Reliability Analysis of Structures with Nonlinear Behavior Using Monte Carlo Simulation and Neural Networks

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Summary

This paper presents a methodology for computing the probability of structural failure by combining Monte Carlo simulation (MCS) with neural networks (NN). In fact, although MCS is well suited to reliability studies, its use for evaluation of very low probabilities of failure, as generally found in structures, implies a great number of structural analyses and is often excessively time consuming. In order to speed up the computation, it is suggested herein to perform MCS using a NN trained to reproduce structural behavior. Nevertheless, it should be assessed whether the NN can approximate accurately complex structural response. To illustrate the approach proposed, a steel frame is analyzed assuming both material and geometrical nonlinear behavior. It is found that the procedure adopted leads to an efficient determination of the probability of failure.

Introduction

The structural designer must verify, within a prescribed safety level, the following inequality: $S < R$, where S represents the action effect and R the resistance. Given the random nature of the variables involved, several approaches with different levels of complexity have been presented to deal with this problem. Among them, MCS is an interesting alternative due to its simplicity. However, MCS has not been used often in structural reliability because it is extremely demanding in terms of computing time. This shortcoming can be removed if NN are used to approximate structural response. Such procedure means that the structural performance can be evaluated at a much faster rate, thus making the application of MCS feasible for practical purposes.

Monte Carlo Simulation

A reliability problem is usually formulated in terms of a failure function $g(X_1,...,X_n)$, where $X_1,...,X_n$ are random variables. MCS allows the determination of an estimate of failure probability through sampling of N independent sets of random variables based on the probability distribution for each variable. Using this method, the failure function is computed for each set and an estimate of the probability of failure, p_f, is given by: $p_f=N_H/N$, where N_H is the number of cases in which failure occurs, ie, when $g(X_1,...,X_n)>0$.

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MCS can consider any type of probability distribution for the random variables and is easy to implement. However, MCS has rarely been used in structural reliability because it requires a great number of structural analyses, one for each sample of random variables. The number of analyses needed to evaluate with satisfactory precision the failure probability of a structure depends on the order of magnitude of that probability. As the values of probability of current structures are normally below 10^{-4}, the number of analyses to be performed must be greater than 10^5[1]. These analyses are generally performed through finite elements codes. Therefore, the computation time can be prohibitively high, especially when the structure is large or exhibits non-linear behavior.

To avoid this drawback, NN can be adopted for reproducing structural response[2, 3]. In fact, a trained NN require only a small fraction of time of the corresponding structural analysis, which makes possible to apply MCS in a more computationally efficient way.

Neural Networks

NN are numerical algorithms inspired by the functioning of biological neurons[4]. Figure 1 represents a neuron m that receives an input vector x_k from L channels and computes a weighted sum of the components of x_k, multiplying each by a coefficient w_{mk} reflecting the importance of each channel k. The neuron m activation, a_m, is given by

$$a_m = \sum_{k=1}^{L} w_{mk} x_k + b_m \tag{1}$$

where b_m is a corrective term, allowing a non-zero activation a_m when all x_k are zero.

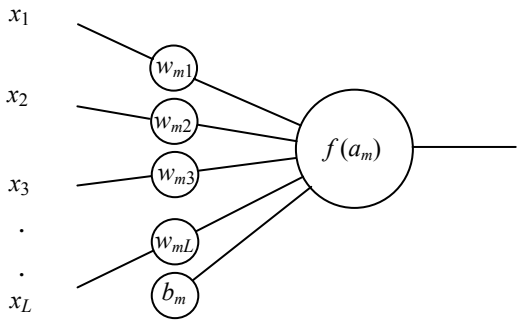

Figure 1 - Artificial neuron

The output signal of neuron m, s_m, results from the computation of an activation function of a_m. In this work, the sigmoid function is adopted:

$$s_m = f(a_m) = \frac{1}{1+e^{-a_m}} \tag{2}$$

The arrangement of several neurons in layers forms a neural network (NN), as shown in Figure 2 where a 3-layer NN is represented: the input layer has 3 neurons, the hidden layer 4 and the output layer 3. It may be proved[5] that this type of NN with sigmoid activation functions in the hidden and output layers can approximate satisfactorily any continuous function, provided that it has enough neurons in the hidden layer.

