Application of Boundary Element Method to Elasto-plastic Analysis of 2-D Orthotropic Plates

Z. Z. Cen[1], X. S. Sun[1], L. X. Huang[1], Y. H. Liu[1],

Summary

In this paper, the Boundary Element Method (BEM) is introduced to analyze the elasto-plastic problems of 2-D orthotropic plates. The fundamental solutions for orthotropic materials and the Hill orthotropic yield criterion are adopted in the elasto-plastic analysis. The initial stress method and tangent predictor-radial return algorithm are used to determine the stress state in solving the nonlinear equation with the incremental iteration method. Numerical example shows that the BEM is effective and reliable in analyzing elasto-plastic problems of orthotropic plates.

Introduction

Elasto-plastic analysis is one of the practical and desirable problems in modern engineering. The BEM is an effective and professional method in some problems such as stress concentration, and it has been successfully exploited to solve many kinds of isotropic problems [1]. The BEM was first introduced to analyze elasto-plastic problems for 3-D isotropic bodies by Swedlow and Cruse [2]. Later, Telles and Brebbia [3] presented the complete BEM formulations for 2-D and 3-D elasto-plastic problems of isotropic materials based on the initial strain method. Cen [4] developed the BEM in coupling with the FEM to solve efficiently 3-D elasto-plastic problems.

As to application of the BEM to orthotropic or anisotropic problems, some studies were mainly focused on elastic problems so far. Green [5] first introduced the fundamental solutions for 2-D orthotropic bodies under a concentrated force. Rizzo and Shippy [6] introduced these fundamental solutions into the boundary integral equations for numerical elastic analysis of stress concentration. Sun and Cen [7] then improved and extended these fundamental solutions into elasto-plastic problems and established the boundary integration and internal

[1] Department of Engineering Mechanics, Tsinghua University, Beijing 100084, P. R. China

stress and displacement integral equations for elasto-plastic analysis of 2-D orthotropic bodies, but no numerical techniques or results were described.

This paper will introduce the application of BEM to 2-D elasto-plastic problems for orthotropic plates, and the discretizated equations and iterative equations for numerical implementation are presented. Numerical example will be presented to demonstrate the validity and reliability of the proposed scheme for analyzing elasto-plastic problem of 2-D orthotropic plates. The results will also be compared to those obtained by the FEM using the commercial code ABAQUS.

Boundary Element Formulations

The boundary integral equation for 2-D orthotropic elasto-plastic problems and corresponding internal virtual elastic stress integral equation can be given with matrix form as [7]:

$$\boldsymbol{c}\dot{\boldsymbol{u}} = \int_{\Gamma} \boldsymbol{u}^{*}\dot{\boldsymbol{t}}\mathrm{d}\Gamma - \int_{\Gamma} \boldsymbol{t}^{*}\dot{\boldsymbol{u}}\mathrm{d}\Gamma + \int_{\Omega} \boldsymbol{u}^{*}\dot{\boldsymbol{f}}\mathrm{d}\Omega + \int_{\Omega} \boldsymbol{\varepsilon}^{*}\dot{\boldsymbol{\sigma}}^{p}\mathrm{d}\Omega \tag{1}$$

$$\dot{\boldsymbol{\sigma}}^{e} = \int_{\Gamma} \left(-\boldsymbol{\sigma}^{*}\right)^{\mathrm{T}}\dot{\boldsymbol{t}}\mathrm{d}\Gamma - \int_{\Gamma} \boldsymbol{t}_{p}^{*}\dot{\boldsymbol{u}}\mathrm{d}\Gamma + \int_{\Omega} \left(-\boldsymbol{\sigma}^{*}\right)^{\mathrm{T}}\dot{\boldsymbol{f}}\mathrm{d}\Omega + \int_{\Omega} \boldsymbol{\varepsilon}_{p}^{*}\dot{\boldsymbol{\sigma}}^{p}\mathrm{d}\Omega + \dot{\boldsymbol{\sigma}}^{eV} \tag{2}$$

and the displacements in internal points can also be written as:

$$\dot{\boldsymbol{u}} = \int_{\Gamma} \boldsymbol{u}^{*}\dot{\boldsymbol{t}}\mathrm{d}\Gamma - \int_{\Gamma} \boldsymbol{t}^{*}\dot{\boldsymbol{u}}\mathrm{d}\Gamma + \int_{\Omega} \boldsymbol{u}^{*}\dot{\boldsymbol{f}}\mathrm{d}\Omega + \int_{\Omega} \boldsymbol{\varepsilon}^{*}\dot{\boldsymbol{\sigma}}^{p}\mathrm{d}\Omega \tag{3}$$

where the variables with the superscript asterisk "*" denote the 2-D orthotropic fundamental solutions (matrix form) in the BEM; matrix $\boldsymbol{c}$ is termed the boundary properties and $\dot{\boldsymbol{\sigma}}^{eV}$ denotes the free term for strong singularity in domain.

