

## Application of the Kalman Filter to Estimate an Elastic Parameter in Order to Improve Controller Performance

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### Summary

With the vast advances in space missions using satellites with rigid and flexible components, where the accuracy of point are very demanding, the requirements for the control system with more efficient controllers has increased. In that context, the validity of the controller performance depends not only on its good design but also on the knowledge of all states to be feed backed in order to improve the control system efficiency. In this paper, a Kalman filter methodology is used to recover all the unmeasured states (elastic displacement and it's rates) considering that only the states associated with rigid motion are measured (angle and angular velocity). In order to investigate the robustness of the filter, one simulates the Kalman filter with a satellite model compose of one, two and three flexible modes. One observes that the fidelity of the estimation process increase with the inclusion of more modes into the satellite model, which in turn not affect the performance of the Kalman filter procedure.

### Introduction

The use of small satellites has been a fast, simple and of a low cost way of reaching the space [1]. However, in the order to conquer the space it's necessary to launch spacecrafts that involves rigid/flexible structures. In that type of spacecrafts, the influence of flexibility plays an important role in the dynamics behavior as well as in the performance of the Attitude Control System (ACS). Other important aspect in the study of the dynamics and control of flexible space structure are: the degree of interaction between the rigid and flexible motion [2], maintenance of a ACS performance in face the uncertainties of the mathematical model [3], damping residual vibrations in order to keep pointing precision and states estimations [4].

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This paper introduces a state estimation procedure using the Kalman filter methodology to recover the flexible coordinates from measurements of the rigid part (angle and velocity angular). Section 2 presents a mathematical model of a simple spacecraft based on a two flexible Euler-Bernoulli beam connected to a rigid hub. The equations of motion are derived considering the torque as input, and angle and angular velocity as outputs. Section 3 presents the Kalman filter state estimation problem. Section 4 presents the simulation of the problem. Section 5 concludes the paper.

### Satellite Mathematical Model

The satellite mathematical model used is composed of a rigid platform with two flexible appendices (see Figure 1). The appendices are identical and opposite, being considered as beam connected to the platform, subject to rotational and vibrational motion. The equations of motion are derived using the Lagrange methodology, starting from the expression of the kinetics and potential energy of the system.

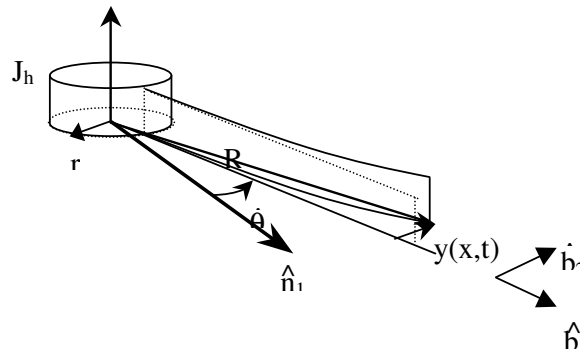


Figure 1: Satellite mathematical model.

The inertial reference system is represented by the axes  $\hat{n}_1, \hat{n}_2, \hat{n}_3$ , which coincides with the center of mass of the rigid body characterized by the axes  $\hat{b}_1, \hat{b}_2, \hat{b}_3$ . The vector  $r$  is the radius of the rigid body. The vector  $x$  represents the position along the axis  $\hat{b}_1$  in no deformed form. The vector position in the appendage relative to the inertial reference system is given by  $\bar{R}$ . The vector of elastic deformation, perpendicular to the axis  $\hat{b}_1$ , is represented by  $y(x,t)$ , and  $\dot{\theta}$  is the satellite angular velocity. Therefore, the vector velocity of any point in the deformed appendage form, relative to the inertial reference system is given for:

$$\dot{\bar{R}} = -\dot{\theta}y\hat{b}_1 + [\dot{\theta}(r+x) + \dot{y}]\hat{b}_2 \quad (1)$$

### Equations of Motion

The total kinetics energy of the system is given by

$$T_T = \frac{1}{2} J_h \dot{\theta}^2 + 2 \int_0^L \rho \left\{ \dot{y}^2 + 2\dot{y}\dot{\theta}(r+x) + \dot{\theta}^2 (r+x)^2 \right\} dx \quad (2)$$

where  $J_h$  is the rotary inertial of the hub,  $\rho$  is the mass density of the appendages,  $L$  is the length of the appendage and  $y(x,t)$  represents the elastic displacement. The potential energy is given by:

