Finite Element Modelling of Viscoelastic Core Sandwich Panels

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Summary

Thin layers of viscoelastic materials are often used in the core of sandwich plates or in surface constrained damping treatments as an effective way to reduce dynamic response of light structures. However the usual approach to model these structures, using a layered scheme of plate and brick finite elements, demands a cumbersome spatial modelling task. In this work a layerwise-based facet-shell finite element model is proposed which is able to describe accurately the stiffness, damping and mass of the composite plate. Some finite element remedies are applied in the finite element formulation in order to improve its membrane formulation, avoid shear locking and introduce the drilling degrees of freedom. Experimental results obtained on several sandwich plates with viscoelastic layers are used to validate the applicability of the proposed model for the simulation of sandwich plates with a single or multiple viscoelastic cores.

Introduction

Viscoelastic layers are usually applied in the core of sandwich panels and as the dissipative layer of constrained damping treatments (Figure 1) as an efective way to improve the dynamic response of light and thin structures [1,2].These structures, usually applied on spacecraft and aeronautical assemblies, are very sensitive to cycling or random loading, which promote high levels of mechanical vibration, noise and fatigue failure, leading to structural disturbance and premature failure.
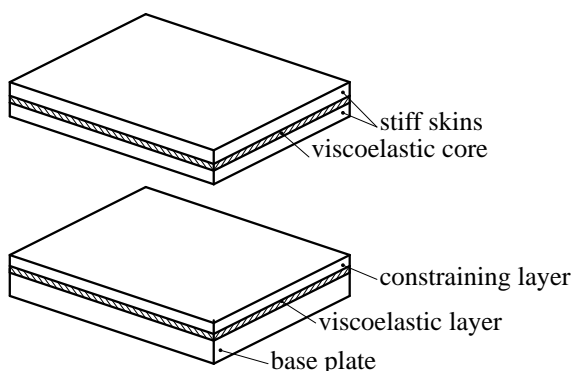

Figure 1: Sandwich plate and constrained viscoelastic treatment

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The viscoelastic materials can provide a very effective vibration energy dissipative mechanism due to the high level of thermal energy that is developed, and partially dissipated, in the viscoelastic layer as a result of the polymeric molecular chain reaction to the imposed cycling deformation.

Though its mechanical performance, low cost and damping efficiency, structures with integrated viscoelastic layers demand a special and complex simulation task in order to properly determine the treatment parameters, like material type, thicknesses, number of layers, location and treatment coverage. The classical laminate theory cannot be used to simulate properly these structures since it is unable to describe accurately the high shear deformation that is developed inside the viscoelastic layers, promoted by the constraining effect of the outside skins or the constraining layer [3].

The usual approach applies a layered scheme of plate and brick finite elements, using a solid brick finite element to model the viscoelastic layer [4-6].This modelling approach can provide a simple and reliable way to simulate the high shear deformation pattern that is developed in the viscoelastic layers of the sandwich structures using the finite element method. However, it demands a cumbersome spatial modelling task, which must be recreated to account for any thickness change, that is not recommended to model multiple layer sandwich panels, using more than one viscoelastic core, and requires special care when simulating non planar structures.

To overcome the layered modelling scheme limitations a layerwise finite element model is proposed which proved to provide results similar to those obtained using the layered approach. With this new model the sandwich plate is spatially modelled by 4-node planar finite elements, using a conventional plate or shell mesh generator, while the layers parameters, namely its thickness and material properties, are simply described in a numerical table that is directly used in the finite element routine. Thus, the redefinition of treatment configuration, i.e. the number of layers, the layering order, thickness of each layer and material properties, is easily allowed by simple redefinition of the input data file.

Layerwise formulation

The layered plate or shell is composed by several different layers of different materials (Figure 2), where each one is individually treated as a thick plate, following the Reissner-Mindlin assumptions, to which are imposed continuity equations within the respective displacement field at interface level.

