Boundary layer in a moderately thick plates under creep-damage conditions

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Summary

The aim of this presentation is to recall the governing equations of creep mechanics, to compare the long-term predictions based on the three-dimensional approach and a two-dimensional plate model, and to discuss the possibilities and limitations of each approach in connection with creep-damage analysis. In the first part we summarize the basic features of creep behavior in metals and alloys, introduce the widely used Kachanov-Rabotnov-Leckie-Hayhurst material model and demonstrate the description possibilities for stress states which are typical for thin-walled structures. In the second part, we discuss the quasi-static initial-boundary value problems of creep. Finally, we use the ANSYS finite element code in order to simulate the time-dependent behavior of different thin structures under creep-damage conditions. Based on the numerical examples we make conclusions regarding the boundary layers as well as the applicability of the shell and the solid type finite elements to the creep analysis of engineering structures.

Introduction

If thin-walled metallic structures (beams, plates, pipes, pipe bends, etc.) operate at elevated temperatures the behavior of metals and alloys is primarily determined by irreversible time-dependent creep-damage processes. In order to estimate the long-term behavior it is important to understand the time-dependent stress redistribution and damage growth, particularly in the zones of nozzles, pipe connections and welds. The aim of this presentation is to analyze the boundary layer stress redistributions in moderately thick plates. Boundary layer solutions are usually discussed for elastic moderately thick plates [10] or laminated composite plates [4].

In what follows we study the evolution of boundary layers as a consequence of creep-damage material behavior. Based on the results of a solid type finite element analysis we show that the "second order effects" such as transversal normal and shear stresses play an important role in long-term failure analysis of thin-walled structures. One possibility to model the creep-damage behavior is the continuum damage mechanics (CDM), see, e.g., [6]. As usual, an important step in the analysis of such structures is to select a structural mechanics model and to specify the type of finite elements. One way is a "three-dimensional approach". This approach seems more preferable for creep analysis, since the existing constitutive models of creep are developed with respect to the Cauchy stress and strain (rate) tensors and the proposed measures of damage (scalars or tensors of different rank) are defined in the three-dimensional space. Another way is the use of the classical structural mechanics equations of beams, plates and shells. This approach often find application because of simplicity of model creation, smaller effort in solving non-linear initial-boundary value problems of creep, and easily interpretable results.

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Basic material behavior equations and structural mechanics models

The commonly used phenomenological model characterizes the secondary creep rate by the power law stress function and includes the effect of tertiary creep by means of the single scalar valued damage parameter [8]

$$
\begin{aligned}
&\dot{\boldsymbol{\varepsilon}}^{cr} = \frac{3}{2} a \left(\frac{\sigma_{vM}}{1-\omega} \right)^n \frac{\mathbf{s}}{\sigma_{vM}}, \quad \dot{\omega} = b \frac{[\alpha \sigma_T + \beta \sigma_m + (1-\alpha-\beta)\sigma_{vM}]^k}{(1-\omega)^l}, \\
&\sigma_{vM} = \sqrt{\frac{3}{2} \mathbf{s} \cdot\cdot\, \mathbf{s}}, \quad \sigma_m = \frac{1}{3} \mathrm{tr} \boldsymbol{\sigma}, \quad \mathbf{s} = \boldsymbol{\sigma} - \sigma_m \mathbf{E}, \quad \sigma_T = \frac{1}{2}(\sigma_I + |\sigma_I|)
\end{aligned}
\tag{1}
$$

In this notation $\dot{\boldsymbol{\varepsilon}}^{cr}$ is the creep strain rate tensor, $\boldsymbol{\sigma}$ is the stress tensor, σ_I is the first principal stress, $\mathbf{E}$ is the second rank unit tensor and ω is the damage parameter. The weighting factor α characterizes the influence of the principal damage mechanisms (σ_T-controlled, σ_m-controlled or σ_{vM}-controlled). a, b, n, k and l are material constants, which are determined from creep tests at a constant temperature. The model (1) ignores the effects of primary creep. The basic mechanisms of the creep-damage processes in metals and alloys at elevated temperatures above 0.4 of the melting temperature are briefly discussed, e.g., [5].

