A finite strain viscoplastic law with anisotropic damage

R. C. Lin[1], W. Brocks[1]

Summary

Based on a dissipation inequality for isotropic behavior and the effective stress concept, a Chaboche-type infinitesimal viscoplastic theory is extended to finite strain cases with anisotropic damage. The anisotropic damage is described by a rank-two symmetric tensor. The constitutive law is formulated in the corotational material coordinate system. Thus, the evolution equations of all internal variables can be expressed in terms of their material time derivatives. The material model has been implemented in a finite element program, and two numerical examples are presented. From the examples it can be seen that the model predictions agree well with the experimental observations.

Introduction

It has been proven that the Chaboche-type viscoplastic laws characterize the viscoplastic constitutive responses of metals very well. But most of these material laws are formulated for infinitesimal deformations, which hinders their application to problems with finite deformations. In addition, deformation or stress induced damage may spread in the material degrading its mechanical behavior. In general, this damage will develop anisotropically. It is therefore also necessary to model and characterize the anisotropic evolution of damage in metals.

In this work, we present a finite strain viscoplastic law with anisotropic damage. It is known from phenomenological observations [7], pp. 1-11, that damage is directly coupled with changes of the elastic behavior. Hence, the damage measure should appear only in the elastic free energy function [7] (pp. 42-47). For isotropic cases, the scalar damage measure can easily included in an elastic free energy function. But for an anisotropic damage the formulation of a coupling form of elastic free energy is difficult [4]. In this work we exploit the effective stress concept and the strain equivalence principle proposed in [6] (see also [7], pp. 13-14). The difficulty of formulating a coupled elastic free energy function is therefore circumvented. Some consequences observed from the dissipation inequality at isotropic damage, for example, the dissipation-conjugate relation between the stress and the inelastic strain rate and the evolution characteristic of the isotropic damage, are generalized for the anisotropic damage cases. The constitutive law is formulated in the corotational material coordinate system, whose rotation is specified by the polar rotation tensor. Thus, the evolution equations of all internal variables can be formulated in terms of their material time derivatives. In the algorithmic aspects the forward Euler integration strategy is exploited for integrating the evolution equations. Although this strategy could lead to loss of the asymptotic quadratic convergence characteristic of Newton's method it makes the numerical integrations quite easy. Two numerical examples are presented for proving the effectivity of the material law and the finite element coding.

Finite strain viscoplastic law with anisotropic damage

First, we observe a viscoplastic body with isotropic damage in the corotational material

[1]Institute of Materials Research, GKSS Research Centre, Max-Planck-Str., D-21502 Geesthacht, Germany

coordinate system. For the isotropic damage cases the free energy function Ψ may be assumed as

$$\Psi = \psi_e(\boldsymbol{H}^e, D) + g(\boldsymbol{\xi}) + h(p). \tag{1}$$

Here, ψ_e, g and h are respectively the elastic free energy function, the free energy functions contributed by the kinematic and isotropic hardening. $\boldsymbol{H}^e$ denotes the elastic logarithmic strain tensor described in the corotational coordinate system. This strain definition can be related to the elastic deformation gradient by the polar rotation tensor. The scalar D denotes the isotropic damage. $\boldsymbol{\xi}$ is a rank-two strain-like tensor described also in the rotated coordinate system and p denotes the accumulated plastic strain. A typical definition of the elastic free energy at isotropic damages is $\psi_e = \frac{1}{2}(1-D)\boldsymbol{H}^e : \mathbb{C} : \boldsymbol{H}^e$, with $\mathbb{C} = 2\mu\mathbb{I} + \lambda\mathbf{1}\otimes\mathbf{1}$ denoting the isotropic elasticity tensor and μ, λ the Lamé constants. Recalling the stress power per unit reference volume at isotropy $W = \boldsymbol{T} : \dot{\boldsymbol{H}}$ (see [1] and [5]), with $\boldsymbol{T}$ being the rotated Kirchhoff stress and $\dot{\boldsymbol{H}}$ the material time derivative of the material logarithmic strain, the dissipation inequality may be expressed as

$$\mathcal{D} = \boldsymbol{T} : (\dot{\boldsymbol{H}} - \dot{\boldsymbol{H}}^e) + \frac{\partial g}{\partial \boldsymbol{\xi}} : (-\dot{\boldsymbol{\xi}}) + \frac{\partial h}{\partial p}(-\dot{p}) + \psi_e^0 \dot{D} \geq 0. \tag{2}$$