In all the layers are imposed the following assumptions:

- extensional and shear deformations of all the layers are accounted;

- deformation through thickness is negligible;

- translational and rotary inertias of all the layers are accounted;

- linear theories of elasticity and viscoelasticity are used;

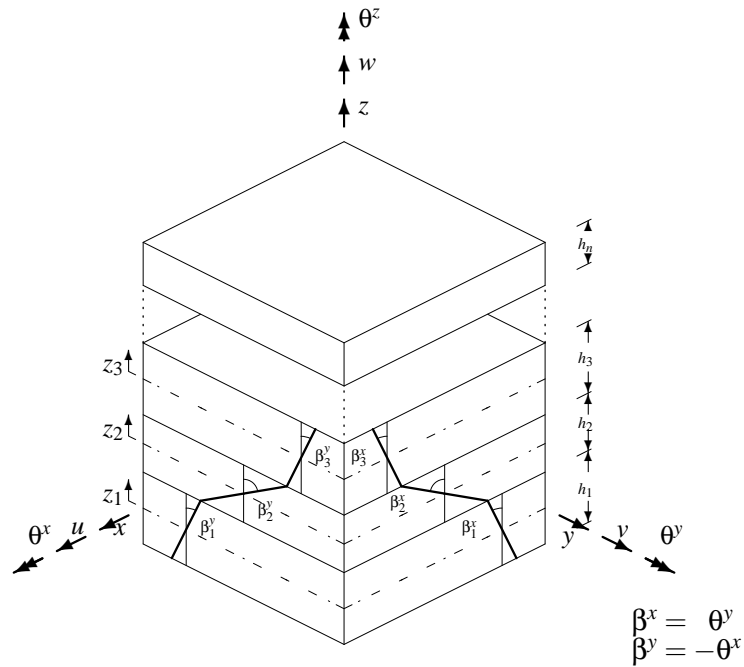- materials are isotropic and homogeneous in each layer.

Figure 2: Layerwise theory - kinematic model

Displacement field

Considering that a generic layer k of the sandwich plate is defined by:

$$\Omega_k = \left\{ (x,y,z_k) \in \mathbf{R}^3 \mid z_k \in \left[-\frac{h_k}{2}, \frac{h_k}{2} \right], (x,y) \in A \subset \mathbf{R}^2 \right\} \tag{1}$$

where Ω_k, A and h_k represent, respectively, the volume, area and thickness of the generic k-th layer, the displacement field $\{\mathbf{u}\}_k$ can be defined as:

$$\{\mathbf{u}\}_k = \left\{ \begin{array}{c} u_k \\ v_k \\ w_k \end{array} \right\} = \left\{ \begin{array}{c} u_0 + \frac{h_1}{2}\beta_1^x + \sum\limits_{j=2}^{k-1} h_j \beta_j^x + \frac{h_k}{2}\beta_k^x + z_k \beta_k^x \\ v_0 + \frac{h_1}{2}\beta_1^y + \sum\limits_{j=2}^{k-1} h_j \beta_j^y + \frac{h_k}{2}\beta_k^y + z_k \beta_k^y \\ w_0 \end{array} \right\} \tag{2}$$

where u_0, v_0 and w_0 are the translations of the reference layer ($k = 1$) and β_k^x, β_k^y are the rotations of the normal about the y and x axes, respectively.

The continuity of the displacement field between the layers is guaranteed through a set of coupling terms in the displacement field definition.

According to Equation (2) the displacement field, $\{\mathbf{u}\}_k$, can be represented through the set of generalized variables as:

$$\{\mathbf{u}\}_k = [\mathcal{N}]_k \{d\} \tag{3}$$

where:

$$\{d\} = \{u_0, v_0, w_0, \beta_1^x, \beta_1^y, \cdots, \beta_j^x, \beta_j^y, \cdots, \beta_n^x, \beta_n^y\}^T \tag{4}$$

represents the generalized displacement field and matrix $[\mathcal{N}]_k$ is defined as:

$$[\mathcal{N}]_k = \begin{bmatrix} 1 & 0 & 0 & \frac{h_1}{2} & 0 & \cdots & h_j & 0 & \cdots & \frac{h_k}{2}+z_k & 0 & \cdots \\ 0 & 1 & 0 & 0 & \frac{h_1}{2} & \cdots & 0 & h_j & \cdots & 0 & \frac{h_k}{2}+z_k & \cdots \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots \end{bmatrix} \tag{5}$$