Assume n boundary elements (one-dimensional element) and m internal cells (two-dimensional element) are discretized in the boundary and domain, respectively. Then after Eq.(1) ~ (3) are discretized and solved, the following equations can be obtained in assembling matrices if body force $\dot{\boldsymbol{f}}$ is absent:

$$\begin{cases} \dot{\boldsymbol{X}} = \dot{\boldsymbol{y}} + \boldsymbol{R}\dot{\boldsymbol{\Sigma}}^p \\ \dot{\boldsymbol{\Sigma}}^e = \dot{\boldsymbol{s}} + \boldsymbol{T}_s\dot{\boldsymbol{\Sigma}}^p \\ \dot{\boldsymbol{U}} = \dot{\boldsymbol{w}} + \boldsymbol{T}_u\dot{\boldsymbol{\Sigma}}^p \end{cases} \tag{4}$$

where $\dot{\boldsymbol{y}}$, $\dot{\boldsymbol{s}}$ and $\dot{\boldsymbol{w}}$ denote the elastic solutions; $\dot{\boldsymbol{U}}$, $\dot{\boldsymbol{\Sigma}}^e$ and $\dot{\boldsymbol{\Sigma}}^p$ denote the arrays of displacements, virtual elastic stress and plastic stress, respectively, which consist of the vectors in all nodes.

Stress Computation in Elasto-plastic Analysis

Eq.(4) consists of a number of nonlinear equations and some numerical techniques are necessary to treat these equations. The techniques to be introduced here are initial stress method and tangent predictor-radial return algorithm in the integration of the elasto-plastic constitutive equations with ideal plasticity, i.e.

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}^e - \frac{\boldsymbol{D}\boldsymbol{a}\boldsymbol{a}^{\mathrm{T}}\boldsymbol{D}}{\boldsymbol{a}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{a}}\dot{\boldsymbol{\varepsilon}} \tag{5}$$

where $\boldsymbol{D}$ is the elastic matrix; vector $\boldsymbol{a} = \partial f / \partial \boldsymbol{\sigma} = 2\boldsymbol{H}\boldsymbol{\sigma}/3$ and $\boldsymbol{H}$ is the Hill orthotropic coefficient matrix termed with orthotropic strength properties.

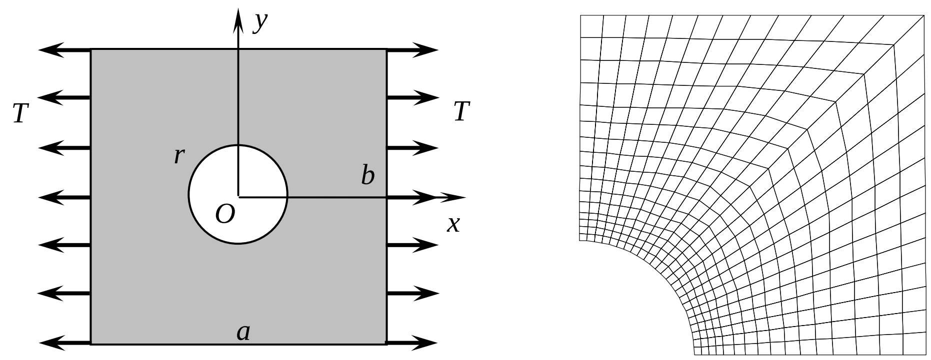
The solution can be implemented through incremental iteration of the load. For elesto-plastic analysis, if the load factor is $\Delta\alpha_i$ in every iterative step, the alternative incremental relations of stress in Eq.(4) is:

$$\Delta\boldsymbol{\Sigma}^e = \Delta\alpha_i\boldsymbol{s} + \boldsymbol{T}_s\Delta\boldsymbol{\Sigma}^p \tag{7}$$

After the increment loops over, the final stress is solved and the accumulated plastic stress $\boldsymbol{\sigma}^p$ is obtained as well. The boundary values can be solved through the first equation in Eq.(4), and the internal displacements can also obtained by the third equation in Eq.(4) if necessary.

Numerical Example

This example is about a square orthotropic plate with hole under uniform uniaxial tension (Fig. 1a). The geometry is $a = b = 60\text{mm}$, $r = 10\text{mm}$ and the magnitude of load is $T = 30\text{MPa}$. The material properties are listed in Table 1.



(a) Model (b) Mesh

Fig. 1 A plate with hole under uniform uniaxial tension

Table 1 Material properties of square-plate with hole

Elastic constants		Strength properties	
E_1	1.2GPa	X	230MPa
E_2	0.6GPa	Y	24MPa
ν_{12}	0.071	S	48.9MPa
G_{12}	0.07GPa		

Due to symmetry, a quarter of the plate is analyzed and the quarter-model is discretized with 425 nodes (including 80 boundary nodes), 40 3-node quadratic boundary elements and 384 4-node quadrilateral linear internal cells (Fig. 1b). The numerical solutions obtained by the BEM are compared with the results by commercial code ABAQUS.