In this equation, $\psi_e^0 = \frac{1}{2}\boldsymbol{H}^e : \mathbb{C} : \boldsymbol{H}^e$ is the elastic free energy function corresponding to the undamaged state and the rotated Kirchhoff stress $\boldsymbol{T}$ can be expressed as $\boldsymbol{T} = (1-D)\boldsymbol{T}^0$, with $\boldsymbol{T}^0 = \partial\psi_e^0/\partial\boldsymbol{H}^e = \mathbb{C} : \boldsymbol{H}^e$ being the effective stress. From the inequality (2) it can be seen that the stress $\boldsymbol{T}$, the back stress $\boldsymbol{\alpha} = \partial g/\partial\boldsymbol{\xi}$ and the isotropic hardening stress $R = \partial h/\partial p$ are dissipation-conjugate to the strain rates $\dot{\boldsymbol{H}} - \dot{\boldsymbol{H}}^e$, $-\dot{\boldsymbol{\xi}}$ and $-\dot{p}$. Since the free energy ψ_e^0 is always nonnegative the damage evolution must satisfy the condition $\dot{D} \geq 0$. This condition means that the damage measure D is a monotonously-increasing function and lies in the interval $[0, 1]$. These conjugate relations and the nonnegative property of damage evolution will be applied to the constitutive law.

Here, we introduce a rank-two symmetric damage tensor $\boldsymbol{D}$ to characterize the anisotropic damage. This type of damage tensor is first proposed in [8]. In the same reference an effective stress $\boldsymbol{\sigma}^0 = \boldsymbol{\sigma}(\mathbf{1} - \boldsymbol{D})^{-1}$ is derived and the related geometric explanation is also presented. Here, $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^0$ are the usual and effective stresses, respectively. $\mathbf{1}$ denotes the rank-two identity tensor. Obviously, this definition reduces to the above isotropic one if the damage tensor $\boldsymbol{D}$ is isotropic (i.e. $\boldsymbol{D} = D\mathbf{1}$). But, this effective stress is unsymmetric. Here, we apply a symmetric and concise definition [4]; i.e.

$$\boldsymbol{T}^0 = \mathbb{M} : \boldsymbol{T}, \tag{3}$$

with $\mathbb{M} = (\mathbf{1} - \boldsymbol{D})^{-1/2} \odot (\mathbf{1} - \boldsymbol{D})^{-1/2}$ being a symmetric rank-four tensor. The tensor operation sign $\odot$ is defined by $(\boldsymbol{A} \odot \boldsymbol{B})_{IJKL} = A_{IK}B_{LJ}$.

Now, we focus on the extension of an infinitesimal Chaboche-type viscoplastic law [3] to the finite strain cases with anisotropic damage. For the effective undamaged state we apply here also the above effective free energy definition. Therefore, we have $\boldsymbol{T}^0 = \mathbb{C} : \boldsymbol{H}^e$. According to [3] the back stress and the isotropic hardening stress are related to their dissipation-conjugate counterparts by $\boldsymbol{\alpha} = \frac{2}{3}C\,\boldsymbol{\xi}$ and $R = Q\{1 - \exp(-bp)\}$. According to the dissipation potential functions given in the same reference and the conjugate relations shown in (2) we obtain the evolution equations for the strain-like variables