$$V_T = \int_0^L (EI) \left( \frac{d^2 y}{dx^2} \right)^2 dx \quad (3)$$

where  $E$  is the modulus of elasticity and  $I$  the moment of inertia of the beam. The discretization of the system is done using assumed mode method [4]. Therefore, the elastic displacement  $y(x,t)$  is given by

$$y(x, t) = \sum_{j=1}^n \phi_j(x) q_j(t) \quad (4)$$

where  $q_j(t)$  are the generalized coordinates and  $\phi_j(x)$  are the admissible functions. The equations of motion for the rigid  $\theta(t)$  and the elastic  $q(t)$  motion, are found using the Lagrange formulation:

$$\frac{d}{dt} \left( \frac{dT_T}{d\dot{x}_i} \right) - \frac{dT_T}{dx_i} + \frac{dV_T}{dx_i} = F_i \quad (5)$$

where  $F_i$  is the generalized force, and  $x_i$  is the  $i$ th element of the vector  $(\mathbf{x})$ . After derivation the equations of motion in matrix form is given by

$$M\ddot{\mathbf{x}} + K\mathbf{x} = D\mathbf{u} \quad (6)$$

where  $M$  represents the mass matrix,  $K$  is the stiffness matrix and  $D$  is known as control influence matrix. Transforming Equation (6) in space state modal form, one has:

$$\tilde{M}\ddot{\eta} + \tilde{C}\dot{\eta} + \tilde{K}\eta = \tilde{D}\mathbf{u} \quad (7)$$

Here  $\tilde{M}$ ,  $\tilde{C}$ ,  $\tilde{K}$  and  $\tilde{D}$  represents mass, damping, stiffness and control influence matrices in modal form, respectively.

### Kalman Filter Methodoly

The complete dynamical model is represented by:

$$\dot{\eta}_1 = A\eta_1 + G\omega \quad (8)$$

where  $\eta_1 = [\eta, \dot{\eta}]^T$  is the modal coordinates,  $\omega$  is white gaussian noise,  $G$  is matrix unitary and  $A$  is the system matrix that relates the state linearly by

$$A = \begin{bmatrix} 0 & I \\ -\tilde{K} & -\tilde{C} \end{bmatrix}$$

The measured model is given by:

$$Y = C\eta_1 + v \quad (9)$$

the output is the angle  $\theta$  and angular velocity  $\dot{\theta}$ , with standard deviation of  $0.05^\circ$  and  $0.005^\circ/s$ , respectively. The matrix is  $C=B^T$ . The term  $v$  represents a white noise vector with the following statical characteristic

$$v_\theta = N(0,0.05^\circ) \quad , \quad v_{\dot{\theta}} = N(0,0.005^\circ/s)$$

In the time update, the states are estimates using

$$\dot{\bar{x}} = A\bar{x} \quad (10)$$

with initial conditions  $\bar{x}_{k-1} = \hat{x}_{k-1}$ , and the covariance is computed by

$$\dot{\bar{P}} = A\bar{P} + \bar{P}A^T + GQG^T - \bar{P}C^TR^{-1}C\bar{P} \quad (11)$$

with initial conditions  $\bar{P}_{k-1} = \hat{P}_{k-1}$ . Equation (11) is known as Riccati equation.

In the measurement update the states and covariance matrix are calculated by

$$K_k = \bar{P}_k C^T (C\bar{P}_{k-1} C^T + R)^{-1} \quad (12)$$

$$\bar{P}_k = (I - K_k C)\bar{P}_{k-1} \quad (13)$$

$$\hat{x}_k = \bar{x}_{k-1} + K_k (y_k - C\hat{x}_{k-1}) \quad (14)$$

where  $K$  represents the Kalman gain, and  $\bar{P}$  and  $\hat{x}$  are the covariance and the state updated. The errors between the actual state and the estimated state is

$$\Delta \varepsilon_i = x_i - \hat{x}_i \tag{15}$$

**Simulations**

In order to investigate the robustness of the filter, it has been done the simulation with a satellite model with one, two and three elastic modes. The structural parameters are: radius  $r = 0.3048\text{m}$ , density  $\rho = 47.89\text{Kg/m}$ , damping  $\zeta = 0.2$ ,  $L = 1.2192\text{m}$ ,  $E = 7.735 \times 10^9 \text{ Kg/m}^2$ ,  $I = 1.293 \times 10^{-10} \text{ Kg} \cdot \text{m}^2$ ,  $J_h = 10.84 \text{ Kg} \cdot \text{m}^2$ ,  $G = [0_{4 \times 4}, I_{4 \times 4}]^T$ ,  $R = [0.05^2, 0.005^2]^T$ , and  $Q = \text{diag}([10^{-4}, 10^{-6}, 10^{-6}, 10^{-6}])$ . The initials conditions,  $\theta_0 = 0.1$ ,  $\dot{\theta}_0 = 0.01$ ,  $P_0 = \text{diag}([10^{-2}]_{n \times n})$ .

