Optimal Reduction of Vibrating Substructures

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Introduction

One of the fundamental problems of linear system theory is the approximation of complex systems by simpler ones. In the context of structural dynamics, the goal is to approximate a given discrete model of a structure by another model which involves a much smaller number of degrees of freedom. Such a reduction is very important in cases where the computational effort associated with the direct analysis of the given system is prohibitive. There is a very large volume of literature on the subject, often called “model order reduction.” Some of this work is related to robust control, where there is a need for the repetitive real-time solution of large structural dynamic problems.

One major way to simplify models is via *modal reduction*. The original linear system is £rst decomposed into its eigenmodes. Then a small number of these eigenmodes is retained to represent the system, whereas all the other modes are discarded. The following question then arises: “*Which* of the modes should be retained?” In structural dynamics it is traditional to retain those modes associated with the *lowest frequencies*. In control theory, a common procedure is the Balanced Realization method proposed by Moore, where a special mode truncation is used to obtain a reduced system with equal amount of controllability and observability. Both these approaches are simple and easy to code, but they are not based on any optimality criterion. Hence, although in many cases they produce very good approximations, they are not guaranteed to do so.

There are also model reduction methods which are not based on modal truncation. One very simple procedure, used mainly in aeroelasticity, is Guyan Reduction which is an approximate dynamical analogue of static condensation. The next level of complexity in structural dynamic condensation is “Component Mode Synthesis.” In this case, the response of the subsystem is projected via a Ritz reduction procedure onto a collection of vectors which are described as rigid body “modes,” dynamic “modes,” and constraint “modes.” In the robust controls literature, we can £nd reduction schemes on the other end of the spectrum of complexity. The Optimal Projection method of Hyland and Bernstein, for example, is an optimization method which guarantees a minimum reduction error using a quadratic error criterion. Both the theoretical considerations and the practical implementation of this method are much more complicated than those of Balanced Realization, Component Mode Synthesis, or Guyan Reduction.

These methods are applied, in most cases, to an entire dynamic system with given boundary conditions, whose reduction is desired. In this paper, we consider a slightly different perspective. We consider a linear subsystem “attached” to a main system. In the context of structural dynamics, such a subsystem may represent a piece of equipment, antenna, etc., connected to the main structure. On the other hand, if an engineer is interested

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in the dynamics of different antenna designs, the "subsystem" may be most of the structure, while the antenna may be treated as the "main" structure. We wish to reduce the subsystem alone, without modifying the main system. When doing this, we are not interested in the accurate representation of the dynamics of the subsystem itself, but in accurately representing the effect this subsystem has on the dynamic behavior of the main structure.

In this presentation we concentrate on the modal reduction of a linear subsystem with no damping. We ask a question similar to the one mentioned above: In the reduction process, *which of the subsystem's modes should be retained?* In contrast to the low-frequency rule dominating structural dynamics, we shall obtain a new criterion for "modal importance." We shall show that the most important modes of the subsystem are those whose coupling matrices, to be de£ned in a particular way, have the highest norm. This will lead to a simple and effective algorithm for optimal modal reduction. We shall explain the new criterion both mathematically and physically and shall demonstrate it via numerical examples.

Outline of the Presentation

We £rst give the detailed statement of the reduction problem under consideration. The formulation of this problem is based on the notion of the Dirichlet-to-Neumann (DtN) map, a concept whose relevance to the present problem we explain. Then we derive the appropriate DtN map, and de£ne the coupling matrix of a mode. This matrix is related to the amount of coupling existing between the subsystem and the main system. Next we use these concepts to derive the solution to the reduction problem. This involves the formulation of a criterion for choosing the subsystem's most important modes. We then present a modal reduction algorithm based on this criterion. Then we provide the physical interpretation of the "modal importance criterion" and illustrate it via examples. We present some numerical results which are compared to those obtained by standard modal reduction (the latter being based on retaining the lowest-frequency modes).