Stiffness and mass matrices formulation

The stiffness matrix of a finite element is computed from:

$$[K]^e = \sum_{k=1}^{n} \int_{\Omega_k^e} [B]_k^{eT} [D]_k [B]_k^e d\Omega_k^e \tag{6}$$

where matrix $[D]_k$, which is based on the Mindlin plate elasticity matrix [7], represents the constitutive matrix of the generic k-th layer. The deformation matrix $[B]_k$ is evaluated from:

$$[B]_k^e = [\mathcal{L}][\mathcal{N}]_k [N]^e \tag{7}$$

Matrix $[N]^e$ represents the shape functions matrix of the finite element and matrix $[\mathcal{L}]$ represents the differential operator matrix that relates the deformation field with the corresponding displacement field.
The mass matrix of the composite finite element is given by:

$$[M]^e = \sum_{k=1}^{n} \int_{\Omega_k^e} [N]^{eT} [J]_k [N]^e d\Omega_k^e \tag{8}$$

where the matrix $[J]_k$ represents the inertia matrix of each generic layer and is obtained from:

$$[J]_k = \rho_k [\mathcal{N}]_k^T [\mathcal{N}]_k \tag{9}$$

Layerwise formulation improvement

The stiffness matrix formulation is improved by three different finite element remedies. Firstly the membrane displacement field is enriched by a set of incompatible modes [8] which provide a good remedy for in-plane shear locking of the bilinear quadrilateral element.The correction issued by Taylor *et al.* [9] is applied to this membrane formulation

improvement.

In order to provide transverse shear locking protection for thin layer condition, while main-taining the proper rank of the quadrilateral finite element, the thick plate formulation of each layer is enriched with a locking protection based on the *MITC* approach [10].

Finally, the drilling degrees of freedom θ_k^z are introduced in the membrane formulation of the plate using a fictitious stiffness stabilization matrix [11]. This remedy provides a generalization of the plate formulation for facet-shell application which suffer from rank deficiency and ill-conditioning problems when adjacent elements are coplanar.

The consistent mass matrix provided by Equation (8) can be converted into a lumped or diagonal mass matrix, which is usually applied on dynamic analysis mainly due to its computational economy. However, in the layerwise model special care must be issued when selecting the proper lumping procedure due to the effect of the coupling terms, be-ing impossible to apply the nodal quadrature using *Lobatto* integration technique [7] or the "*row-sum*" technique [7]. The "*HRZ*" procedure [12], which is based on a diagonal-scaling technique, is efficiently applied in the proposed finite element.

Experimental validation

The applicability of the proposed layerwise finite element to the simulation of the dynamic response of sandwich plates with soft viscoelastic cores is assessed by comparing the numerical results achieved with this finite element and the experimental data obtained for several specimens with viscoelastic layers [6].

The numerical simulation is developed using a direct frequency analysis [6] along with the complex modulus approach [5,6] in order to account for the frequency-dependent shear modulus of the viscoelastic layers.

Figure 3 represents the direct frequency response function, measured and simulated, of a 300x200 mm sandwich plate made of two aluminium plates, with 1mm of thickness each, bonded together with a thin (0.125 mm) 3M ISD112 [13] viscoelastic layer.
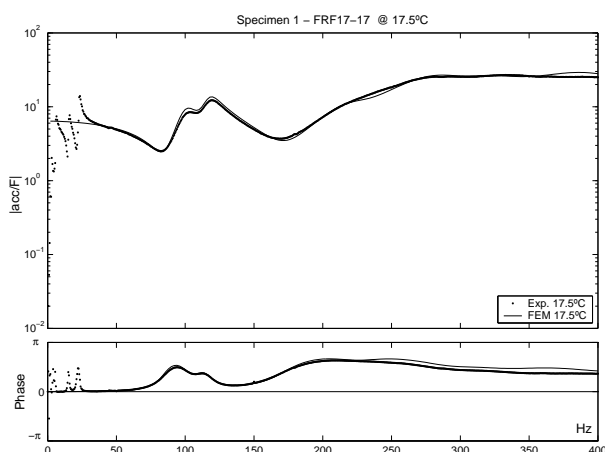

Figure 3: Direct frequency response function—experimental vs. numerical

Conclusion

The proposed finite element reveals to be suitable to model viscoelastic core sandwich panels and permits to avoid the laborious modelling task when using the layered models. The frequency response functions obtained with the proposed finite element model agree well with the experimental ones for the magnitude and phase.

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