The circumferential stress is shown in Fig. 2, and the elastic results are plotted as well. It can be seen that the results obtained by the present BEM are in very good agreement with those calculated by ABAQUS. Fig. 2 also shows that the area around the hole's boundary near x-axis is much easier to yield than that near y-axis. Fig 3 shows the stress distributions along the direction of 45 degree from x-axis, i.e. the diagonal direction in the plate.

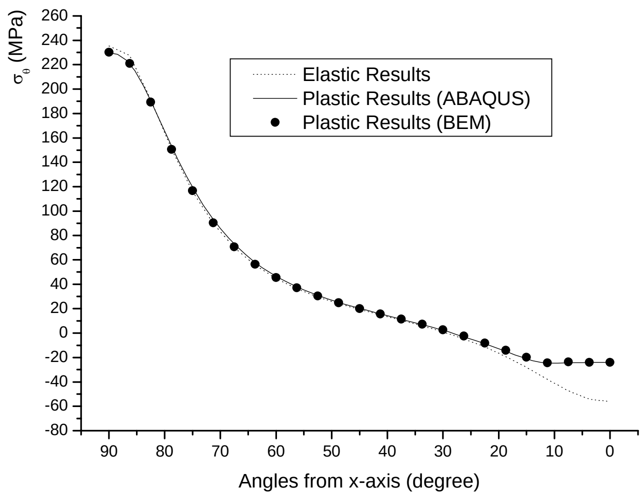


Fig. 2 Stress distribution along the circumference of the hole

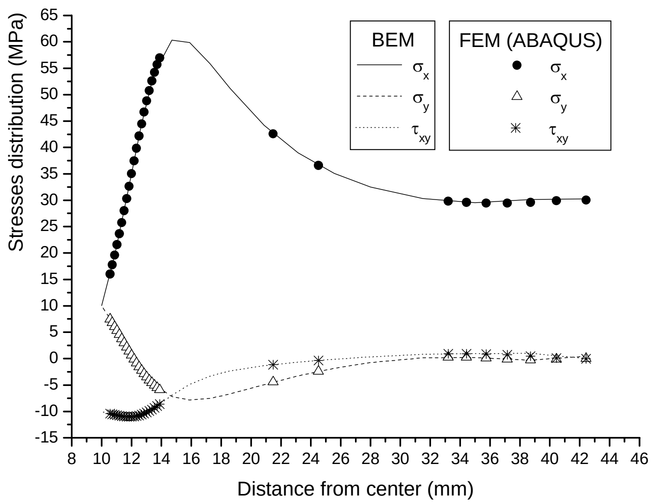


Fig. 3 Stresses distribution along the diagonal direction of the plate

Fig. 4 contours the equivalent stress distributions throughout the plate, which clearly shows the plastic zones and the stress states throughout the plate.

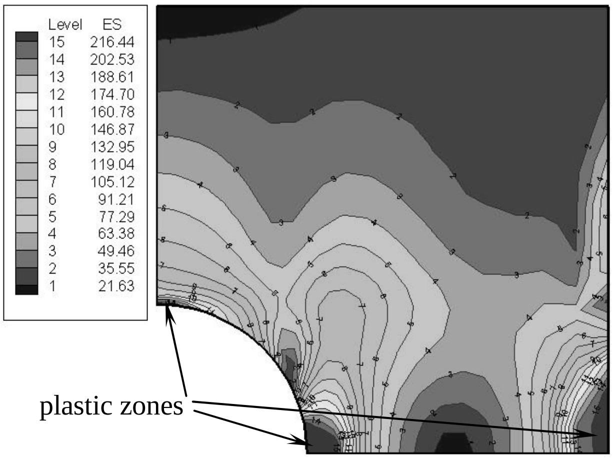


Fig. 4 The equivalent stress contour in the plate

Reference

1. Brebbia, C. A., Telles, J. C. F. and Wrobel, L. C. (1984): *Boundary Element Techniques: Theory and Applications in Engineering*, Springer-Verlag Press.
2. Swedlow, J. L. and Cruse, T. A. (1971): "Formulation of Boundary Integral Equation for Three-dimensional Elasto-plastic Body", *International Journal of Solids and Structures*, Vol. 7, pp. 1673-1683.
3. Telles, J. C. F. and Brebbia, C. A. (1979): "On the Application of the Boundary Element Method to Plasticity", *Applied Mathematical Modelling*, Vol. 3, pp. 466-470.
4. Cen, Z. Z. (1984): *Three-dimensional elasto-plastic analysis with coupling method of the FEM and the BEM*, Ph.D. dissertation, Department of Engineering Mechanics, Tsinghua University.
5. Green, E. (1943): "A Note on Stress Systems in Aeolotropic Materials", *Philosophical Magazine*, Vol. 34, pp. 416-418.
6. Rizzo, F. J. and Shippy, D. J. (1970): "A Method for Stress Determination in Plane Anisotropic Elastic Bodies", *Journal of Composite Materials*, Vol. 4, pp. 36-61.
7. Sun, X. S. and Cen, Z. Z. (2002): "Further Improvement on Fundamental Solutions of Plane Problems for Orthotropic Materials", *Acta Mechanica Solida Sinica*, Vol. 15, pp. 171-181.