpha} = \mathbf{K}\underline{\alpha} \tag{7}$$

where $\underline{\alpha} = [\alpha_1\alpha_2\ldots\alpha_N]^T \in R^{N\times 1}$ and N number of λ and $\underline{\alpha}$, i.e., λ^k and $\underline{\alpha}^k$, $k = 1,\ldots,N$, can be obtained. The ij^{th} entity of $\mathbf{K}$ matrix $\in R^{N\times N}$, $\mathbf{K}_{ij}$, is defined as:

$$\mathbf{K}_{ij} \equiv \underline{\phi}(\underline{x}_i)\cdot\underline{\phi}(\underline{x}_j) = k(\underline{x}_i,\underline{x}_j) \tag{8}$$

In Eq. (8), kernel, $k(\underline{x}_i,\underline{x}_j)$, is used simply to replace the dot product based on the Mercer's theorem and the nonlinearity is achieved in a relatively easy way by the calculation of this kernel. This kernel method enables the dot products to be calculated in the lower dimensional input space, even without involving nonlinear transformation $\underline{\phi}(\cdot)$. Popular kernels are polynomial, radial basis function, and sigmoid kernels. Further discussion on kernels can be found in [7,8]. Since $\mathbf{K}$ matrix based on Mercer's theorem is positive-semidefinite, the eigenvalues are non-negative. Then, normalization of the corresponding eigenvectors is performed for the first p nonzero eigenvalues to satisfy:

$$\left(\underline{v}^k\cdot\underline{v}^k\right) = 1 \rightarrow N\lambda^k\left(\underline{\alpha}^k\cdot\underline{\alpha}^k\right) = 1 \tag{9}$$

where λ^k and $\underline{\alpha}^k$ are the k^{th} eigenvalues and eigenvectors computed from Eq. (7), respectively, and $k = 1,\ldots,p$.

Finally the k^{th} principal components for feature vector $\underline{x}_j$ are computed as the projection onto the corresponding k^{th} eigenvector:

$$(KPC(\underline{x}_j))^k = \underline{\phi}\left(\underline{x}_j\right)\cdot\underline{v}^k = \sum_{i=1}^{N}\alpha_i k\left(\underline{x}_i,\underline{x}_j\right) \tag{10}$$

where $(KPC(\underline{x}_j))^k$ and $\underline{v}^k$ represent the k^{th} principal component for feature vector $\underline{x}_j$ and eigenvector in Eq. (5), respectively, and $k(\underline{x}_i,\underline{x}_j)$ is the chosen kernel. In practice, since the transformed feature, $\underline{\phi}(\underline{x})$, are not centered usually, $\mathbf{K}$ matrix in Eq. (7) needs to be modified to $\mathbf{K}_c$ as follows:

$$\mathbf{K}_c = \mathbf{K} - \mathbf{1}_N\mathbf{K} - \mathbf{K}\mathbf{1}_N + \mathbf{1}_N\mathbf{K}\mathbf{1}_N \tag{11}$$

where $\mathbf{1}_N \in R^{N\times N}$ is a matrix with the entity $(\mathbf{1}_N)_{ij} = 1/N$. The detailed derivation of $\mathbf{K}_C$ can be found in [9].

Aforementioned Kernel PCA can be also derived in the context of unsupervised support vector machine (SVM) to consider minimization of the classification errors as well as regularization to prevent over-fitting [9,10,11].

Novelty Index

A novelty index plays a role in indicating the degree of divergence of data from those in an undamaged normal condition [6]. This index is defined as:

$$\text{Novelty Index}(\underline{x}^{pd}) = ||\underline{x}^{u} - \underline{x}^{pd}|| \tag{12}$$

where $\underline{x}^{u}$ and $\underline{x}^{pd}$ represent the features extracted from the data measured from undamaged and possibly damaged conditions of the structure being monitored, respectively.

Since this index uses the signals measured from a structure being monitored, a structural model is not required. This novelty detection provides how different the currently-obtained data are from baseline data, i.e., data obtained from an undamaged structure. If this index alarms possible indications of damage, then the detailed inspections of damage can be followed in order to investigate the location and their severity.