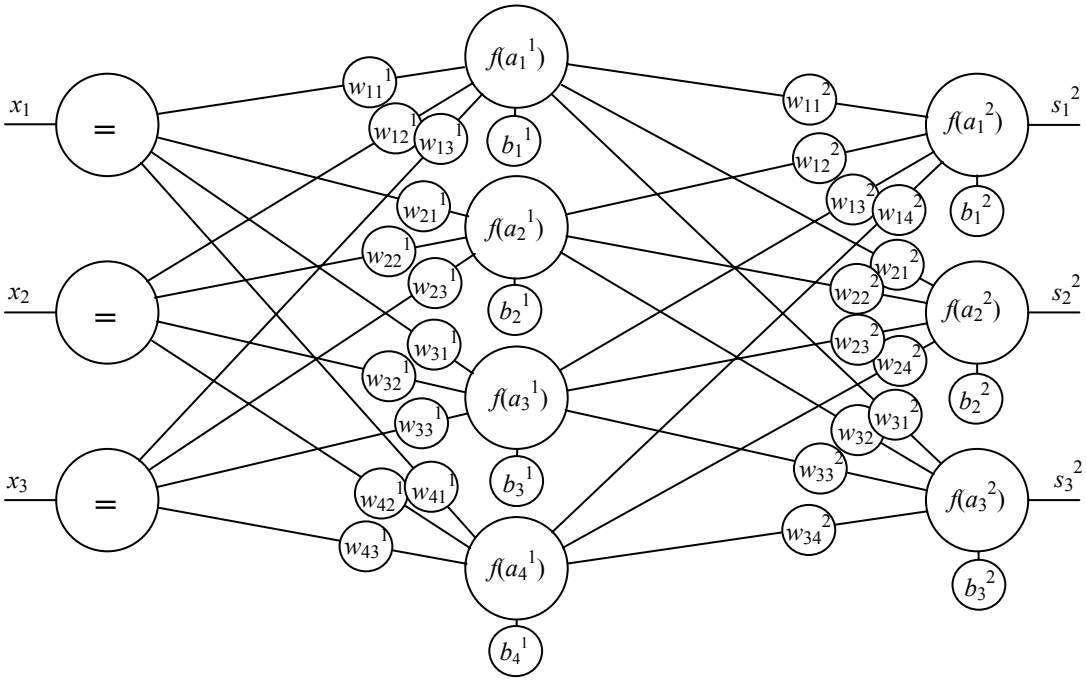

Figure 2 - Multi-layer neural network

The process of obtaining the unknown coefficients w_{mk} and b_m required to approximate the prescribed function is called training. During training, the coefficients are gradually adjusted in order to minimize the error between the NN output and the exact value of the function. To perform the training, the exact value of the function must be known for a given set of input values, known as the training set.

In this work, the error minimization associated to the NN training is initially performed using a genetic algorithm and the solution found is used subsequently as a starting point by a conjugate gradient algorithm. The process is repeated, restarting successively the genetic algorithm while restraining the domain of each coefficient to a closer vicinity of the optimal solution obtained in the previous iteration.

Structural Analysis Model

Since it is important to confirm the adequacy of a NN based approach for complex structural behavior, a nonlinear model is adopted herein. For this purpose, use is made of a object oriented computer program for the analysis of steel frames[6], that can take into account both material and geometrical nonlinearity.

In this code, elasto-plastic analysis is carried out using the plastic-zone approach[6], in which gradual cross-sectional yielding and longitudinal spread of plasticity is allowed throughout the members. The structure is modeled considering only one element per member, which reduces the degrees of freedom involved and hence the computing time.

The gradual plastification of the cross-section of each member subjected to the combined action of axial force and bending moments is described by moment-rotation curves of Ramberg-Osgood type. The effect of the axial force on the reduction of the plastic moment of the section is considered by means of standard strength interaction curves, such as the AISC LRFD beam-column interaction equations[7].

Geometrical nonlinear local effects are incorporated using stability stiffness functions in a beam-column approach[6]. At each load increment, the length, flexural stiffness and axial force of each element are updated. Using an updated lagrangean formulation, the global geometrical effects (large deflections) are also considered by updating the rotation matrix corresponding to the transformation equations from local to global coordinates.

An incremental-iterative procedure is adopted to evaluate the equilibrium path under loading. At each increment a modified constant arc-length method is applied to compute the nonlinear load-deformation path, including ultimate load and post-critical response.

Numerical Example

To exemplify the approach proposed to compute the probability of failure, the 6-story steel frame shown in Figure 3 is analyzed. Only the vertical loads, the horizontal loads and the steel yield stress are taken as random variables. All other parameters (Young's modulus, initial sway, members lengths and cross-sections) are considered as constants.