$$
\begin{aligned}
\dot{\boldsymbol{H}} - \dot{\boldsymbol{H}}^e &= \frac{3}{2}\frac{\mathbb{M}:(\boldsymbol{s}^0-\boldsymbol{\alpha})}{J_2(\boldsymbol{s}^0-\boldsymbol{\alpha})}\dot{\lambda}_p, \\
\dot{\boldsymbol{\xi}} &= \frac{3}{2}\frac{\boldsymbol{s}^0-\boldsymbol{\alpha}}{J_2(\boldsymbol{s}^0-\boldsymbol{\alpha})}\dot{\lambda}_p - \frac{3\gamma(p)\boldsymbol{\alpha}}{2a}\dot{\lambda}_p - \frac{3d}{2C}\frac{\boldsymbol{\alpha}}{J_2(\boldsymbol{\alpha})}\dot{\lambda}_s, \qquad (4)\\
\dot{p} &= \sqrt{\tfrac{2}{3}(\dot{\boldsymbol{H}}-\dot{\boldsymbol{H}}^e):(\dot{\boldsymbol{H}}-\dot{\boldsymbol{H}}^e)} = \frac{J_2^M(\boldsymbol{s}^0-\boldsymbol{\alpha})}{J_2(\boldsymbol{s}^0-\boldsymbol{\alpha})}\dot{\lambda}_p,
\end{aligned}
$$

with definitions $\boldsymbol{s}^0 = \boldsymbol{T}^0 - \frac{1}{3}(\mathrm{tr}\,\boldsymbol{T}^0)\mathbf{1}$, $J_2(\bullet) = \sqrt{\frac{3}{2}(\bullet):(\bullet)}$, $J_2^M(\bullet) = \sqrt{\frac{3}{2}[\mathbb{M}:(\bullet)]:[\mathbb{M}:(\bullet)]}$ and

$$
\gamma(p) = \gamma_\infty + (1-\gamma_\infty)\exp(-\omega p), \quad \dot{\lambda}_p = \left[\frac{\langle f(\boldsymbol{T}^0,\boldsymbol{\alpha},R)\rangle}{K}\right]^m, \quad \dot{\lambda}_s = \left[\frac{J_2(\boldsymbol{\alpha})}{a}\right]^r. \qquad (5)
$$

Here, we note that $\langle\cdot\rangle$ is the McCauley bracket with definition $\langle y\rangle = \frac{1}{2}(y+|y|)$. Based on the strain equivalence principle [7] (pp. 15-17) the yield function f is formulated in terms of $\boldsymbol{T}^0$, $\boldsymbol{\alpha}$ and R; i.e.

$$
f(\boldsymbol{T}^0,\boldsymbol{\alpha},R) = J_2(\boldsymbol{s}^0-\boldsymbol{\alpha}) - k - R = \sqrt{\tfrac{3}{2}(\boldsymbol{s}^0-\boldsymbol{\alpha}):(\boldsymbol{s}^0-\boldsymbol{\alpha})} - k - R. \qquad (6)
$$

In the above equations $a, b, C, d, Q, K, k, m, r, \gamma_\infty$ and ω are material constants. k denotes the yield strength at zero plastic strain. Further, the following evolution equations can be derived

$$
\dot{\boldsymbol{\alpha}} = \left[\frac{\boldsymbol{s}^0-\boldsymbol{\alpha}}{J_2(\boldsymbol{s}^0-\boldsymbol{\alpha})} - \frac{\gamma(p)\boldsymbol{\alpha}}{a}\right]C\dot{\lambda}_p - \frac{d\,\boldsymbol{\alpha}}{J_2(\boldsymbol{\alpha})}\dot{\lambda}_s, \qquad \dot{R} = b(Q-R)\dot{\lambda}_p. \qquad (7)
$$

Essentially, the condition $\dot{D} \geq 0$ for the isotropic damage can be regarded as a condition satisfied by the three identical eigenvalues of the isotropic damage tensor $D\mathbf{1}$. For the anisotropic damage this condition should be also satisfied by the three disparate eigenvalues of $\boldsymbol{D}$. According to the inequality (2) the evolution of these eigenvalues should be driven by the effective elastic free energy ψ_e^0. But, for the anisotropic damage tensor $\boldsymbol{D}$ we cannot take this free energy as the driving force of its evolution. We take here formulation for stress driven damage proposed in [2] and formulate the evolution equation in the corotational coordinate system; i.e.

