**_Advances in Computational & Experimental Engineering & Science_**


# An Improved Inverse Design Method for Blades of Finite Thickness

J. C. Páscoa[1], A. C. Mendes[1], L. M. C. Gato[2]

## Summary

The paper describes an enhanced turbomachinery inverse aerodynamic design formulation for cascades of blades of finite thickness. The prescribed conditions are the aerodynamic load and the blade thickness. The inverse method starts with an initial analysis of the flow on a preliminary geometry that will be later modified by changing the blade camber-line. The flow analysis is performed by a time-marching procedure based on a Finite Volume discretization. The numerical scheme is able to perform the analysis of supersonic as well as transonic compressible flow with shock waves. The presented results emphasise the numerical efficiency attained with the new inverse design formulation.

## Introduction

There are usually two kind of approaches for turbomachinery design, which are the direct and inverse methods. In an inverse method we seek the blade geometry from some prescribed conditions for the flow. This kind of inverse approach can be formulated in different ways, which differ in the prescribed design variables. These can be the prescription of the static pressure in both surfaces of the blade [1] or the specification of the aerodynamic load [2]. The classical design condition considers the imposition of the pressure distribution around the blade surface. This condition is used because it allows the control of the diffusion along the blades, avoiding flow separation for the desired design conditions. However, this design perspective is best fitted to the two-dimensional case, where the streamlines lie in the two-dimensional plan of the computation. Therefore, for three-dimensional flow this design approach is no longer valid, particularly for certain kind of internal flow problems. For a purely three-dimensional design case the prescription of the velocity, or pressure, in both surfaces of the blades looses its advantage, because the location of the streamline is not known in advance. Without a knowledge of the location of the streamlines, the idea of controlling the pressure gradient is without meaning [3]. Besides, Demeulenaere and Braembussche [4] have reported some problems in defining a target pressure distribution resulting in a blade shape respecting the mechanical constraints, particularly regarding the blade thickness. In order to circumvent some of the limitations highlighted above, in the present approach we prescribe as a design variable the mean tangential velocity (that is related to the aerodynamic load) and the blade thickness. An immediate consequence of this choice is that the designer looses is direct control over the velocity, or pressure, distribution on the blade surface. This weakness is although compensated in several ways. First, the prescription of the thickness distribution ensures always a realistic blade shape. Besides, and since that the mechanical stresses are explicitly related with the thickness of the blade, the ability to prescribe the thickness allows the designer a straight control on

[1]Dep. Eng. Electromecânica, Universidade da Beira Interior, R. Marquês D'Ávila e Bolama, 6200 Covilhã, Portugal
[2]Dep. Eng. Mecânica, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

*Advances in Computational & Experimental Engineering & Science*
Copyright 2004 Tech Science Press

the mechanical stresses. The ability to control the mechanical stresses, just before starting the aerodynamic design task, allows a reduction in global design time. This is achieved due to a reduction in the number of aerodynamic and structural design task iterations. Second, the specification of the blade load, instead of the velocity, or pressure distribution in both surfaces, will decrease the occurrence of ill-posed problems, see Volpe and Melnik [5]. Numerical experiments have suggested that with our formulation, based on blade load, there is always a solution [6], therefore our design approach seems more robust than the one specifying the velocity in both sides of the blades.

A main drawback of the method is the need to perform a dozen of analysis cycles until reaching convergence. This can result in high computation times, particularly for three-dimensional problems. In this work we shall present a new approach that seeks a reduction in the amount of design iterations, by reformulating the method for blades of finite thickness.

## Formulation of the Inverse Method for Blades of Finite Thickness

The design procedure is composed of two main components:

—-The analysis code (coupled to a mesh generator).

—-The blade modification algorithm (camber line generator).

The flow analysis algorithm performs the intensive component of the computation. A change in geometry is obtained by modifying the camber line until reaching convergence, as is presented in figure 1. As stated previously, the distribution of mean tangential velocity, $\bar{V}_y(x)$, throughout the cascade is chosen as the design variable, since it is related to the force made by the flow on the blade section. Borges *et al.* have shown in [3] that the derivative of the mean tangential velocity can be expressed as,

$$-\frac{d}{dx}\left[\frac{\int_{y_s}^{y_p}\rho u v dy}{\int_{y_s}^{y_p}\rho u dy}\right]=\frac{\Delta p}{\dot{m}} \quad ; \quad \bar{V}_y(x)=\frac{\int_{y_s}^{y_p}\rho u v dy}{\int_{y_s}^{y_p}\rho u dy}. \tag{1}$$

