Modelling of anisotropic ductile damage using gradient theory

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Summary

The present paper deals with the numerical analysis of large elastic-plastic deformation and localization behavior of anisotropically damaged ductile solids within the framework of nonlocal continuum mechanics. The idea of bridging the length-scales is realized by using higher-order gradients in the evolution equations of the equivalent inelastic strain measures. Estimates of the stress and strain histories are obtained from a numerical integration algorithm which employs an inelastic predictor followed by an elastic corrector step. This leads to a system of elliptic partial differential equations which is solved via the finite difference method at each iteration of the loading step. Numerical simulations demonstrate the efficiency of the formulation and show the influence of several model parameters on the deformation prediction of tension specimens.

Introduction

The accurate and realistic description of inelastic behavior of ductile materials and structures as well as the development of associated efficient and stable numerical solution techniques are essential for the solution of numerous boundary-value problems occurring in engineering. Large inelastic deformations of metals are usually accompanied by damage processes due to the nucleation, growth and coalescence of micro-defects. The proper understanding of these mechanisms and their mechanical description are of importance in discussing the mechanical effects of the material deterioration on the macroscopic behavior of solids as well as in elucidating the mechanisms leading to final fracture. The proposed continuum damage model uses a continuous variable which is related to the density of the defects in order to describe the deterioration of the material. In this context, nonlocal effects seem to be important when deformation mechanisms governed by microscopic phenomena as well as scale effects are considered in order to explain and predict certain experimentally observed critical phenomena. For example, gradient theories [1] represent a constitutive framework on the continuum level that is used to bridge the gap between the micro-mechanical level and the classical continuum level through the incorporation of intrinsic material length parameters into the constitutive model [2].

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Fundamental governing equations

A nonlocal generalization of the framework presented by Brünig [3,4] is proposed to describe the inelastic deformation behavior of ductile metals. The anisotropic continuum damage model is based on the introduction of damaged as well as fictitious undamaged configurations related via metric transformations which allow for the introduction of damage strain tensors. The modular structure is accomplished by the kinematic decomposition of strain rates into elastic, plastic and damage parts which take into account the physics of these deformation processes. Furthermore, respective Helmholtz free energy functions are formulated with respect to fictitious undamaged and to current damaged configurations. In addition, to be able to minimize the analytical and numerical difficulties associated with general nonlocal formulations [5] the nonlocal concept is applied only to those parameters which cause material softening while the elastic behavior is still assumed to be governed by a local formulation. Thus, the kinematic relations as well as the balance equations remain local, and the distribution of the stresses and displacements is still governed by the standard differential equations of equilibrium and the associated boundary conditions. On the other hand, to be able to address equally the two physically distinct modes of irreversible changes, i.e. plastic flow and ductile damage, a macroscopic nonlocal yield condition is employed to adequately describe the onset and continuation of plastic flow of ductile metals observed experimentally and the concept of a nonlocal damage surface is used to designate the onset and evolution of anisotropic damage. Nonlocal plastic and damage effects are described via additional length quantities which play the role of material parameters bridging the gap between microscopic variables and the classical continuum variables considering weighted averages of the corresponding local plastic and damage variables over a material volume of the body.

The effective specific free energy $\bar{\phi}$ of the undamaged matrix material is decomposed into an effective elastic and an effective plastic part

$$\bar{\phi} = \bar{\phi}^{el}\left(\bar{\mathbf{A}}^{el}\right) + \bar{\phi}^{pl}\left(\gamma, \hat{\gamma}\right), \tag{1}$$

where $\bar{\mathbf{A}}^{el}$ is the effective elastic strain tensor, γ and $\hat{\gamma}$ denote an internal plastic variable and its nonlocal counterpart, respectively. In addition, plastic yielding of the matrix material is described by the nonlocal yield condition

$$f^{pl}\left(\bar{I}_1, \bar{J}_2, c\right) = \left(1 - \frac{a}{c}\bar{I}_1\right)^{-1}\sqrt{\bar{J}_2} - c\left(\gamma, \hat{\gamma}\right) = 0 \ , \tag{2}$$

where $\bar{I}_1 = \mathrm{tr}\bar{\mathbf{T}}$ and $\bar{J}_2 = \frac{1}{2}\mathrm{dev}\bar{\mathbf{T}} \cdot \mathrm{dev}\bar{\mathbf{T}}$ are invariants of the effective stress tensor $\bar{\mathbf{T}}$, c denotes the strength coefficient of the matrix material and a represents the hydrostatic stress coefficient [6].