Lognormal distributions are assumed for all random variables. Their statistical parameters, given in Table 1, are such that the corresponding characteristic values equal the values depicted in Figure 3. The characteristic values of the loads and of the yield stress are respectively the percentiles 98 and 5 of the probability distributions adopted.

Table 1 - Characterization of random variables (*: values for the top story)

variable	mean	standard deviation	variation coefficient	characteristic value
vertical load (kN/m)	33.3 (21.5*)	6.66 (4.30*)	0.20	49.1 (31.7*)
horizontal load (kN)	10.76 (5.38*)	3.76 (1.88*)	0.35	20.4 (10.2*)
yield stress (MPa)	280	28	0.10	235

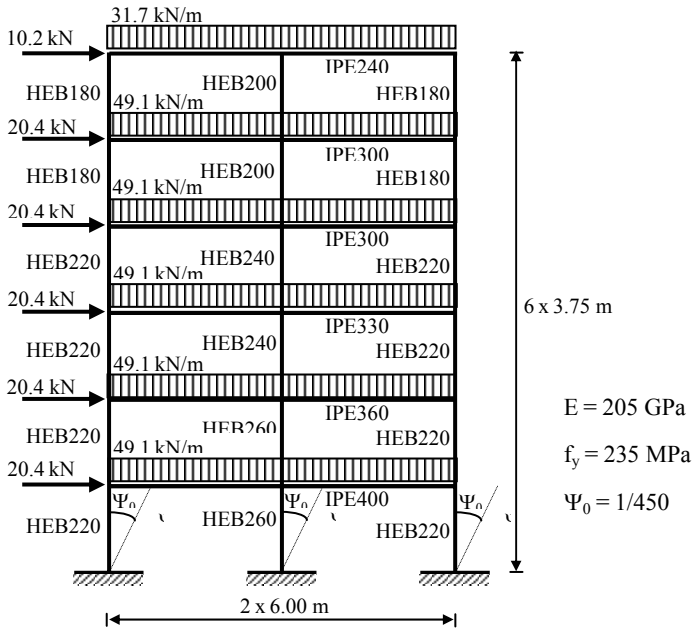

Figure 3 - Steel frame

With respect to the NN that approximates the structural response, the number of input neurons is equal to the number of random variables, i.e., 3. The number of neurons in the output layer is equal to the number of failure functions, i.e., 1 (frame structural collapse).

The NN training set is composed of 512 elements, resulting from the combination of 8 possible values for each random variable. More relevance is given to values situated in the upper extreme of the loads distributions and in the lower extreme of the yield stress distribution, because it is important to achieve a very good fit in those regions, as they influence mostly the probability of failure. A test set containing 343 elements is also created, to measure the precision of the response obtained with the NN.

Several NN are trained, considering either 4, 8 or 12 neurons in the hidden layer. It is found that the NN can approximate the structural response with great precision, since the output given by the NN (failure or non failure) matches the output of the structural analysis program for all cases analyzed, both from the training set and from the test set.

MCS is performed using the trained NN on 10^7 samples. Due to the randomness of the problem, several simulations are carried out for each NN, enabling to obtain a mean and a standard deviation for the probability of failure, p_f. Nevertheless, the variation of p_f is very small, as shown in Table 2, which also presents the training and simulation times required by the NN. Note that, had MCS been performed on the same computer using the structural analysis code instead of NN, it would take several years to complete the corresponding 10^7 runs. This clearly shows the advantage of the approach followed.

Table 2 - MCS using NN: probabilities of failure and computing times

No. hidden neurons	p_f (mean)	p_f (st. deviation)	training time (s)	simulation time (s)
4	2.726×10^{-4}	3.326×10^{-6}	506	81
8	2.489×10^{-4}	7.062×10^{-6}	5830	113
12	2.426×10^{-4}	3.910×10^{-6}	23517	141

Conclusion

Whenever it is possible to model adequately the random variables that influence structural behavior, the use of MCS allows a rational design. However, the amount of time required by this method represents a significant drawback.

The use of NN to approximate structural behavior leads to results similar to the ones obtained with a conventional MCS but requires much less computational effort, even considering the NN training phase. The increase in speed is so much bigger as the structural analysis model adopted becomes more complex. Therefore, the application of the methodology proposed seems to be of great interest in structural reliability.

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