Equation 1 shows without ambiguity that the tangential component of the force on the blade is due to the pressure difference between the suction, $y_s$, and pressure, $y_p$, surfaces. We will now present the method of blade modification, this is based on summing to the camber line, $f(x)$, the thickness distribution, $t(x)$. The contour of the upper, $S^+$, and lower, $S^-$, surfaces of the blade is given by,

$$S^+(x)=y_0+f(x)+\frac{t(x)}{2} \quad ; \quad S^-(x)=y_0+f(x)-\frac{t(x)}{2}. \tag{2}$$

Here $y_0$ refers to an arbitrary location for the leading edge of the cascade of blades with pitch $s$. And $f(x)$ is the camber line of a blade with imposed thickness $t(x)$. The slope of the streamline on the upper and lower surfaces of the blade is given by $dS^{\pm}/dx = V_y^{\pm}/V_x^{\pm}$,

***Advances in Computational & Experimental Engineering & Science***

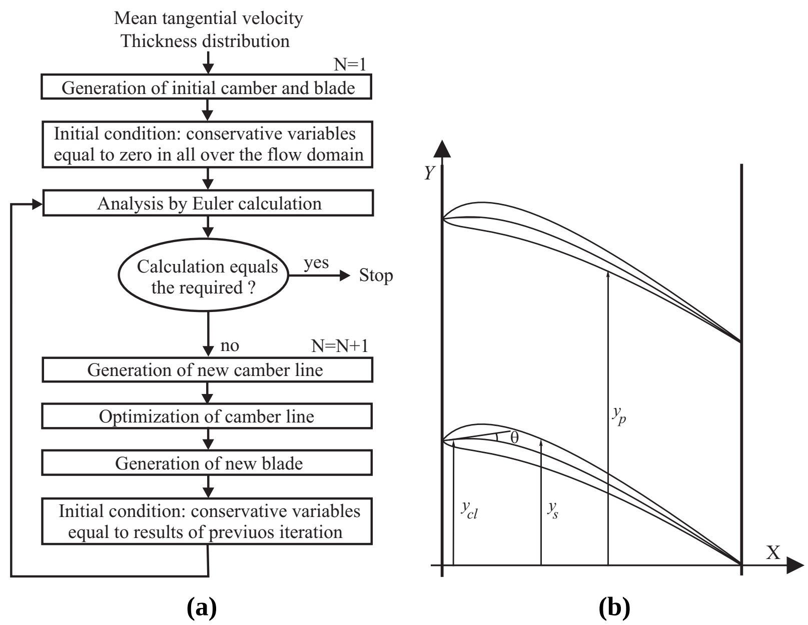




Figure 1: **(a)** Flowchart of the design method and, **(b)** nomenclature of the cascade of blades.

therefore:

$$\begin{cases} V_x^+ \frac{\partial f}{\partial x} + \frac{V_x^+}{2} \frac{\partial t}{\partial x} = V_y^+ \\ V_x^- \frac{\partial f}{\partial x} - \frac{V_x^-}{2} \frac{\partial t}{\partial x} = V_y^- \end{cases} . \tag{3}$$

Summing up both equations of (3) we come up with an expression that combines the slip boundary condition in both surfaces of the blade. Applying the resulting differential equation to the target $(*)$ and to the actual $(n)$ design iterations we get,

$$\begin{cases} \frac{df^*}{dx} = \frac{\overline{V}_y^*}{\overline{V}_x^{/*}} + \frac{1}{4} \left( \frac{\Delta V_x^*}{\overline{V}_x^{/*}} \right) \frac{dt}{dx} \\ \frac{df^n}{dx} = \frac{\overline{V}_y^n}{\overline{V}_x^{/n}} + \frac{1}{4} \left( \frac{\Delta V_x^n}{\overline{V}_x^{/n}} \right) \frac{dt}{dx} \end{cases} . \tag{4}$$

We have considered a linear velocity profile between the upper and lower surfaces, with a mean value given by $\overline{V}_x^/ = (V_x^+ + V_x^-)/2$ and also the jump in axial velocity as $\Delta V_x = (V_x^+ - V_x^-)$. Subtracting both differential equations of (4) we get an ordinary differential equation that allows us to compute the new camber line approximation $(N+1)$, as a function of the

**Advances in Computational & Experimental Engineering & Science**


kinematic variables and blade thickness derivative.

