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A Contemporary Approach to Space/Time Discretization for F= ma: Newtonian, Lagrangian and Hamiltonian Mechanics

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Summary

A contemporary, unified and consistent mathematical setting and computational framework for numerical computations of the equations of motion arising in Computational Dynamics and under the umbrella of F=ma is presented. *It is to be noted that typical Lagrangian and Hamiltonian descriptions do not yield any new physics in contrast to the Newtonian mechanics; they are simply descriptions that provide different viewpoints and in certain cases yield either simpler or more complicated mathematical representations based on the problem at hand, and all indeed are supposed to yield the same answers for displacement, velocity and accelerations.* As such, the philosophical argument here is that: a) the space discrete formulations for all descriptions should indeed yield the same equivalent semi-discretized equations of motion, and b) consequently, the associated time discretization procedures should also yield the same time integration approximations which are all equivalent. The presumption that one description yields something different than the other is not physical or meaningful.

Most finite element space discretization procedures are typically based on those derived from the Cauchy equations of motion describing F=ma and leading to the semi discretized equations which are then integrated in time via time integration procedures. In particular, the predominant approaches either seeks to invoke variational formulations or the weighted residual methods in the process of discretization. This is what a majority of texts and the research literature describe. Also, since Newtonian mechanics requires the concept of force and consequently does not have the notion of energy built into the model equations, to-date, the conservative properties of space/time discretizations for conservative systems, are mostly established through relating them with the Lagrangian or Hamiltonian framework. Firstly, without resorting to these other routes, we introduce the concept of total energy in the Cartesian coordinate description itself, which directly yields Newton's second law, and to consistently enable the space/time discretization. In contrast to starting from the Cauchy equation, emanating from Hamilton's law, we provide a unified and consistent mathematical framework via the concept of space discrete total energy such as that introduced earlier in the Cartesian description, or equivalently describe the concept of space discrete Lagrangian or Hamiltonian by invoking the notion of total energy density, Lagrangian density, or Hamiltonian density

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respectively for each of the underlying mechanics descriptions to derive the semi-discretized equations of motion; this is followed by a consistent time discretization framework of the resulting equations of motion which readily yield a systematic design methodology for conducting the time discretization process. The resulting designs naturally enable conserving/preserving algorithm designs, or designs with controllable numerical dissipation as well based on the desired needs. The overall framework consistently explains space/time discretization concepts in a unique mathematical setting for the broad fields of computational dynamics.