Advances in Computational & Experimental Engineering & Science
Copyright 2004 Tech Science Press

Wave Scattering to The Crack Between Piezoelectric Medium and Substrate or Matrix

Yu Shou-wen*, Gu Bin and Wang Xuyue

Summary

Using the integral transform method, the scattering problem of Love wave and plane wave to the interfacial crack between a piezoelectric layer and a homogeneouse substract is investigated in this paper. The dynamic stress intensity factors under mode I, II and III of the left and the right crack tip are deduced. The numerical results show that the maximal value and the oscillation of the dynamic stress intensity factors increase with the increasing of the crack size, and that the different material combinations may promote or impede the growth of the crack. The wave function expansion method and singular integral equations are used to investigate the solutions of near and far fields for the scattering of SH waves by a cylindrical piezoelectric inclusion partially debonded from piezoelectric matrix.

Introduction

Recently, the dynamic fracture of piezoelectric materials has attracted more and more attention because of the wide application, the demand of design and the loading environment of piezoelectric structures. Li and Mataga [1,2] have obtained the solutions of the dynamic anti-plane problem of propagating crack in infinite piezoelectric medium under the permeable electrical boundary conditions and the impermeable electrical boundary conditions. Chen and Yu [3], Chen and Karihaloo [4] have investigated the anti-plane fracture problems of piezoelectric materials under impact loads. Further, Wang and Yu [5] have solved the mode-I impact problem of piezoelectric layer containing a crack. As to scattering problems of wave, Narita and Shindo [6] have analyzed the scattering of Love wave to the edge crack in the piezoelectric layer bonded with infinite homogeneous medium. In the present paper, the scattering problems of love wave and plane wave by the interfacial crack between the piezoelectric layer and elastic half-space. The dynamic stress intensity factors (DSIF) of the left and the right crack tip are derived, and the effects of the dimension of crack, the material combinations and the incident direction of wave are discussed.

Basic Equations

For transversely isotropic piezoelectric medium and elastic medium, the linear constitutive equations read [7]

$$\underline{\boldsymbol{\sigma}} = \underline{\mathbf{c}} \cdot \underline{\boldsymbol{\varepsilon}} - \underline{\mathbf{e}} \cdot \underline{\mathbf{E}}, \quad \underline{\mathbf{D}} = \underline{\mathbf{e}}^{\mathrm{T}} \cdot \underline{\boldsymbol{\varepsilon}} + \underline{\boldsymbol{\kappa}} \cdot \underline{\mathbf{E}},$$
$$\underline{\boldsymbol{\sigma}}^{e} = \underline{\mathbf{c}}^{e} \cdot \underline{\boldsymbol{\varepsilon}}^{e}. \tag{1}$$

Then, the governing equation can be expressed as [7]

$$\nabla \cdot \underline{\boldsymbol{\sigma}} = \rho \ddot{\underline{\mathbf{u}}}, \quad \nabla \cdot \underline{\mathbf{D}} = 0,$$
$$\nabla \cdot \underline{\boldsymbol{\sigma}}^{e} = \rho^{e} \ddot{\underline{\mathbf{u}}}^{e}, \tag{2}$$

- Correspondence author, Fax:++86 10 62781824, E-mail address: yusw@mail.tsinghua.edu.cn
  Key Lab. of Faillure Mechanics of MoE, Tsinghua University, Beijing, 100084, China

Proceedings of the 2004 International Conference on Computational & Experimental Engineering & Science 26-29 July, 2004, Madeira, Portugal

where the superscript e denotes the quantities of elastic medium. $\underline{u}$, $\underline{\boldsymbol{\sigma}}$, $\underline{\boldsymbol{\varepsilon}}$, $\underline{\mathbf{D}}$ and $\underline{\mathbf{E}}$ stand for the displacement, the stress, the strain, the electrical displacement and electrical intensity, respectively. $\underline{\mathbf{c}}$, $\underline{\mathbf{e}}$ and $\underline{\boldsymbol{\kappa}}$ are the elastic moduli, the piezoelectric and dielectric constants. ρ is the mass density.

Scattering of Love Wave [8]

Consider the configuration as shown in Fig. 1. Under the Cartesian coordinate system (x, y, z), *z*-axis is the pole axis, and the center crack lies on the interface along the *x*-axis. Love wave is assumed to propagate from distant along *x*-axis.