An example of application of Eqs. (1) is presented for the 316 stainless steel at 650°C in [2]. It was shown that the considered stress states and the stress values provide the same secondary creep with the same minimum creep rates. The tertiary creep responses are quite different and depend significantly on the kind of the applied stress state [7]. The shear strain behavior depends not only on the value of the applied shear stress, but more significantly on the value of the normal stress. The change of sign of σ leads to the considerable change of the shear strain response.

The stress states with combined action of normal tensile (compressive) stress and small shear stress are typical for transversely loaded beams, plates and shells. The creep response of transversely loaded beams is discussed in [2], [9]. It was shown that the through-the-thickness distribution of the transverse shear stresses differs from the classical parabolic one. Nevertheless, by formulation of suitable constitutive equations for the shear force, the Timoshenko type beam theory can be applied to the creep-damage analysis [1]. The results for the life-time estimation agree well only in the case of the von Mises equivalent stress σ_{vM} controlled tertiary creep. For the case of the σ_T or σ_m controlled damage evolution the shell and the solid models lead to significantly different results.

Various approaches to derive a shell theory have been developed within the assumption of elastic or viscoelastic material behavior. As far as we know, a ”closed form” shell theory in the case of creep does not exist at present. The principal problem lies in establishing the constitutive equations of creep with respect to the shell type strain measures, i.e. the membrane strains, changes of curvature and transverse shear strains. Although, a general structure of such equations can be found based on the direct approach, e.g. [3], the open question is the introduction of appropriate damage measures as well as the identification of damage mechanisms under the shell type stress states. Depending on the type of the applied variational equation (e.g. displacement type or mixed type) and the type of incorporated cross-section assumptions, different two-dimensional versions of general equations of the theory of elasticity with a different order of complexity can be obtained (i.e. models with forces and moments or models with higher order stress resultants). Various types of finite elements which were developed for the inelastic analysis of shells are reviewed in [12]. Let us note that if studying the creep behavior coupled with damage, the type of assumed cross-section approximations may have a significant influence on the result. For example, if we use a mixed type variational equation and approximate both the displacements and stresses, a parabolic through-the-thickness approximation for the transverse shear stress or a linear approximation for the in-plane stresses is in general not suitable for the creep damage estimations [2].

FEA of a plate based on solid and shell elements

In order to compare both shell and three-dimensional models we perform a FEA of a plate. As an example we selected a square plate with $l_x = l_y = 1000$ mm, $h = 100$ mm, loaded by a pressure $q = 2$ N/mm^2 uniformly distributed on the top surface. The edges $x = 0$ and $x = l_x$ are simply supported and the edges $y = 0$ and $y = l_y$ are clamped. Note that the displacement based finite element method allows only to prescribe the kinematical boundary conditions. According to the first order shear deformation type plate model we can specify the vectors of midplane displacements $\mathbf{u}(x,y) = \mathbf{u}_T(x,y) + \mathbf{n}w(x,y)$ and cross-section rotations $\boldsymbol{\varphi}(x,y)$ on the lines $x = const$ or $y = const$. Applying such a model and assuming infinitesimal cross-section rotations the displacement vector $\mathbf{U}(x,y,z)$ is usually assumed to be $\mathbf{U}(x,y,z) \approx \mathbf{u}(x,y) + z\boldsymbol{\varphi}(x,y) \times \mathbf{n}, \boldsymbol{\varphi} \cdot \mathbf{n} = 0$. In the case of the three-dimensional model the displacement vector $\mathbf{U} = \mathbf{U}_T(x,y,z) + W(x,y,z)\mathbf{n}$ can be prescribed on the planes x_c, y, z or x, y_c, z of the plate edges $x = x_c$ or $y = y_c$. Here we discuss two types of the clamped edge conditions. For the first type (TYPE I) we assume the vector of in-plane displacements $\mathbf{U}_T$ to be zero. The deflection W is zero only in the points of the plate mid-surface. In the second type (TYPE II) the whole displacement vector $\mathbf{U}$ is assumed to be zero in all points which belong to the plate edges.