$$
\dot{\boldsymbol{D}} = [\beta\mathbf{1}\otimes\mathbf{1} + (1-\beta)\mathbb{I}] : \left\langle\frac{\hat{\boldsymbol{T}}}{B_0}\right\rangle^n = [\beta\mathbf{1}\otimes\mathbf{1} + (1-\beta)\mathbb{I}] : \sum_{A=1}^{3}\left\langle\frac{\hat{T}_A}{B_0}\right\rangle^n \hat{\boldsymbol{n}}_A\otimes\hat{\boldsymbol{n}}_A. \qquad (8)
$$

Here, $\hat{\boldsymbol{T}} = \hat{\mathbb{M}} : \boldsymbol{T}$ is another effective stress, with $\hat{\mathbb{M}} = (\mathbf{1}-\boldsymbol{D})^{-q} \odot (\mathbf{1}-\boldsymbol{D})^{-q}$. $\hat{T}_A$ and $\hat{\boldsymbol{n}}_A$ are the eigenvalues and eigenvectors of the effective stress. Four material constants $B_0 > 0$, n, q and $0 \leq \beta \leq 1$ with $\beta = 1$ for pure isotropy are involved in the damage evolution equation. Due to the isotropy of $\mathbf{1}\otimes\mathbf{1}$ and $\mathbb{I}$ (the rank-four identity tensor), the conditions $\beta \geq 0$, $1-\beta \geq 0$ as well as the nonnegative characteristic of the McCauley bracket, it may be easily seen that the eigenvalues of $\dot{\boldsymbol{D}}$ are nonnegative. Obviously, this characteristic is consistent with the condition $\dot{D} \geq 0$ mentioned above.

Table 1: Material constants of the model for IN738 LC at 850°C

Material constants of the viscoplastic law [9]			
$E = 149650$ MPa	$\nu = 0.33$	$k = 153$ MPa	$C = 62511$ MPa
$b = 317$	$Q = -153$ MPa	$K = 397$ MPa·h$^{1/m}$	$m = 7.7$
$a = 311$ MPa	$r = 4.8$	$d = 81.72$ MPa·h^{-1}	$\gamma_\infty = 1.1$
$\omega = 0.04$			
Material constants of the damage law [2]			
$q = 0.4$	$\beta = 0.0 \sim 1.0$	$n = 14$	$B_0 = 613$ MPa·h$^{1/n}$

Numerical examples

This finite strain viscoplastic law has been implemented as a subroutine UMAT of the general-purpose finite element program ABAQUS. For the numerical integration of the evolution equations the forward Euler integration strategy is exploited. In the following, two numerical examples are presented. In the first example the present material law is checked by comparing its predictions with experimental results from the Ni-based super alloy IN738 LC at 850°C [9]. The used material parameters of the viscoplastic law with anisotropic damage are listed in Table 1. As the parameter β, characterizing the fraction of anisotropic damage cannot be determined uniquely due to the lack of biaxial test data, we assume $\beta = 0.5$. The comparison between the numerical simulations and the experimental observations is shown in Figure 1. Here, we note that ε_{22} and D_{22} denote the true strain and the damage component in the uniaxial tension direction respectively. It can be seen that the model predictions coincide satisfactorily with the experimental observations. For small tensile stress, $\sigma = 335$ MPa, the experimentally observed creep strains lie a little bit below those predicted by the model, damage having little effect. At the highest stress level, $\sigma = 410$ MPa, the experimental data lie between the simulations with and without damage, and the medium level, $\sigma = 392$ MPa, both predictions overestimate the experimental values. No significant effects of damage can be concluded from the experiments, anyway, so that the data are mainly suited for a basic check of correct simulation of deformations by the UMAT, only. The simulations predict a significant evolution of damage, D_{22}, for the two higher stress levels, $\sigma = 392$ and 410 MPa, nevertheless. More significant tests have to be performed and analyzed to check the predictive capabilities of the model.