$$\frac{df^{N+1}}{dx} = \frac{df^{N}}{dx} + K\left(\frac{\overline{V}_y^*}{\overline{V}_x^{/*}} - \frac{\overline{V}_y^N}{\overline{V}_x^{/N}}\right) + \frac{1}{4}\frac{dt}{dx}\left(\frac{\Delta V_x^N}{\overline{V}_x^{/N}} - \frac{\Delta V_x^{N-1}}{\overline{V}_x^{/N-1}}\right) \tag{5}$$

In (5) we determine the jump in axial velocity with a time delay, because this value is not known for the design conditions. For the case of infinitely thin blades, equation (5) results in the expression used in [3].

$$\bar{V}_x(x) = \frac{\int_{y_s}^{y_p} \rho u dy}{\int_{y_s}^{y_p} \rho dy} \quad ; \quad \tan\beta = \frac{\bar{V}_y(x)}{V_{x1}} \quad ; \quad \tan\beta^* = \frac{\bar{V}_y^*(x)}{V_{x1}^*} \tag{6}$$

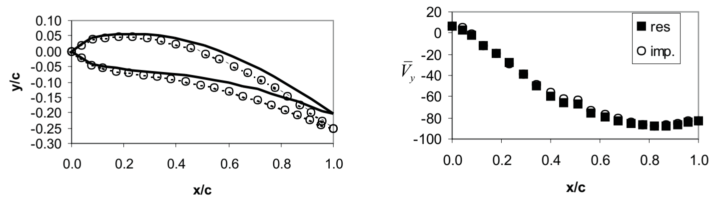
As outlined in Fig. 1(a) we compute the mean tangential and axial velocity components after the first analysis of the flow on the initial cascade. In equation (5) we have also changed the assumption of a linear velocity profile and computing the mean velocity by integration of the first equation of (6). The mean axial velocity component is only known at entrance to the cascade, therefore the flow angle of the present iteration, $\beta$, and of the target iteration, $\beta^*$, are determined using the mean axial velocity at entrance. In this approach $\bar{V}_{x1}^N$ and $\bar{V}_{x1}^*$ are only used to nondimensionalize the tangential component of the velocity and, usually, they change during the design iteration until the mass flow attains the design value. The values obtained through equation (5) are further optimized by means of a polynomial interpolation, in order to ensure a monotone behavior for the camber line. The relaxation constant $K$ can usually be chosen between 0.2 and 0.6. The convergence criteria is determined by computing the maximum relative variation of camber line ordinates, $\Phi = \max\left(y_{cl}^{N+1} - y_{cl}^N\right)/y_{cl}^N$, that usually should be less than 0.1%.

In each design iteration the inverse method calls the flowfield analysis routine. The analysis code (direct) is based on an improved time-marching algorithm described in Páscoa *etal*. [6]. It is an explicit time-marching method that solves the Euler system of equations in 2D with a Finite Volume central spatial discretization. The method is enhanced with state-of-the-art artificial viscosity components based on the limiter theory. In [7] a throughout validation of the scheme is performed involving benchmark test cases.

### Results Obtained with the New Inverse Design Formulation

We will now present the results obtained with the improved design formulation. The test case refers to a cascade of blades composed of NACA 0012 thickness distribution given analytically. This is chosen to give us a straightforward implementation of the thickness derivative needed by equation (5). The pitch/chord ratio is 0.5. The inlet flow conditions are $Ma_{-\infty} = 0.6$, with an inlet flow angle $\alpha_{-\infty} = 2^o$ and a stagger angle $\lambda = 13.7^o$. The H-type mesh is composed of 20×60 nodes, 40 of them over the blade section. In figure

***Advances in Computational & Experimental Engineering & Science***
Copyright 2004 Tech Science Press




**(a)** **(b)**

Figure 2: **(a)** Blade geometry: original (···), first design iteration (—) and redesigned (○○○); **(b)** Values of mean tangential velocity imposed and obtainded in the last design iteration.