The creep-damage analysis has been performed using the ANSYS finite element code after incorporating the material model (1). The time step based calculations were performed up to $\omega = \omega_* = 0.9$, where ω_* is the selected critical value of the damage parameter. Figure 1 illustrates the results of the computations, where the maximum deflection and the maximum value of the damage parameter are plotted as functions of time. From Fig. 1a we observe that the starting values of maximum deflection as well as the starting rates of the deflection growth due to creep are approximately the same for the shell and the two solid models. Consequently the type of the elements (shell or solid) and the type of the applied boundary conditions in the case of the solid elements has a small influence on the description of the steady-state creep process. However, the three used models lead to quite different life time predictions. The difference can be clearly seen in Fig. 1b. The shell model overestimates the time to failure, while the result based on the solid model depends significantly on the type of the clamped edge boundary conditions. In the case of the TYPE II clamped edge the damage parameter vs. time curve is too abrupt and the predicted time

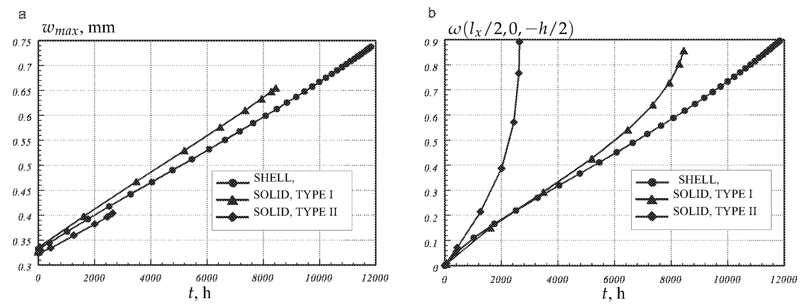

Figure 1: Time variations: **a** maximum deflection; **b** damage parameter

to failure is four times shorter compared to those based on the shell model. All considered models predict the zone of maximum damage to be in the midpoint of the clamped edge on the plate top surface.

The creep response of a structure is connected with the time-dependent stress redistribution. If the applied load and the boundary conditions are assumed to be constant and the effect of tertiary creep is ignored, then an asymptotic stress state exists, which is known as the state of stationary creep [11]. If tertiary creep is considered, then stresses change with time up to the critical damage state. It is clear that the damage growth and the tertiary creep behavior of the considered plate is controlled by the local stress state in the vicinity of the clamped edges. Figure 2 illustrates the stress states in the midpoint of the clamped edge with the coordinates are $x = l_x/2, y = 0$. Four components of the stress tensor (two remaining components are zero due to symmetry conditions) are plotted as functions of the normalized thickness coordinate. The starting elastic distributions (solid lines) as well as creep solutions at the last time step (dotted lines) are presented. The maximum starting stresses obtained by use of three considered models are the normal in-plane stresses σ_{yy} and σ_{xx} (the stresses which results in the maximum bending and twisting moments in the clamped edge), Fig. 2. These in-plane stresses remain dominant during the whole creep process for the used shell and solid elements. Therefore, all the applied models predict the damage evolution in the zone of the clamped edge on the plate top side. However, the influence of the "second order" stresses (stresses which are usually neglected in the plate theories) is different and depends on the type of the boundary conditions. For the TYPE I clamped edge the effect of the transverse normal stress σ_{zz} decreases with time and has negligible influence on the stress state. In contrast, for the TYPE II clamped edge the initial transverse normal stress remains approximately constant, while σ_{yy} relaxes with time as consequence of creep. The transverse normal stress becomes comparable with the bending stress and cannot be considered as the second order effect anymore.

