Effects of gravity variations on thermocapillary convection

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Summary

We study, from the numerical point of view, a thermal convection problem in a cylindrical annulus where a dynamic flow is imposed through a non-zero temperature gradient at the bottom. Both theoretical and experimentally many interesting dynamical behaviours have been discovered in this system, which are controlled by heat related parameters and buoyant and thermocapillary instability mechanisms. By changing the gravity constant g, we explore how instabilities driven by surface tension effects, are affected. We find that for large g values, waves are the only instability. It is remarkable the apparition of several oscillatory-oscillatory codimension two bifurcations controlled by heat-related parameters.

Introduction

Instabilities and pattern formation in buoyant-thermocapillary flows have been extensively studied in the last few years. Classically heat is applied uniformly from below [1] where the conductive solution becomes unstable for temperature gradients beyond a certain threshold. A more general set-up considers thermoconvective instabilities where a basic dynamic flow is imposed through non-zero horizontal temperature gradients [2], [3]. In [10], results on this problem are obtained which address the importance of heat related parameters to develop the instabilities. In [8] a great diversity of bifurcations is controlled by the Biot number. Previous works [2] have addressed the importance of surface tension effects as the origin of waves. In [9] results are obtained for large surface tension effects. In this paper we study how those results are affected by increasing buoyancy effects enlarging the values of the gravity constant g.

Governing Equations

The physical set-up is shown in Fig. 1. A horizontal fluid layer of depth d (z coordinate) is in a container limited by two concentric cylinders of radii a and $a+\delta$ (r coordinate). The values of a, δ and d are fixed, $a = 0.008$ m, $\delta = 0.135$ m and $d = 0.0019$ m. The bottom plate is rigid and a linear temperature profile is imposed where the inner part has a temperature $T_{\max}$ whereas the outer one is

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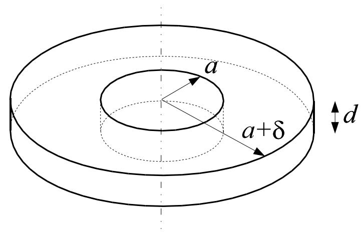


Figure 1: Problem set-up.

at $T_{\min}$. The top is open to the atmosphere which is at temperature T_0. We define $\triangle T = T_{\max} - T_0$ and assume $\triangle T \neq 0$. In the equations governing the system u_r, u_ϕ and u_z are the components of the velocity field u, T is the temperature, p is the pressure, r is the radial coordinate and t is the time. The system evolves according to the momentum and the mass balance equations and to the energy conservation principle. After rescaling the magnitudes, $\mathbf{r}' = \mathbf{r}/d$, $t' = \kappa t/d^2$, $u' = du/\kappa$, $p' = d^2 p/(\rho_0 \kappa \nu)$, $\Theta = (T - T_0)/\triangle T$ the equations in dimensionless form are,

$$\nabla \cdot u = 0, \tag{1}$$

$$\partial_t \Theta + u \cdot \nabla \Theta = \nabla^2 \Theta, \tag{2}$$

$$\partial_t u + (u \cdot \nabla) u = \mathrm{Pr}\left(-\nabla p + \nabla^2 u - b + \mathrm{R}\Theta e_z\right), \tag{3}$$

where the operators and fields are expressed in cylindrical coordinates and the Oberbeck-Bousinesq approximation has been used. Here e_z is the unit vector in the z direction, κ is the thermal diffusivity, ν the kinematic viscosity of the liquid and ρ_0 is the mean density at the environment temperature T_0. The following dimensionless numbers have been introduced: the Prandtl number Pr$= \nu/\kappa$, which is ∞ in our problem, $b = d^3 g/(\kappa\nu)$ and the Rayleigh number R$= g\alpha\triangle T d^3/\kappa\nu$, which represents the buoyant effect. In these definitions α is the thermal expansion coefficient and g is the gravity constant, which takes several values in this work. The boundary conditions (bc) for velocity are

$$\begin{aligned} u_r &= u_\phi = u_z = 0, \text{ on } r = a^*, \; r = a^* + \delta^*, \\ u_r &= u_\phi = u_z = 0, \text{ on } z = 0, \\ u_z &= 0, \text{ on } z = 1, \\ \partial_z u_r + \mathrm{M}\partial_r \Theta &= 0, \; \partial_z u_\phi + \frac{\mathrm{M}}{r}\partial_\phi \Theta = 0, \text{ on } z = 1, \end{aligned}$$

where $a^* = a/d$ and $\delta^* = \delta/d$. Here M$= \gamma\triangle T d/(\kappa\nu\rho_0)$ is the Marangoni number and γ is the constant change rate of surface tension with temperature. For temperature we consider

$$\partial_z \Theta = -B\Theta, \text{ on } z=1, \tag{4}$$

$$\Theta = \left(-\frac{r}{\delta^*}+\frac{a}{\delta}\right)\frac{\triangle T_h}{\triangle T}+1, \text{ on } z=0, \tag{5}$$

$$\partial_r \Theta = 0, \text{ on } r=a^* \text{ and } r=a^*+\delta^*. \tag{6}$$

A thorough explanation on those bc can be found in [6],[7].