Figure 2 shows the difference between the ideal state and the estimate state the “error” for the satellite model with one, two and three modes. It can be seen that angular velocity estimated, remains in all modes, under the limits of standard deviation. But for the angle it is necessary 50 seconds for the filter to adapt and have a good performance.

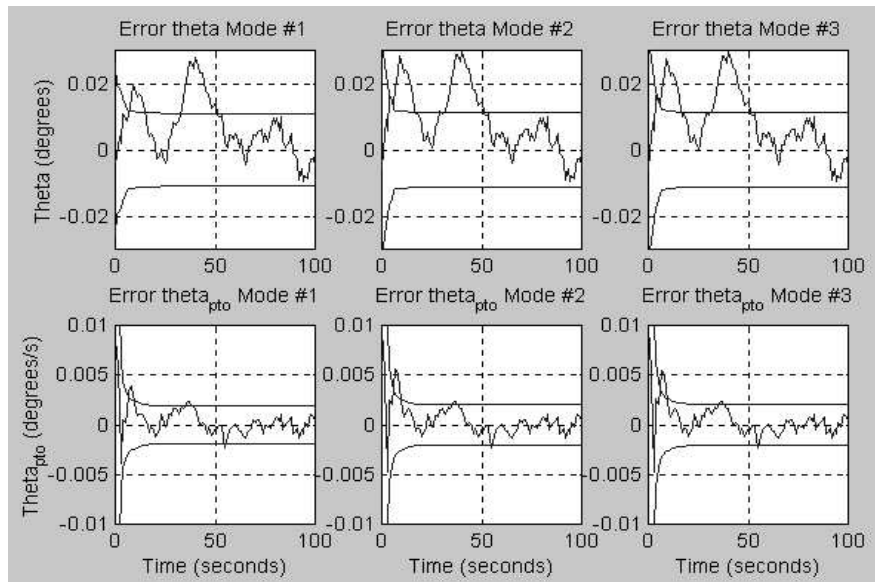


Figure 2 Errors for angle and angular velocity.

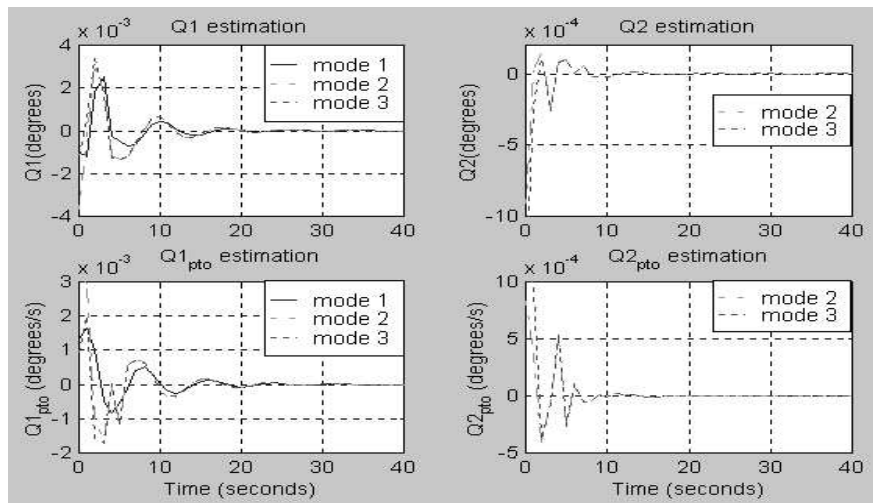


Figure 3 Estimation of the elastic displacement  $q_1$  and  $q_2$ .

Figure 3 shows a significant difference between the model with one and two modes in the flexible coordinate  $q_1$  and  $q_2$ . However, that difference is negligible for the model with two and three modes, which means that the satellite can be modeled at most with two modes without lost of accuracy. This is correct because, when more modes are included, the dynamics of the system tend to stationary values.

### Conclusions

In this work, one applies the Kalman filter Methodoly to estimated the elastic displacement, considering that the angle and the angular velocity of a flexible satellite are sensed. Having in mind the complexity of putting a sensor on the elastic parts of the satellite, the application of the Kalman filter mythology has been showed a good approach to estimate indirectly the flexible parameters of a rigid-flexible satellite. That approach becomes more promising when it is necessary to feedback the elastic measurements into the control system in order to assure better pointing conditions and/or better system performance.

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