Let us compare the finite element results for the mean stress and the von Mises equivalent stress. Figure 3a shows the corresponding time variations in the element A of the solid

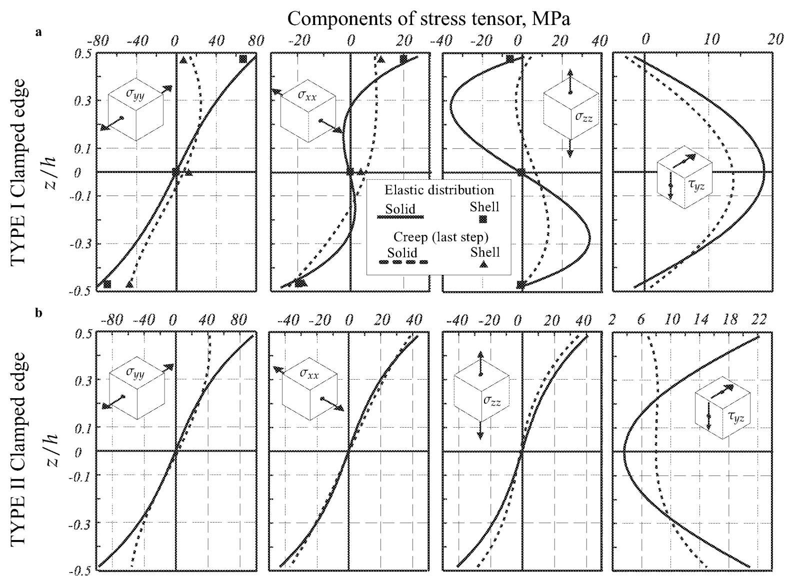

Figure 2: Local stress state in a midpoint of the clamped edge vs. thickness coordinate: **a** TYPE I clamped edge; **b** TYPE II clamped edge

model for the TYPE I and TYPE II boundary conditions. TYPE II boundary condition leads to a lower starting value of the von Mises stress and a higher starting value of the mean stress when compared with those for the TYPE I boundary condition. In addition, for the TYPE II clamped edge the mean stress rapidly decreases within the short transition time and after that remains constant while the von Mises stress relaxes during the whole creep process. With the relaxation of σ_{vM} the stress state tends to $\boldsymbol{\sigma} = \sigma_m \mathbf{E}$. The relatively high constant value of σ_m is the reason for the obtained increase of damage and much shorter time to fracture in the case of the TYPE II clamped edge (see Fig. 1b). Note that the above effect of the mean stress has a local character and is observed only in the neighborhood of the edge. As Fig. 3b shows the value of the transverse normal stress decreases rapidly with increased distance from the boundary.

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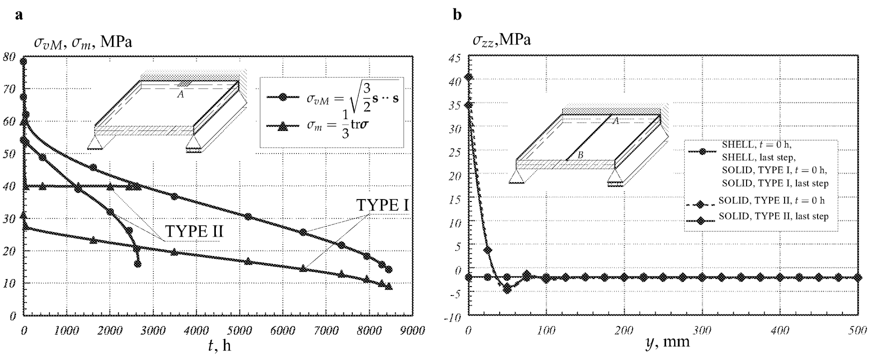

Figure 3: Transversely loaded plate: **a** von Mises equivalent stress and hydrostatic stress vs. time, element A of the clamped edge; **b** transverse normal stress σ_{zz} in elements along the line AB

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