For convenience we redefine three dimensionless parameters as R= $C_R \triangle T$, M= $C_M \triangle T$, P= $C_P/\triangle T$, where $C_M = \gamma d/(\kappa\nu\rho_0)$, $C_R = g\alpha d^3/\kappa\nu$ and $C_P = \triangle T_h$. We notice that for a given set-up in experiments bifurcation thresholds are controlled by moving $\triangle T$ and therefore the three dimensionless quantities defined above are changed simultaneously. In our numerical results we maintain $\triangle T$ as the control parameter, noticing that in classical convection studies the parameter P= $C_P/\triangle T$ does not appear. In reference [7], the influence of different C_P values on bifurcation thresholds and structure, by keeping C_R and C_M constant, was reported. In reference [9] the influence of large C_M values on bifurcations was studied. In this work we analyze how those results may be modified by changing C_R. In particular for the constants defining C_R we consider similar values to those used in experiments of Ref. [4]: $d = 0.0019$ m, $\rho_0 = 885$ Kg m^{-3}, $\nu = 3,2\cdot 10^{-6}$ m^2/s, $\kappa = 7\cdot 10^{-8}$ m^2/s, $\gamma = 8\cdot 10^{-5}$ N m^{-1}/ ^{o}C and $\alpha = 9,8\cdot 10^{-4\,o}$C^{-1}. The value of g is increased from its usual value 9.81 m/s^2 to 50 m/s^2 and 100 m/s^2.

Basic state and linear stability analysis

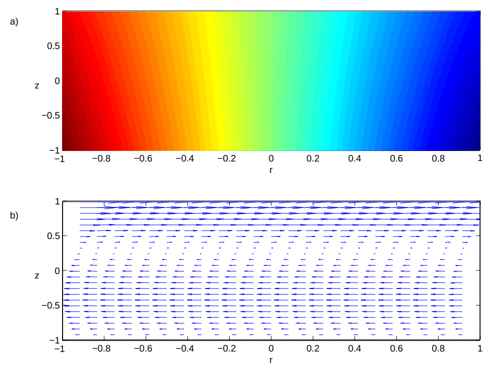
As in [7] we solve these equations with a collocation method by expanding the fields with Chebyshev polynomials. The basic states are stationary solutions for this problem assuming radial symmetry. The stability of the basic state is studied by perturbing it with fields depending on the r,ϕ and z coordinates, in a fully 3D analysis:

$$X(r,\phi,z) = X^b(r,z) + \bar{X}(r,z)\,e^{im\phi+\lambda t}, \tag{7}$$

Here X stands for a generic field, the superscript b denotes the basic state and the bar refers to the perturbation. We have considered Fourier modes expansions in the angular direction as along it boundary conditions are periodic.

We replace Eqs. (7) into the basic equations and after linearizing the resulting system and rescaling the coordinates for computational convenience, we obtain the

Figure 2: Isotherms and velocity fields of basic states at an oscillatory instability threshold $\triangle T_c = 0.66$ K, for $g = 50$ m/s^2, $B = 0.2$ and $\triangle T_h = 1$ K.



following eigenvalue problem (the bars have been dropped):

$$\Delta_{\mathrm{m}} u_r - A\partial_r p - G^2 u_r - 2G^2 \mathrm{i} \mathrm{m} u_\phi = 0, \tag{8}$$

$$\Delta_{\mathrm{m}} u_\phi - G\mathrm{i}\mathrm{m} p + 2G^2 \mathrm{i}\mathrm{m} u_r - G^2 u_\phi = 0, \tag{9}$$

$$\Delta_{\mathrm{m}} u_z - 2\partial_z p + \mathrm{R}\Theta = 0, \tag{10}$$

$$G u_r + A\partial_r u_r + G\mathrm{i}\mathrm{m} u_\phi + 2\partial_z u_z = 0, \tag{11}$$

$$\Delta_{\mathrm{m}}\Theta - u_r A\partial_r \Theta^b - u_r^b A\partial\Theta - 2u_z^b \partial_z \Theta - 2u_z \partial_z \Theta^b = \lambda\Theta, \tag{12}$$

where $\Delta_{\mathrm{m}} = A^2\partial_r^2 + GA\partial_r - m^2G^2 + 4\partial_z^2$, $A = 2d/\delta$ and $G(r) = 2d/(2a + \delta + r\delta)$. The problem is completed with the appropriate boundary conditions as explained in [6], [7]. Following those references eigenfunctions and thresholds of the generalized eigenvalue problem are computed with a collocation method, so that a dispersion relation $\lambda \equiv \lambda(\mathrm{m}, \mathrm{R}, \mathrm{M}, B, u^b, \Theta^b, p^b)$, is obtained. If $\mathrm{Re}(\lambda) < 0$ the basic state is stable while if $\mathrm{Re}(\lambda) > 0$ the basic state becomes unstable.