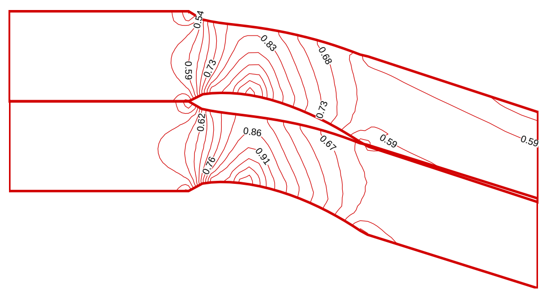


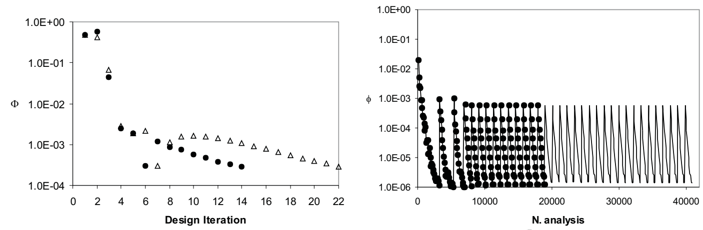
Figure 3: Comparison between the iso-Mach lines in the original blade (below) and obtained in the last redesign iteration (up).

2(a) we can see a comparison between the blade used in the initial analysis of the flow and the blade resulting from the redesigned cascade. This later was obtained imposing the mean tangential velocity obtained on the analysis of the initial cascade, see figure 2b. This is a supercritical cascade of blades and in figure 3 we compare the iso-Mach lines for the original and redesigned cascade of blades. It worth mention that the flow is almost entirely subsonic apart from a small supersonic bubble. Even with this crude mesh (20$\times$60) the method is able to redesign the original blade with remarkable accuracy. Figure 4 present the numerical efficiency attained with the new inverse design formulation for blades of finite thickness, as compared with the method of Borges *et al.*. We can see a substantial reduction in the computing time. The number of design iterations took a reduction from 22 to 14. If we compare the time-steps of time-marching scheme we can see a reduction of around 50% in the overall computing time, see figure 4(b).

## Conclusion

We have presented a formulation for the inverse design of transonic turbomachinery cascades. The method is based on a modification of the camber line. We have introduced a new camber line generator for blades of finite thickness. This new formulation lead to a remarkable increase in numerical efficiency. Besides, this formulation is completely independent of the flow analysis method and thus it can be applied with other analysis codes and even for different classes of flow. This is a better approach compared to other design methods [8] that impose a change in the boundary conditions of the analysis code.

*Advances in Computational & Experimental Engineering & Science*
Copyright 2004 Tech Science Press




**(a)** **(b)**

Figure 4: Comparison between our formulation (●) for blades of finite thickness and the one (△) presented in [3]: **(a)** convergence of the design criteria; **(b)** convergence of the time-marching scheme during all the flow analysis iterations.

**Acknowledgment**

This work as been partly funded by UBI, IDMEC-IST, and by the FCT project contract POCTI/EME/39717/01.

## Reference

1. Léonard, O. and Van den Braembussche, R. A. (1992): "Inverse Design of Compressor and Turbine Blades at Transonic Flow Conditions", International Gas Turbine and Aeroengine Congress, Germany, *ASME Paper 92-GT-430*.
2. Borges, J. E. T. (1990): "A Three-dimensional Inverse Method for Turbomachinery: Part 1", *ASME Journal of Turbomachinery*, Vol. 112, pp. 346-354.
3. Borges, J. E. T., Gato, L. M. C. and Pereira, R. M. R. J. (1996): "Iterative Use of a Time-marching Code for Designing Turbomachine Blade Rows", *Computers & Fluids*, Vol. 25, No. 2, pp. 197-216.
4. Demeulenaere, A. and Van den Braembussche, R. A. (1998): "Three-dimensional Inverse Method for Turbomachinery Blading Design", *Journal of Turbomachinery*, Vol. 120, pp. 247-255.
5. Volpe, G. and Melnik, R. E. (1981): "Role of Constraints in Inverse Design for Transonic Airfoils", *AIAA Journal*, Vol. 22, No. 12, pp. 1770-1778.
6. Páscoa, J. C., Mendes, A. C. and Gato, L. M. C. (2004): "An Inverse Method for Turbomachinery Cascades of Blades - Investigation of the Existence and Uniqueness of Solution", *International Series on Advances in Fluid Mechanics*, Vol. 40, pp. 3–12, WIT Press, U.K.
7. Páscoa, J. C., Mendes, A. C. and Gato, L. M. C. (2002): "An Efficient Finite Volume Scheme with Improved Artificial Viscosity Suitable for Transonic-flow Calculations", *International Series on Advances in Fluid Mechanics*, Vol. 32, pp. 505-514, WIT Press, U.K.
8. Dang, T. and Qiu, X. (2000): "Euler-based Inverse Method for Turbomachinery Blades, Part 2: Three-dimensional flows", *AIAA Journal*, Vol. 38, No. 11, pp. 2007-2013.