Imperfection's Effect of Beam Element for Elastic Buckling Analysis

R. Adman [1], H. AFRA [2]

Summary

In this paper, a versatile and efficient solution of the differential equilibrium equation for an initially curved beam element under high axial load and arbitrary boundary conditions is derived. The initial lateral curvature is included by a sine curve approximation. The present paper focused the effect of the initial imperfection which can be expressed as an equivalent nodal force with respect of nodal generalized coordinates. By using this approach, the initial curvature can readily be introduced in the global equivalent load vector. This formulation, offers a significant practical advantages for the elastic buckling analysis of an imperfect beam element.

Introduction

In the recent publications about the geometric nonlinear behaviour of an elastic beam element under high axial load, it appears clearly the need to formulate an accurate element with an initial geometric imperfection. This imperfection is adopted in various national design codes as an initial curvature of which the magnitude is commonly taken as 0.1% of the member length. The beam-column method which corresponds to the analytical resolution of the differential equilibrium equation gives the well-known stability functions. However, the stability functions derived from this approach neglected the initial member curvature, therefore its application to design and analysis becomes limited. Recently, Chan and Gu**[1]**, proposed an exact solution for an imperfect beam-column members by using the stability functions approach. The authors used Thimoshenko's theory (Timoshenko and Gere 1961) **[2]**, and extended this theory to take into account the effect of the initial curvature along the element length.

In the present paper, a versatile and efficient displacement function introducing the effect of the initial imperfection, expressed with respect of nodal generalised coordinates is derived analytically from the differential equilibrium equation.

[1]Built Environment Research Laboratory, Faculty of civil engineering, University of science and technology Houari Boumediene, BP 32 El-Alia, Bab Ezzouar, 16111, Algiers, ALGERIA.

[2] National Center of Building Research (CNERIB), Souidania, Algiers, ALGERIA

This formulation offers a significant practical advantages to overcome limitations arising from the classical ones .

Element formulation

Consider a beam segment (ij) with initial curvature given in Fig. 1.

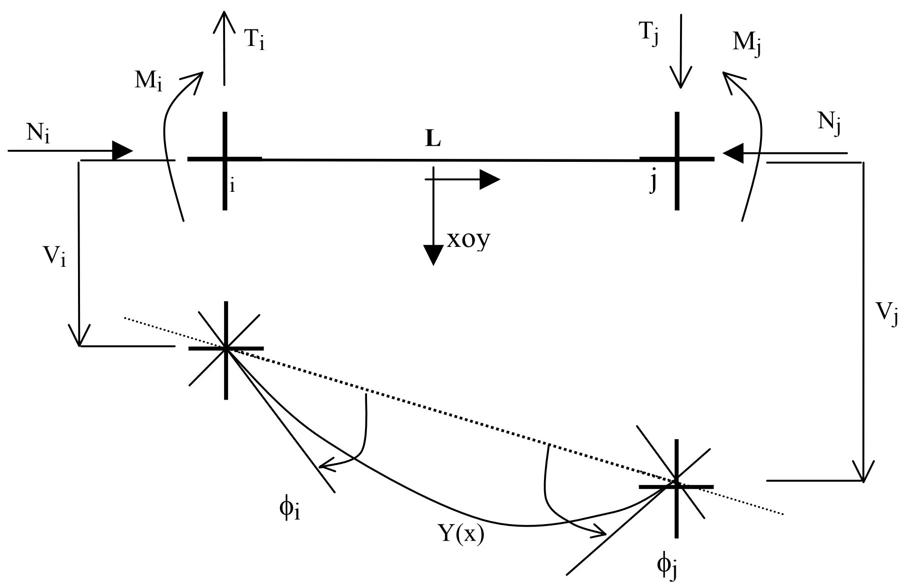

Figure1: Beam-segment model

The equilibrium equation along the element length can be expressed in compressive case as:

$$\hat{W}_{,xx} + K^2 (\hat{W} + Wo) = - Mo/ EI \qquad \textbf{(1)}$$

Where: $\hat{W}$ is the deflection superimposed to the initially curved beam under high compressive load.***((,xx)** indicate the second derivative operator.)*

And: $$Wo = Vo \ . \ \sin\left(\frac{\pi x}{L}\right) \quad \textbf{(2)}$$

W_O is the initial curvature. Vo is the magnitude of the initial curvature.

$$\mathbf{K}=\sqrt{\frac{|\mathbf{N}|}{\mathbf{EI}}} \quad \textbf{(3)}$$

$$Mo = M_i + T_i \ . \ x \quad \textbf{(4)}$$

$$T_i = (M_j - M_i) / L \quad \textbf{(5)}$$

E=Young modulus of elasticity, I=moment of inertia, M_i and M_j are the nodal moments, T_i and T_j are the nodal shears.