Moreover, the Helmholtz free energy function of the damaged material sample is assumed to consist of three parts:

$$\phi = \phi^{el}\left(\mathbf{A}^{el}, \mathbf{A}^{da}\right) + \phi^{pl}\left(\gamma, \hat{\gamma}\right) + \phi^{da}\left(\mu, \hat{\mu}\right) \quad . \tag{3}$$

In particular, the elastic free energy ϕ^{el}, which is an isotropic function of the elastic and damage strain tensors, $\mathbf{A}^{el}$ and $\mathbf{A}^{da}$, is used to describe the decrease of the elastic material properties and of the stored energy of the damaged material at the current state of deformation and material damage compared to the response of the virgin undamaged material. In addition, the energies ϕ^{pl}, due to plastic hardening, and ϕ^{da}, due to damage strengthening, only take into account the respective internal state variables, γ and μ, as well as their nonlocal counterparts $\hat{\gamma}$ and $\hat{\mu}$ which are taken to be volume averages of γ and μ, respectively.

Furthermore, evolution of damage is described by the nonlocal damage criterion

$$f^{da}\left(I_1, J_2, \tilde{\sigma}\right) = I_1 + \tilde{\beta}\sqrt{J_2} - \tilde{\sigma}\left(\mu, \hat{\mu}\right) = 0 \quad , \tag{4}$$

where $\tilde{\beta}$ represents the influence of the deviatoric stress state on the damage condition and $\tilde{\sigma}$ denotes the equivalent damage stress measure.

Numerical aspects

On the numerical side a key aspect in the numerical treatment of inelastic continuum models using the finite element method is the numerical integration of the nonlinear constitutive equations governing the flow and damage behavior as well as the evolution of internal state variables. Estimates of the irreversible strain histories are obtained via a gradient-enhanced version of the inelastic predictor method. In the inelastic predictor step the entire incremental deformation is assumed to be inelastic. This leads to an overestimation of the equivalent inelastic strains which have to be corrected. In the elastic corrector step the nonlocal plastic and damage correctors, $\Delta_{er}\hat{\gamma}$ and $\Delta_{er}\hat{\mu}$, are expanded in respective Taylor series, e.g.

$$\Delta_{er}\hat{\gamma} = \Delta_{er}\gamma + d_{pl}\frac{\partial^2 \Delta_{er}\gamma}{\partial \mathbf{x} \cdot \partial \mathbf{x}} + ... = \Delta_{er}\gamma + d_{pl}\nabla^2\left(\Delta_{er}\gamma\right) + ..., \tag{5}$$

where only terms up to second order are retained and d_{pl} takes into account a plastic internal length scale [6]. This leads to the system of coupled elliptic partial differential equations for the estimates of the correctors of the equivalent plastic and damage strains

$$m_{pl}d_{pl}\frac{\partial c}{\partial \gamma}\nabla^2\left(\Delta_{er}\gamma\right) + \left[\sqrt{2}G_1k_1 + \frac{\partial c}{\partial \gamma}\right]\Delta_{er}\gamma + \sqrt{2}G_1k_2\Delta_{er}\mu = c_{pr} - c(t) \tag{6}$$

and

$$m_{da}d_{da}\frac{\partial \sigma}{\partial \mu}\nabla^2\left(\Delta_{er}\mu\right) + \left[\sqrt{2}G_2k_4 + \frac{\partial \sigma}{\partial \mu}\right]\Delta_{er}\mu + \sqrt{2}G_2k_3\Delta_{er}\gamma = \sigma_{pr} - \sigma(t). \tag{7}$$

where G_1 and G_2 as well as $k_1,...,k_4$ represent modified material parameters [4] and m_{pl} and m_{da} denote the relative weights of the nonlocal effects compared to the local ones. Equations (6) and (7) are solved via a standard finite difference method employing an overlay mesh defined by the Gaussian integration points of the underlying finite element mesh. The global weak equilibrium equations, on the other hand, are solved using standard displacement-based finite elements and the associated linearized variational equations are derived from a consistent linearization algorithm. At the end of each time step equivalent plastic and damage strain increments are computed simultaneously which then lead to the corresponding tensorial quantities employing an integration scheme with an exponential shift.