Illustrative Example: Computer Hard Disk

The proposed method of kernel PCA is applied to an illustrative example of computer hard disk [12]. This case study demonstrates the capability of kernel PCA incorporated with a novelty detection method to identify damage in the system. The computer hard disk model can be expressed by using Newton's law:

$$J\frac{d^2\theta}{dt^2} + C\frac{d\theta}{dt} + K\theta = K_i i \tag{13}$$

where J is the inertia of the head assembly, C is the viscous damping coefficient of the bearings, K is the return rotational spring constant, K_i is the motor torque constant, θ is the angular position of the head, and i is the input current. The feedback compensator of the hard disk is omitted in this example for the purpose of the simplicity and comparison with the previous results in [5]. The operational variability is assumed to depend on temperature, T, and can be reflected on the values of J, C, K, and K_i such as:

$$K(T) = \frac{6}{87}(0.1 \times \mathrm{T} - 1.5)^3 + \frac{4}{87}(0.1 \times \mathrm{T} - 1.5) + 10 \tag{14}$$

$$K_i(T) = \frac{0.01}{30}\left[(0.1 \times \mathrm{T} - 1.5)^3 + (0.1 \times \mathrm{T} - 1.5)^2 + (0.1 \times \mathrm{T} - 1.5) + 1\right] + \frac{0.14}{3} \tag{15}$$

$$J(T) = 0.01\left(\frac{\mathrm{T}}{15} + 9\right) \tag{16}$$

$$C(T) = 0.004 \times \tanh\left(\frac{\mathrm{T}}{30}\pi - \frac{\pi}{2}\right) \tag{17}$$

The temperature dependencies of these variables are arbitrarily assumed, since the proposed method does not require any physical modeling. Discretization of the Laplace transformation of Eq. (13) becomes:

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} \tag{18}$$

and the coefficients of a_1 and a_2 are used for features, since they alone are related with dynamic characteristics of the system being monitored.

Principal Components Estimated by Kernel PCA

Baseline data of a_1 and a_2 representing an undamaged state are simulated by varying the temperature in the range of [-15°C, 45°C]. For each temperature, the coefficients of $K(T)$, $K_i(T)$, $J(T)$, and $C(T)$ are computed from equations (14)-(17), and then a_1 and a_2 are extracted from Eq. (18). The total number of baseline data is 121 and the data set is scaled so that each variable ranges from -1 to 1 in order to be equally weighted (Figure 1 (a)).

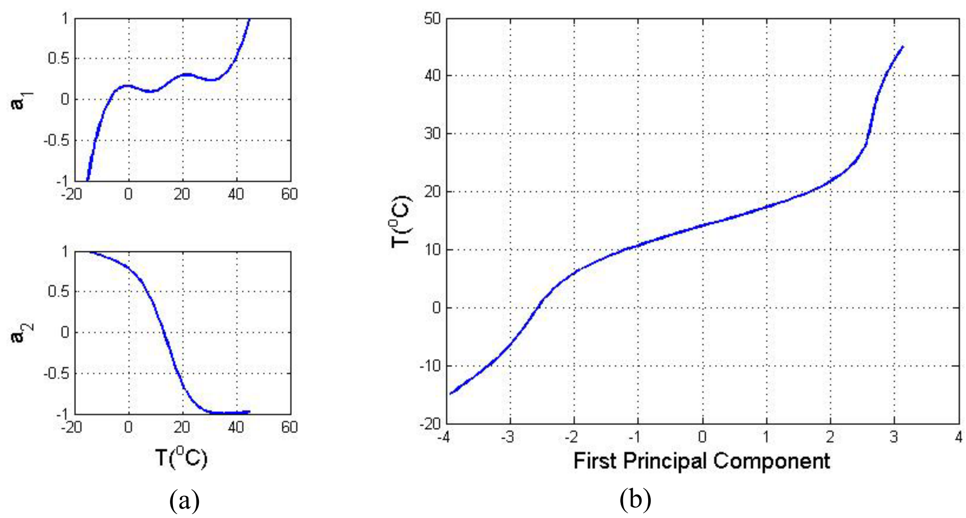

Figure 1: Scaled variables of a_1 and a_2 (a) and correlation between the first principal component and temperature (b)

Additional data sets are generated as prediction data for verification of the algorithm and damage diagnosis. For verification, 600 temperatures are simulated from a uniform distribution in the range of [-15°C, 45°C] and the corresponding a_1 and a_2 are prepared through the same procedure as those used for baseline data simulation. These data are utilized to verify the performance of the proposed kernel PCA.

For damage diagnosis, 2,400 data, i.e., 4 Cases×600 data, are simulated according to each damage scenario listed in Table 1. For example, the possibly damaged return rotational spring constants, K^{pd}, and damping coefficients, C^{pd}, for damage case (a) are simulated from $[0.85K(20), 0.95K(20)]$ and $C(20)$, respectively.

Since the parameter affecting the operational variability is temperature alone in this example, only the first principal component is selected for computing novelty index. The correlation between the first principal component and the temperature is shown in Figure 1 (b): this figure shows monotonic increases of the first principal component as temperature, T, increases.