The second example of a 3D cubic body pulled in 2-direction with a strain rate of 1% per hour, is a purely numerical parameter study investigating the effects of parameter changes, B_0, β, n, q, on the damage evolution. The respective results are shown in Figure 2. D_{22} and D_{11} denote the damage components in the tensile and the perpendicular stress-free direction. For all $\beta < 1$, D_{11} has to be smaller than D_{22}, obviously, only in the purely isotropic case, $\beta = 1$, we obtain $D_{11} = D_{22}$. In the other limiting case, $\beta = 1$, i.e. purely anisotropic behavior, $D_{11} = 0$. The results for D_{22} do not depend on β, i.e. the respective curves for $\beta = 0$, 0.5 and 1, marked by the solid lines in Figure 2(b), are identical. Increasing B_0 and n reduce damage evolution, whereas increasing q accelerates damage.

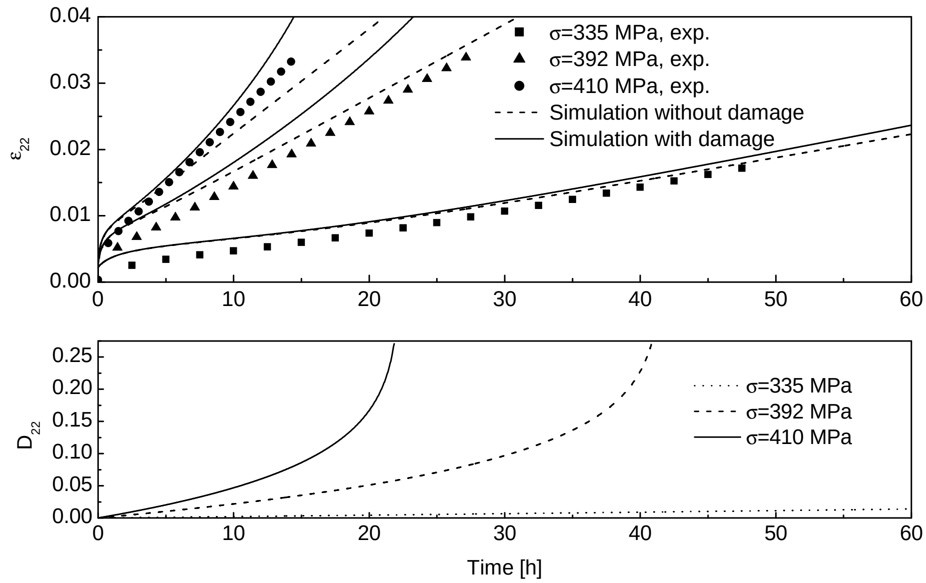

Figure 1: Experimental observations and numerical simulations of uniaxial creep (IN738 LC, 850°C)

Conclusions

A finite strain viscoplastic law coupled with anisotropic damage is presented. Some conclusions from the dissipation inequality at isotropic damage are applied to the formulation of the material law. The effective stress concept and the strain equivalence principle are exploited, so that the formulation of a complicated elastic free energy function including anisotropic damage effect can be circumvented. The viscoplastic formulation is an extension of a Chaboche-type viscoplastic law, the anisotropic damage evolution equation is based on that proposed in [2]. The entire material law is formulated in the corotational material coordinate system, and all evolution equations are expressed in terms of the material time derivatives of the internal variables. Two numerical examples are presented and the model predictions are compared with the experimental observations [9]. These comparisons prove that the model is working reliably.