Numerical example

The numerical example deals with the finite deformation and localization behavior of uniaxially loaded rectangular specimens with clamped ends. The corresponding load-deflection curves are shown in Fig. 1 based on the gradient-enhanced elastic-plastic material model without damage, including isotropic damage and including anisotropic damage discussed above, respectively. In particular, Fig. 1 shows that with the onset of damage the numerical calculations including isotropic and anisotropic damage show rapid loss in load carrying capacity with increasing elongation of the specimen which agrees quite well with experimental observations.

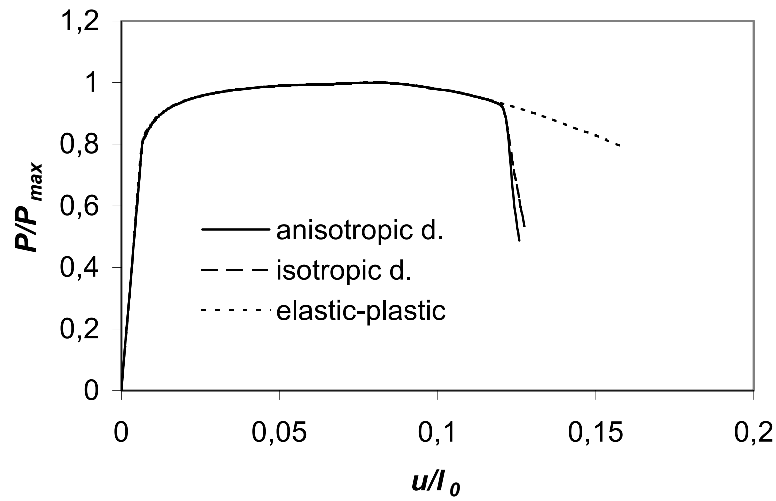

Fig. 1: Load-deflection curves

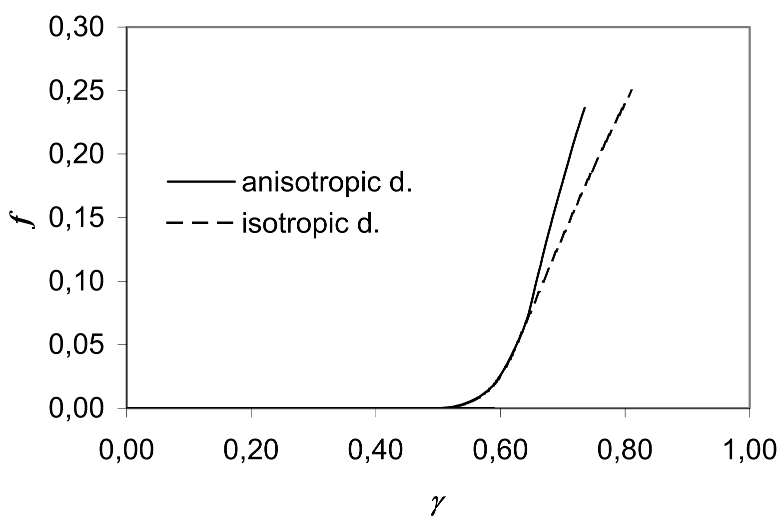

Fig. 2: Evolution of void volume fraction

Fig. 2 shows the evolution of the void volume fraction f with increasing equivalent plastic strain γ. The numerical calculation for the isotropic damage model shows an increase in void volume fraction after the onset of damage. The point where the two curves based on the isotropic and the anisotropic model separate characterizes the onset of anisotropic damage which leads to an even larger increase in void volume fraction with growing plastic strain.

Conclusions

A gradient-enhanced anisotropic damage theory for ductile metals and its numerical treatment have been discussed. The proposed large strain damage theory is a robust and efficient framework to develop structural models capable for providing practical solutions of general problems in engineering.

References

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