Table 1: Damage scenarios in this study

Cases	Spring constant (K^{pd})	Damping coefficient (C^{pd})
(a)	$[0.85K(20), 0.95K(20)]$	$C(20)$
(b)	$K(20)$	$[0.90C(20), 1.10C(20)]$
(c)	$[0.95K(20), 1.05K(20)]$	$C(20)$
(d)	$[0.95K(20), 1.05K(20)]$	$[0.90C(20), 1.10C(20)]$

Novelty Detection

The values of novelty index are calculated by using prediction data generated for verification and damage diagnosis. Since the algorithm of kernel PCA does not work in auto-associative mode, the baseline data, $\underline{x}^u$, whose first principal component has the minimum Euclidean distance from that of prediction data, $\underline{x}^{pd}$, is used for index calculation. For each damage scenario, the computed index values are shown in Figure 2. Solid (darker) lines corresponding to the first 600 number of data are for verification, while dashed ones are for damage diagnosis. Case (a) shows the distinct changes of index, while Case (b) does not. Case (c) and (d) produce noticeable changes even if they are not as distinct as Case (a).

Advantage of Kernel PCA

The obtained results of damage diagnosis by kernel PCA are very similar to what were obtained by AANN; novelty index shows noticeable changes in Cases (a), (c), and (d) for both methods [6].

In summary, the advantages of kernel PCA in comparison with AANN are: (1) the simple formation of an eigenvalue problem instead of solving a complex nonlinear optimization problem with a possibility of getting trapped in local minima, (2) evasion of over-fitting problems by employing regularization [11], and (3) flexibility in computing multiple principal components without redesigning kernel PCA. (For AANN, the number of nodes in the bottleneck layer should be known *a priori*, and the network needs to be designed accordingly).

Conclusions

In this paper, integration of kernel principal component analysis (kernel PCA),

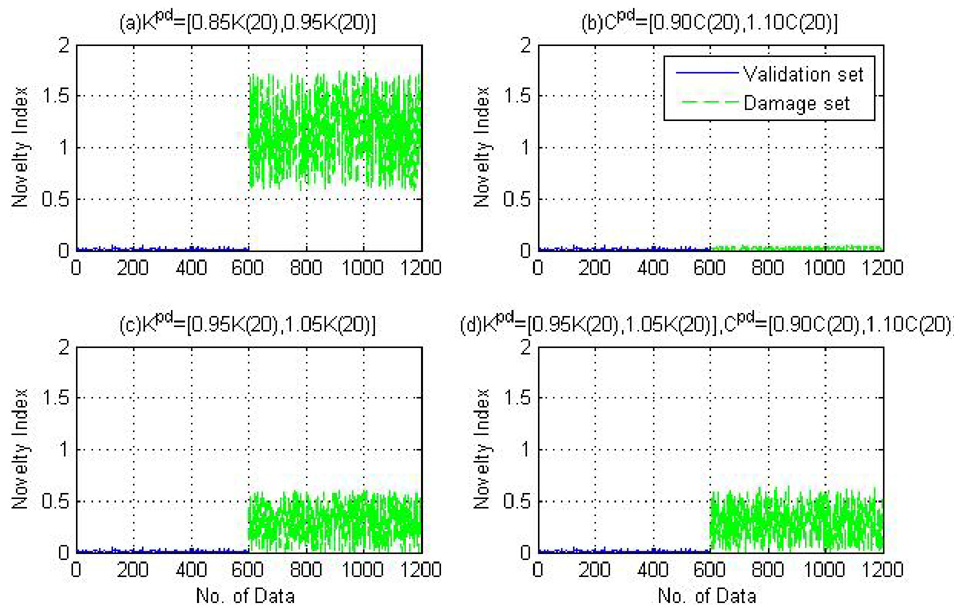

Figure 2: Novelty indices evaluated at four different damage cases

also known as unsupervised support vector machine based PCA, with novelty detection is proposed for damage diagnosis at the presence of time-varying environmental and operational conditions. The kernel PCA method performs nonlinear PCA by adopting a kernel method that replaces dot products with kernels and by solving a simple eigenvalue problem. Then, novelty detection is employed to assess the health condition of the structure by measuring the degree of the discrepancy between the data obtained from a possibly damaged state and those obtained from the undamaged state of the structure.

The computer hard disk example presented in this study demonstrates that the proposed method is able to detect damage under time-varying temperature conditions without the knowledge of the temperature at hand. Baseline data are generated from a wide temperature range of the computer hard disk's intact condition, and data corresponding to various damage scenarios are simulated by perturbing the intact system's stiffness and damping values obtained at 20°C.

The scope of this study is limited to the application of the proposed damage detection to a simplified numerical model. Additional studies are underway to investigate the robustness of the proposed method in harsh field environments. In particular, the sensitivity of the novelty index is further examined taking into account the uncertainties such as measurement noises.

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