As well known, the present problem can be treated as the superposition of the scattering problem of the incident wave in a structure of the same configuration except in the absence of the crack and the problem where the incident wave is applied to the crack surface and there is no incident wave in the far field.

As to the incident fields without the crack, the stress on the position of crack can be easily to obtained as

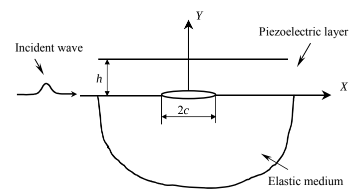

Fig. 1 An interface crack subjected to an incident Love wave

$$\tau(x,t) = \sigma_{yz}^{e\ c}(x,0,t) = \tau_0 \exp[i(kx - \omega t)] \qquad (-c < x < c), \tag{3}$$

where the superscript c stands for quantities of the incident field, *k* is the wave-number and ω is the circular frequency. The time factor $\exp(-i\omega t)$ is common to all the field variables in a steady-state regime, and so will be omitted in the sequel.

Introducing the electrical potential ϕ by $\underline{\mathbf{E}} = -\nabla\phi$ and applying the Fourier transform to the variable x in Eqs. (3). Taking the transform $u = t/c$ and $r = x/c$ and using the Gauss-Chebyshev integral formula in Ref. [5], Finally, one obtains the DSIF of the left and right crack tips by

$$K_{\mathrm{III}}^{\mathrm{L}} = \lim_{x \to -c^-} \sqrt{2(x+c)}\sigma_{yz}^{\mathrm{s}}(x,0) = \sqrt{c}\ MR(-1),$$

$$K_{\mathrm{III}}^{\mathrm{R}} = \lim_{x \to c^+} \sqrt{2(x-c)}\sigma_{yz}^{\mathrm{s}}(x,0) = -\sqrt{c}\ MR(1). \tag{4}$$

It should be noticed that for the integrand of Eqs. (8), there exist some poles along the integral path along the *x*-axis. The numerical results can be obtained by referring to the method of path deviation in Ref. [10]. Two material combinations, PZT-4/Al and $BaTiO_3$/Al, are considered.

It is seen from Figs. 2 that as the value of c/h increases, both the peak values and the vibration magnitudes of the DSIF increases in the two material combinations, and that the DSIF in the material combination of PZT-4/Al is smaller than that of $BaTiO_3$/ Al. This provides the possibility of retarding the crack growth by specifying an appropriate material combination.

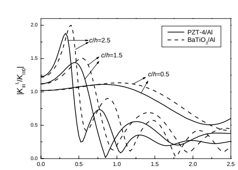

Fig. 2 Normalized DSIF versus normalized ω for different material combinations

Scattering of Plane Wave [9]

Consider the configuration as shown in Fig. 3 and assume that a wave propagates from the elastic substrate with an angle θ of incidence with respect to the z-axis. As the same as the problem of Love wave, the stresses induced by the incident wave on the crack surfaces are obtained

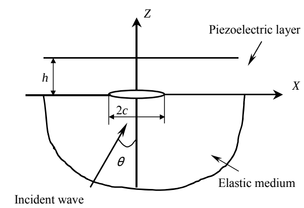

Fig. 3 Crack configuration of the plane wave scattering

$$\sigma^{\mathrm{c}}_{xz}(x,t)=\tau_0(\omega,k)e^{i(kx-\omega t)},\quad \sigma^{\mathrm{c}}_{zz}(x,t)=\sigma_0(\omega,k)e^{i(kx-\omega t)}\quad (-c<x<c)\,. \tag{5}$$

Introducing the displacement potential φ and ψ, and then utilizing Fourier integral transform, the solutions of the scattering fields are expressed as

$$u^{\mathrm{s}}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\sum_{j=1}^{6}A_j(\omega,s)\,e^{\lambda_j z}]e^{-isx}\,\mathrm{d}s\,,$$
$$w^{\mathrm{s}}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\sum_{j=1}^{6}a_j(\omega,s)A_j(\omega,s)\,e^{\lambda_j z}]e^{-isx}\,\mathrm{d}s\,,$$
$$\phi^{\mathrm{s}}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\sum_{j=1}^{6}b_j(\omega,s)A_j(\omega,s)\,e^{\lambda_j z}]e^{-isx}\,\mathrm{d}s\,,$$
$$\varphi^{\mathrm{s}}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}B_1(\omega,s)e^{\lambda_{\mathrm{p}}z}e^{-isx}\mathrm{d}s\,,$$
$$\psi^{\mathrm{s}}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}B_2(\omega,s)e^{\lambda_{\mathrm{s}}z}e^{-isx}\mathrm{d}s\,, \tag{6}$$