Reference

1. Atluri, S. N. (1984): "Alternate stress and conjugate strain measures, and mixed variational formulations involving rigid rotations, for computational analyses of finitely deformed solids, with application to plates and shells–I: theory", *Computers and Structures*, Vol. 18, pp. 93-116.
2. Brocks, W. and Qi, W. (2002): "Numerical investigation of creep damage development in the Ni-based superalloy IN738 LC at 850 °C", *CMES*, Vol. 3, pp. 313-320.
3. Chaboche, J. L. (1993): "Cyclic viscoplastic constitutive equations, part I: a thermodynamically consistent formulation, part II: stored energy–comparison between models

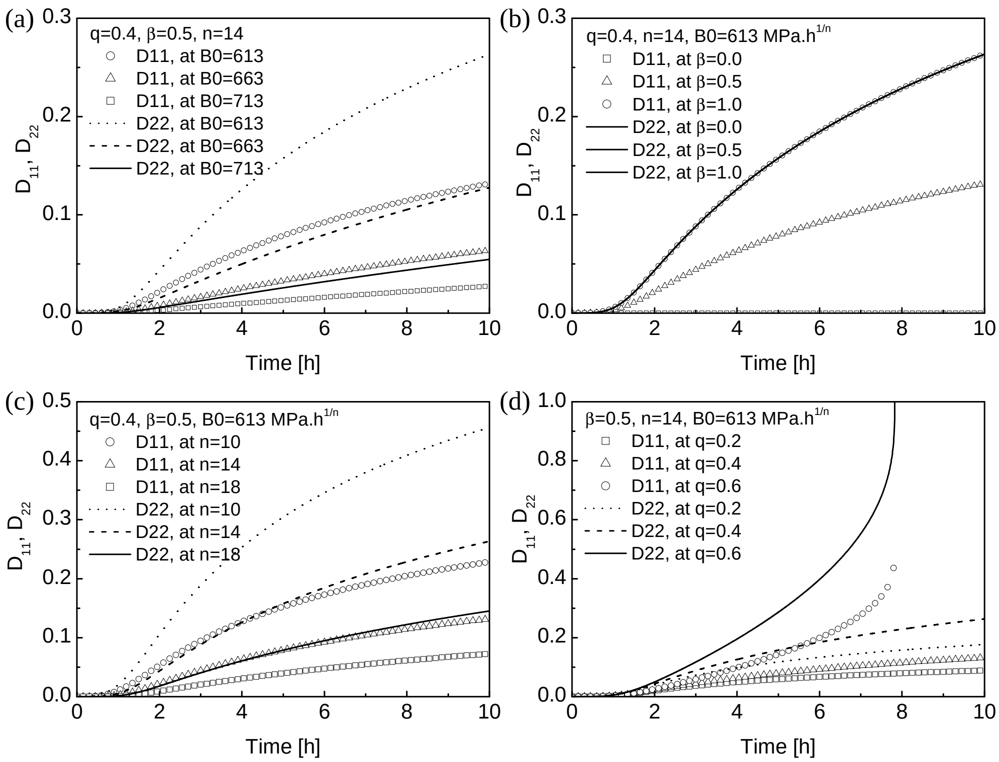

Figure 2: Influences of the changes of (a) B_0, (b) β, (c) n and (d) q on damage evolution

and experiments", *Journal of Applied Mechanics*, Vol. 60, pp. 813-828.

4. Cordebois, J. P. and Sidoroff, F. (1982): "Endommagement anisotrope en élasticité et plasticité", *J.M.T.A., Numéro spécial*, pp. 45-60.
5. Hoger, A. (1987): "The stress conjugate to logarithmic strain", *International Journal of Solids and Structures*, Vol. 23, pp. 1645-1656.
6. Lemaitre, J. (1971): "Evaluation of dissipation and damage in metals submitted to dynamic loading". In: *Proceedings of ICM-1*, Kyoto, Japan.
7. Lemaitre, J. (1996): *A Course on Damage Mechanics*, Springer-Verlage, Berlin.
8. Murakami, S. and Ohno, N. (1981): "A continuum theory of creep and creep damage". In: Ponter A. R. S. and Hayhurst, D. R. (eds), *Creep in Structures*, Springer Verlage, Berlin.
9. Olschewski, J., Sievert, R., Meersmann, J. and Ziebs, J. (1990): "Selection, calibration and verification of viscoplastic constitutive models used for advanced blading methodology". In: Nachelet et al. (eds.), *High Temperature Materials for Power Engineering*, Kluwer Academic Publishers, pp. 1051-1060.