The general solution of Eq. (1) is:

$$\hat{W} = C_1 \cos(Kx) + C_2 \sin(Kx) + C_3\, x + C_4 + \frac{(KL)^2}{(\pi)^2 - (KL)^2} \cdot \sin(\frac{\pi}{L}\, x) \cdot Vo \quad \textbf{(6)}$$

C_1, C_2, C_3, C_4, are constants depending on the boundary conditions of the element(ij).

One poses: $\beta = KL$ **(7)**

$X = x / L$ **(8)**

Let Dr expressed by: $Dr = -2 + 2\cos(\beta) + \beta\sin(\beta)$ **(9)**

one poses: $\hat{W} = W + W_1$ **(10)**

The lateral displacement W brings out exclusively the coupling between the compressive force N and the deflection of the beam element(without taking into account the effect of the initial curvature).

W_1 .yields for lateral displacement part associated with the magnitude Vo of the initial curvature.

Superimposing the deflection $\hat{W}$ to the initial curvature Wo, we obtain the final deflection of the element, written as:

$$W_F = \hat{W} + Wo \tag{11}$$

From W_F we can deduce the displacement function part associated with the initial curvature which is given by:

$$W_v = W_0 + W_1 = N_v .Vo \tag{12}$$

Where:

$$N_v = B_1 \cos(\beta X) + B_2 \sin(\beta X) + B_3 .(\beta X) - B_1 + \frac{1}{1-(\beta/\pi)^2} \sin(\pi X) \tag{13}$$

B_1, B_2, B_3 are expressed by:

$$B_1 = \frac{1}{Dr} \frac{(\beta/\pi)}{1-(\beta/\pi)^2} \left((\sin(\beta) - \beta \cos(\beta)) + \cos(\pi)\,(\beta - \sin(\beta))\right) \tag{14}$$

$$B_2 = \frac{1}{Dr} \frac{(\beta/\pi)}{1-(\beta/\pi)^2} \left((1 - \cos(\beta) - \beta \sin(\beta)) + \cos(\pi)\,(\cos(\beta) - 1)\right) \tag{15}$$

$$B_3 = \frac{1}{Dr} \frac{(\beta/\pi)}{1-(\beta/\pi)^2} \left((1 - \cos(\beta)) + \cos(\pi)\,(1 - \cos(\beta))\right) \tag{16}$$

The function W_v associated with the magnitude Vo is used to derive the analytic expression for the equivalent nodal forces relating to the initial curvature.

Discussion

For the present approach, the solution is derived from the equilibrium equation by using arbitrary boundary conditions with respect of nodal generalised coordinates. We have noted a significantly difference between the present solution and their corresponding one, proposed in reference **[1]** which is expressed by:

$$N_v = \frac{1}{1-(\beta/\pi)^2}\sin(\pi X) \tag{17}$$

The case of hinged-hinged beam, with the properties cited below:

Young modulus: E=1E7

Moment of inertia: I=.101322

Length of the beam: L=100

Imperfection's amplitude: V0=.1

Which correspond to Euler's critical load: Ncr=1000, is considered.

The results of the present approach are compared to those cited in reference**[1]** in table1.

TABLE 1. $N_{\phi i}$(x) for the present study and reference [1]

β	Wv(x=L/2) **present**	Wv(x=L/2) **Reference:[1]**
Π/2	.1057	.1333
Π	.1282	.1236E5
3Π/2	.2097	-.0800
2Π	.1035E5	-.0333

We can readily observe from table 1 that the buckling capacity of the hinged-hinged beam deviates from Euler's buckling load (**βcr=Π**) and tends to the limit which corresponds to the case of a fixed-fixed beam (**βcr=2Π**)

Conclusion

In the present paper, we present a new formulation for the elastic buckling analysis of an imperfect beam element. The contribution of a sinusoidal imperfection is derived from the resolution of the differential equilibrium equation with arbitrary boundary condition, and the results obtained from this approach are compared to those in reference **[1]**, where restricted boundary conditions are considered.

The main observed result is that the buckling capacity of the hinged-hinged beam deviates from Euler's buckling load (**$\beta cr=\Pi$**) and tends to the limit which corresponds to the case of a fixed-fixed beam (**$\beta cr=2\Pi$**)

Reference

1 Chan S.-L., Jian-Xin Gu(2000): *Exact tangent stiffness for imperfect beam-column members,* Journal of Structural Engineering, 1094-1107.

2 Timoshenko S.P. and Gere J. M.(1961): *Theory of elastic stability,* 2^{nd} Ed.,McGraw-Hill, New York.