where the parameters $A_j(\omega,s)\ (j=1,...,6)$, $B_1(\omega,s)$ and $B_2(\omega,s)$ are unknown.
The boundary conditions are

$$\sigma^{\mathrm{s}}_{xz}(x,0)=\sigma^{\mathrm{s}}_{xz}(x,0)=-\sigma^{\mathrm{c}}_{xz}(x)\,,\quad \sigma^{\mathrm{s}}_{zz}(x,0)=\sigma^{\mathrm{s}}_{zz}(x,0)=-\sigma^{\mathrm{c}}_{zz}(x)\,,$$
$$D^{\mathrm{s}}_{z}(x,0)=0\quad(-c<x<c)\,,$$
$$u^{\mathrm{s}}(x,0)=u^{\mathrm{e\,s}}(x,0)\,,\quad w^{\mathrm{s}}(x,0)=w^{\mathrm{e\,s}}(x,0)\,,$$
$$\sigma^{\mathrm{s}}_{xz}(x,0)=\sigma^{\mathrm{e\,s}}_{xz}(x,0)\,,\quad \sigma^{\mathrm{s}}_{zz}(x,0)=\sigma^{\mathrm{e\,s}}_{zz}(x,0)\,,\quad D^{\mathrm{s}}_{z}(x,0)=0\quad(|x|>c)\,,$$
$$\sigma^{\mathrm{s}}_{xz}(x,h)=\sigma^{\mathrm{s}}_{zz}(x,h)=0,\quad D^{\mathrm{s}}_{z}(x,h)=0\quad(-\infty<x<\infty)\,, \tag{7}$$

and the dislocation density functions are defined as

$$f(x)=\mathrm{d}[u^{s}(x,0)-u^{es}(x,0)]/\mathrm{d}x\,,\quad g(x)=\mathrm{d}[w^{s}(x,0)-w^{es}(x,0)]/\mathrm{d}x\,. \tag{8}$$

The Cauchy singular integral equations of the second kind can be derived from the boundary conditions

$$\underline{\mathbf{A}}\ \underline{\boldsymbol{\varphi}}(x)+\frac{1}{\pi}\int_{-c}^{c}\underline{\mathbf{B}}\frac{\underline{\boldsymbol{\varphi}}(t)}{t-x}\mathrm{d}t+\frac{1}{\pi}\int_{-c}^{c}\underline{\mathbf{Q}}(x,t)\ \underline{\boldsymbol{\varphi}}(t)\mathrm{d}t=\underline{\mathbf{L}}(x)\,, \tag{9}$$

with the single value conditions $\int_{-c}^{c}\underline{\boldsymbol{\varphi}}(t)\mathrm{d}t=0$.

Adopting to the approximate method , Eq. (14) is transformed to be

$$\sum_{k=1}^{N}[T_{lk}^{11}A_k+T_{lk}^{12}B_k]=L_l^{e^1},$$
$$\sum_{k=1}^{N}[T_{lk}^{21}A_k+T_{lk}^{22}B_k]=L_l^{e^2}, \tag{10}$$

Finally, the DSIF of mode-I and mode-II at the left and right crack tips can be deduced

$$\begin{bmatrix}K_{\mathrm{II}}^{\mathrm{L}}\\K_{\mathrm{I}}^{\mathrm{L}}\end{bmatrix}=\sqrt{c}\,\underline{\mathbf{BR}}\sum_{k=1}^{N}\begin{bmatrix}\dfrac{-(1+\gamma_1^{\,2})^{1/2}}{\sqrt{2}}(-2)^{\alpha_1}P_k^{(\alpha_1,\beta_1)}(-1)A_k\\\dfrac{-(1+\gamma_2^{\,2})^{1/2}}{\sqrt{2}}(-2)^{\alpha_2}P_k^{(\alpha_2,\beta_2)}(-1)B_k\end{bmatrix},$$
$$\begin{bmatrix}K_{\mathrm{II}}^{\mathrm{R}}\\K_{\mathrm{I}}^{\mathrm{R}}\end{bmatrix}=\sqrt{c}\,\underline{\mathbf{BR}}\sum_{k=1}^{N}\begin{bmatrix}\dfrac{-(1+\gamma_1^{\,2})^{1/2}}{\sqrt{2}}2^{\beta_1}P_k^{(\alpha_1,\beta_1)}(1)A_k\\\dfrac{-(1+\gamma_2^{\,2})^{1/2}}{\sqrt{2}}2^{\beta_2}P_k^{(\alpha_2,\beta_2)}(1)B_k\end{bmatrix}. \tag{11}$$