Numerical Results

Figure 2 displays the temperature and velocity fields of a basic state solution for conditions at the threshold of an oscillatory bifurcation for $g = 50$ m/s^2. It is noticed that the velocity is very close to a return flow predicted by Smith and Davis [2].

For large surface tension effects and large gravitatory effects ($g = 50$ m/s^2 and $g = 100$ m/s^2), at the considered Biot values, we have found that only os-

Table 1: Critical $(\mathrm{m}_c, \triangle T_c)$ values at different B and $\triangle T_h$.

	$g = 50\mathrm{m/s}^2$			$g = 100\mathrm{m/s}^2$		
	$\triangle T_h = 1\mathrm{K}$	$\triangle T_h = 2\mathrm{K}$	$\triangle T_h = 3\mathrm{K}$	$\triangle T_h = 1\mathrm{K}$	$\triangle T_h = 2\mathrm{K}$	$\triangle T_h = 3\mathrm{K}$
$B = 0.05$	$(15, 2.11)$	$(23, 2.37)$	$(25, 2.62)$	$(21, 1.81)$	$(23, 2.06)$	$(25, 2.28)$
$B = 0.1$	$(15, 1.14)$	$(17, 1.31)$	$(19, 1.47)$	$(15, 0.98)$	$(18, 1.12)$	$(20, 1.28)$
$B = 0.2$	$(15, 0.66)$	$(17, 0.77)$	$(15, 0.87)$	$(15, 0.57)$	$(17, 0.67)$	$(15, 0.76)$
$B = 0.3$	$(15, 0.49)$	$(14, 0.58)$	$(15, 0.66)$	$(15, 0.43)$	$(13, 0.51)$	$(15, 0.57)$

cillatory bifurcations appear. Table 1 presents the critical values of $(\mathrm{m}_c, \triangle T_c)$ for $g = 50, 100\mathrm{m/s}^2$. In previous works [8] oscillatory and stationary bifurcations are present depending on heat conditions represented by the Biot number B. Now we find that there exist conditions where only oscillatory bifurcations are possible.

A codimension two bifurcation (CTB) oscillatory-oscillatory has been observed. This is a new result with respect to those obtained in [8] where proven CTB are oscillatory-stationary or stationary-stationary. Figure 3a) proves that for $\triangle T_h = 2\mathrm{K}$ and $B = 0.06$ two complex eigenvalues with different wave numbers, $\mathrm{m}_c = 18$ and $\mathrm{m}_c = 23$ cross the real axis at the same threshold $\triangle T = 1.73\mathrm{K}$. In Figure 3b) we display a temperature contour plot at $z = 1$ for one of the growing eigenfunctions at the codimension two bifurcation point.

Conclusions

We have studied a thermal convection problem in a cylindrical annulus where a dynamic flow is imposed through a non-zero temperature gradient at the bottom. We have found that for large surface tension effects and large g values, waves are the only instability present. It is remarkable the apparition of oscillatory-oscillatory codimension two bifurcations controlled by heat-related parameters.

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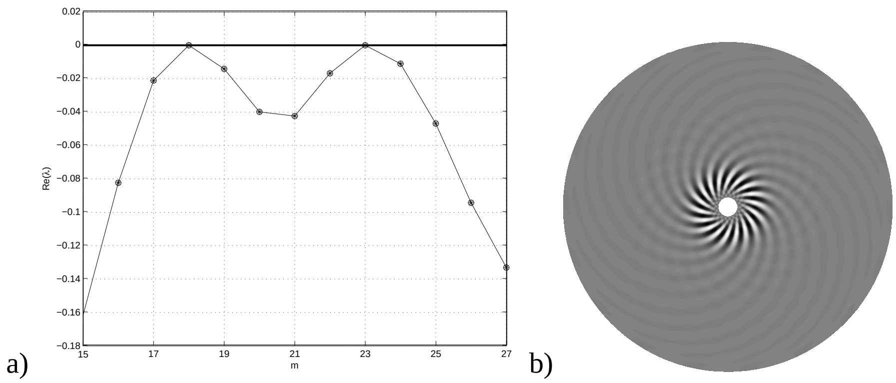

Figure 3: a) Eigenvalues λ as a function of the wave number m at a codimension two bifurcation point. The physical parameters are $\triangle T_h = 2$K $B = 0.06$ and $\triangle T = 1.73$K. b) Growing perturbation for $g = 100$ m/s^2 at the codimension two bifurcation point. The rest of conditions are: $B = 0.0612$, $\triangle T = 1.7310$K, m=18, $\triangle T_h = 2$K.

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