To illustrate the basic features of the solutions, numerical calculations have been carried out for two different material pairs, PZT-5H/Al and $BaTiO_3$/Al.

Figs. 4 indicates that with the increase in the value of c/h, the maximal value of the mode-I DSIF increases for both the material combinations of $BaTiO_3$/Al and PZT-5H/Al when a P-wave is incident. It can also be seen from Fig. 4 that for $BaTiO_3$/Al and PZT-5H/Al, the maximal values and the resonant circular frequency of the DSIF are different though they exhibit the same changing tendency. This means that the DSIF may be impeded or accelerated by specifying different material combinations. It can be found from Fig. 5 that In the case of P-wave, the mode-I DSIF of the left crack tip decreases while that of the right crack tip increases as the value of θ increases.

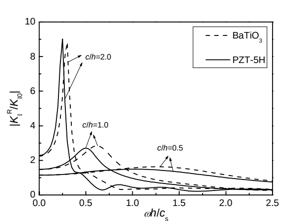

Fig. 4 Normalized DSIF versus normalized ω for different material combinations under an incident P-wave

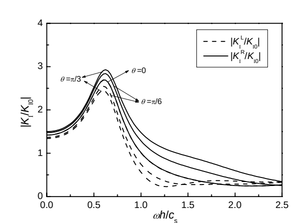

Fig. 5 Normalized DSIF versus normalized ω for different crack tip under an incident P-wave ($BaTiO_3$/Al)

Scattering of SH Waves By An Arc-shaped Crack Between a Cylindrical Piezoelectric Inclusion And Matrix: Near Fields and far fields [13,14]

The wave function expansion method and singular integral equations are used to investigate the scattering of SH waves by a cylindrical piezoelectric inclusion partially debonded from piezoelectric matrix. For impermeable crack case, dynamic stress intensity factors of near fields solutions are obtained and numerical results show the phenomenon of low frequency resonance for various debonding sizes and incident angles. Though the resonance frequency and the peak value of normalized dynamic stress intensity factor depend on the incident angle θ_0 and debonding size in some certain ways as shown in Fig. 6, the conspicuous feature lies in that the main resonance peaks all appear at low frequencies.

The scattered far field pattern (SFFP) and scattered cross section (SCS) are obtained based on the work of the solutions of near fields. Different from the near field case, the numerical results indicate that the resonance occurs at relatively high frequency, and on the whole keeps in a wake of decreasing resonance peaks. .Therefore, the investigation on the far field of debonded piezoelectric inclusion under dynamic loading may be helpful to those who are developing such techniques as nondestructive detection. Using methods given by Pao [12], we present SFFP and SCS of a partially debonded piezoelectric cylindrical inclusion following the work of the solution of near field [14]. Numerical results show the phenomenon of relatively high frequency resonance for various cases.

the resonance frequency of SCS is relatively high compared with the near field case; on the other hand, there exists a relatively wide frequency band over which the resonance peaks are distributed in a decreasing way on the whole.

Example. Two dissimilar materials are considered where

PZT-4: $c_{44} = 2.56e10N/m^2, e_{15} = 12.7C/m^2, \varepsilon_{11} = 64.6e-10C/Vm, \rho = 7500kg/m^3$

$BaTiO_3$: $c_{44} = 4.4e10N/m^2, e_{15} = 11.4C/m^2, \varepsilon_{11} = 128.3e-10C/Vm, \rho = 5700kg/m^3$

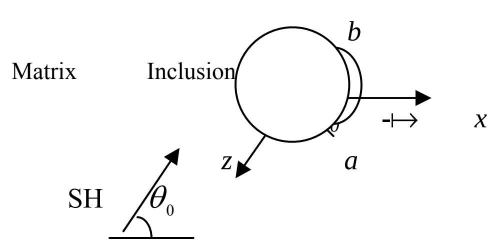

Fig. 6 The incident waves on a partially debonded cylindrical piezoelectric inclusion.

Conclusion

The scattering problems of Love wave and plane wave to an interfacial crack between a piezoelectric layer and homogeneous substrate are solved by means of the integral transform technique and the singular integral equations method. The DSIF of the left and the right crack tip are deduced. The results show that the dimension of crack, the material combination and the incident direction have great influence on the DSIF, and that the crack growth may be impeded by choosing proper material combination. The wave function expansion method and singular integral equations are used to investigate the scattering of SH waves by a cylindrical piezoelectric inclusion partially debonded from piezoelectric matrix. The scattered far field pattern (SFFP) and scattered cross section (SCS) are obtained based on the work of the solutions of near fields.

Acknowledgements

This work supported by NSFC-(90205022,10172050) and Sino-German Sci.Center-GZ050/02;SRFDP-3604;Key grant project of MoE-No-0306.

References

1 Li, S.F. and Mataga, P.A.,(1996): Dynamic crack propagation in piezoelectric materials-Part I: Electrode solution. *J. Mech. Phys. Solids*, Vol. 44: 1799–1830
2 Li, S.F. and Mataga ,P.A.,(1996): Dynamic crack propagation in piezoelectric materials-Part II: Vacuum solution. *J. Mech. Phys. Solids*, Vol. 44: 1831–1866
3 Chen ,Z.T. and Yu, .S.W.,(1997): Antiplane dynamic fracture mechanics of piezoelectric materials. *Int. J. Fract.*, Vol. 85: L3–L12
4 Chen, Z.T. and Karihaloo, B.L.,(1999): Dynamic response of a cracked piezoelectric ceramic under arbitrary electro-mechanical impact. *Int. J. Solids Struct.*, Vol. 36: 5125–5133
5 Wang ,X.Y. and Yu, .S.W.(2000): Transient response of a crack in piezoelectric strip subjected to mechanical and electrical impacts: mode-I problem. *Mech. Mater.*, Vol. 33: 11–20
6 Narita, F. and Shindo, Y.,(1998): Scattering of Love waves by a surface-breaking crack in a piezoelectric layer medium, *JSME Int. J.*, Vol. 41: 40–48
7 Qin, Q.H.,(2000): *Fracture Mechanics of Piezoelectric Materials.* Southampton: WIT Press.
8 Gu, Bin , Yu ,Shouwen , Feng ,Xie-qiao and Mai Yiu-wing,(2002): Scattering of Love wave by an interface crack of a Griffith crack between a piezoelectric layer and an elastic substrate, *Acta Mechanica Solida Sinica,* 2002, 15(2):111-118..
9 Bin Gu, Shouwen Yu and Xiqiao Feng,(2002) : Elastic wave scattering by an interface crack between a piezoelectric layer and elastic substrate, *Int. J. Fracture*, Vol. 116: L29-L34.
10 Yang ,H.J. and Bogy ,D.B., (1985): Elastic wave scattering from an interface crack in a layered half-space. *J.Appl. Mech.*, Vol. 52: 42–50.
11 Shen, S.P. and Kuang, Z.B.(1998):. Wave scattering from an interface crack in laminated anisotropic media. *Mech. Res. Comm.*, Vol. 25: 509–517
12 Pao, Y. and Mow, C.,(1973): *Diffraction of Elastic Waves and Dynamic Stress Concentrations.* Crane and Russak, New York.
13 Wang ,X.Y. and Yu, S.W.,(1999): Scattering of SH wave by an arc-shaped crack between a cylindrical piezoelectric inclusion and matrix –II: far field ,*Int. J. Fract*., Vol.100, No.4, L35-L40.
14 Wang, X.Y.and Yu, S.W,(2003): Scattering of SH wave by an arc-shaped crack between a cylindrical piezoelectric inclusion and matrix :Near field , *Key Engineering of Materials*, Vol.,251-252:215-220..
15 Shou-wen Yu and Bin Gu,(2002): Wave scattering to the crack between piezoelectric layer and semi-infinite elastic substrate, *Proc. of the 4th Int. Conf. on Nonlinear Mechanics*, Shanghai, Eds. By Chien WC,388-393