Wave Dispersion in concrete due to Microstructure

D. G. Aggelis[1], T. P. Philippidis[1], S. V. Tsinopoulos[1], D. Polyzos[1]

Summary

Experiments shown that longitudinal pulses propagating through concrete and mortar specimens exhibit a dispersion behavior at low frequencies. In the present work the dipolar gradient elastic theory of Mindlin is employed and the experimentally observed dispersion in concrete and mortar is successfully explained. Both concrete and mortar are considered as homogeneous isotropic solids with microstructural effects and these effects are taken into account with the aid of the simple dipolar gradient elastic theory proposed by Mindlin.

Introduction

Concrete is a highly non-homogeneous material with a complex microstructure containing random inhomogeneities over a wide range of length scales. Its structure can be considered as a composite material where large aggregates are embedded in a mortar matrix. Similarly, the mortar consists of small aggregates dispersed in a cement paste medium. Understanding of how a stress wave propagates through such a medium is of paramount importance for many non-destructive testing techniques like ultrasonics and acoustic emission [1,2].

Experiments performed in [3] with the aid of an experimental setup explained in [4], show that traveling longitudinal waves in concrete as well as in mortar undergo dispersion only at low frequencies where the material microstructure is much smaller than the wavelength of the incident wave. This fact reveals that microstructural effects on the propagating pulses appear to be dominant in cementitious material. On the other hand, it has been shown in [3] that the scattering of traveling waves by the embedded inhomogeneities [5,6] is not the mechanism of the observed dispersion.

Considering concrete as a linear elastic material with microstructure, its dynamic mechanical behavior cannot be described adequately by the classical theory of linear elasticity, which is associated with concepts of homogeneity and locality of stresses. When the material exhibits a non-homogeneous behavior, microstructural effects become important and the state of stress has to be defined in a non-local manner. These microstructural effects can be successfully modeled in a macroscopic framework by employing higher order gradient elastic theories like those proposed by Cosserat brothers, Mindlin, Eringen, Aifantis and Vardoulakis. For a literature review on the subject of these theories one can consult [7-11].

During the last fifteen years, a variety of elastodynamic boundary value problems were solved either analytically or numerically by employing mainly simplified forms of the above mentioned enhanced continuum theories. Here one can mention the simple

[1] Mechanical Engineering and Aeronautics Department, University of Patras and FORTH/ICE-HT

gradient elastic theory of Altan and Aifantis [12] used by Altan et al. [13], Tsepoura et al. [10] and Polyzos et al. [14] to treat one and three dimensional vibration problems in gradient elastic bars, the gradient elastic theory with surface energy proposed by Vardoulakis and Sulem [11] employed by Georgiadis and Vardoulakis [15] to explain the dispersion behavior of Rayleigh waves propagating on the free surface of an elastic half space with microstructure, etc.and the dipolar gradient elastic theory of Mindlin [16] used by Georgiadis [17] to solve Mode-III crack problems in dynamic gradient elasticity.

In the present work, the dipolar gradient elastic theory introduced by Mindlin [16] is employed in order to explain the dispersion of longitudinal ultrasonic pulses observed experimentally in [3]. It is the simplest possible dynamic form of Mindlin's higher order gradient theory. It is called dipolar since besides the classical Lame constants, two new material constants are introduced, which correlate the microstructure with the macrostructure of the considered gradient elastic continuum. The key idea of the present study is that a proper determination of the two microstructural material constants enables one to explain remarkably well the low frequency dispersive nature of concrete and mortar.

Dipolar Gradient Elastic Theory and Wave Propagation

Taking into account the non-local nature of microstructural effects, Mindlin [16] considered that the density of strain density is not only a function of strains, as in the classical case, but also a function of the gradients of the strains. In the dipolar version of his theory, this is expressed as

$$W = \frac{1}{2}\lambda\left(tr\tilde{\mathbf{e}}\right)^2 + \mu\tilde{\mathbf{e}} : \tilde{\mathbf{e}} + \frac{1}{2}\lambda g^2 \nabla\left(tr\tilde{\mathbf{e}}\right)\cdot\nabla\left(tr\tilde{\mathbf{e}}\right) + \mu g^2 \nabla\tilde{\mathbf{e}} \,\vdots\, \nabla\tilde{\mathbf{e}} \tag{1}$$

where $\tilde{\mathbf{e}}$ and $tr\tilde{\mathbf{e}}$ are the classical strain tensor and its trace, respectively, ∇ represents the gradient operator, the dot, the double dots and the column of three dots indicate inner product between vectors and tensors of second and third order, respectively, (λ, μ) are the classical Lame constants and g^2 is a new material constant (units of m^2) called volumetric strain gradient energy coefficient, which correlates the microstructure with macrostructure.

Extending the idea of non-locality to the inertia of the continuum with microstructure, Mindlin proposed a new expression for the kinetic energy density function [16] where the gradients of the velocities are taken into account, i.e.

$$T = \frac{1}{2}\rho\dot{\mathbf{u}}\cdot\dot{\mathbf{u}} + \frac{1}{2}\rho h^2 \nabla\dot{\mathbf{u}} : \nabla\dot{\mathbf{u}} \tag{2}$$

where ρ is the mass density, u is the displacement vector, $\dot{\mathbf{u}} = d\mathbf{u}/dt$ and h^2 is the second new material constant (units of m^2) called velocity gradient coefficient, which is always smaller than the volumetric strain gradient energy coefficient g^2. Taking the variation of strain and kinetic energy, according to the Hamilton's principle, one concludes to the equation of motion of a continuum with microstructure, which in terms of displacements is written as follows:

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u} + g^2\nabla^2\left(\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u}\right) = \rho\ddot{\mathbf{u}} - \rho h^2\nabla^2\ddot{\mathbf{u}} \tag{3}$$

Copyright 2004 Tech Science Press

The Helmholtz vector decomposition implies that $\mathbf{u}$ can be written as a sum of irrotational and solenoidal fields according to the relation:

$$\mathbf{u} = \nabla\phi + \nabla\times\mathbf{A} \tag{4}$$

with $\nabla\phi$, $\nabla\times\mathbf{A}$ denoting volumetric and shape with no volume changes, respectively. In terms of wave propagation this means that $\nabla\phi$ corresponds to longitudinal waves, while $\nabla\times\mathbf{A}$ represents shear waves propagating through the medium, i.e.

$$\nabla\phi = \hat{\mathbf{k}}e^{i(k_p\hat{\mathbf{k}}\cdot\mathbf{r}-\omega t)}$$
$$\nabla\times\mathbf{A} = \hat{\mathbf{b}}e^{i(k_s\hat{\mathbf{k}}\cdot\mathbf{r}-\omega t)} \tag{5}$$

where $\hat{\mathbf{k}}$ represents the direction of incidence, $\hat{\mathbf{b}}$ is the polarization vector for the shear wave, $\mathbf{r}$ stands for position vector, k_p, k_s are the wave numbers of the longitudinal and shear disturbances, respectively, while ω is the frequency of the propagating waves. Representing by C_p, C_s the classical phase velocities of longitudinal and shear waves, respectively, and inserting Eqs.(5) into Eq. (3) one obtains the following relation for longitudinal waves (similarly for shear).

$$\omega^2 = C_p^{\,2}\frac{k_p^{\,2}\left(1+g^2k_p^{\,2}\right)}{1+h^2k_p^{\,2}},\quad C_p^{\,2} = \frac{\lambda+2\mu}{\rho} \tag{6}$$

Equation (6) reveal that longitudinal stress waves undergo dispersion when they travel in solids with microstructure. Their dispersion is entirely due to the presence of the two microstructural material constants g^2 and h^2. It is easy to see that by zeroing g^2 and h^2, Eq.(6) lead to the linear expressions:

$$\omega^2 = C_p^{\,2}\cdot k_p^{\,2} \tag{7}$$

which characterize propagation of non dispersive waves in a classical elastic medium. Solving Eq.(6) for the wave number k_p respectively, one yields the following relation:

$$k_p^{\,2} = \frac{-\left(C_p^{\,2}-\omega^2h^2\right)+\sqrt{\left(C_p^{\,2}-\omega^2h^2\right)^2+4\cdot C_p^{\,2}g^2\omega^2}}{2C_p^{\,2}g^2} \tag{8}$$

Then the phase velocity of longitudinal and shear plane waves propagating in a dipolar gradient elastic continuum has the following form:

$$V_p = \frac{\omega}{k_p} = \frac{\omega}{\sqrt{\dfrac{-\left(C_p^{\,2}-\omega^2h^2\right)+\sqrt{\left(C_p^{\,2}-\omega^2h^2\right)^2+4\cdot C_p^{\,2}g^2\omega^2}}{2C_p^{\,2}g^2}}} \tag{9}$$

Results and Discussion

The experimental setup is described in detail in [4]. The phase velocity was calculated by the difference in phase between the input and output signal as described in

[18]. The cases presented herein concern mortar with water to cement ratio, w/c=0.55 and 0.65 and concrete with w/c=0.375 and 0.45. The aggregates in total (sand and coarse aggregates occupy approximately 60% of the material volume. Since concrete can be considered as a particulate composite, a first approach concerning the volumetric constant g, is to relate its value to the particule size. Therefore, it was calculated as the ratio of mean aggregate size (2mm for mortar, 10mm for concrete) to the applied wavelength. This way the g constant is proportional to frequency and results in phase velocity vs frequency curves close to the experimentally obtained, as seen in Fig.1. As to the acceleration parameter h, that rules mainly the dispersion (difference between minimum and maximum velocity), it was found that being proportional and somehow lower than g, facilitates the fitting of the experimental curves. Specifically for the mortar cases it was set equal to g/1.034 and g/1.016 while for concrete it is lowered to g/1.216 and g/1.175.

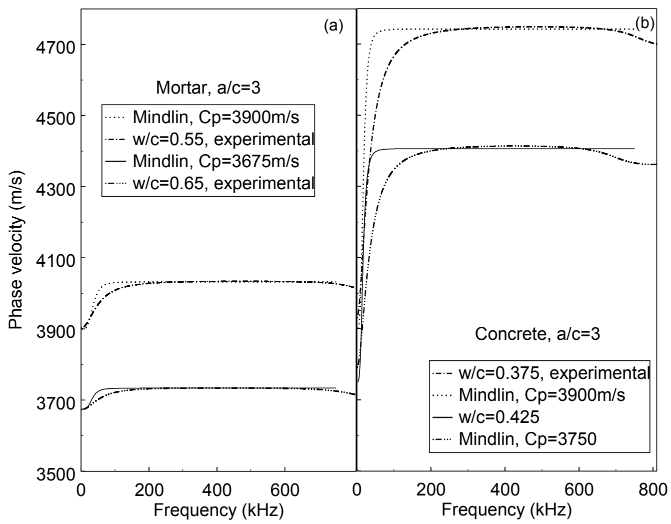

Fig.1. Comparison of experimental phase velocity and Mindlin's prediction for (a) mortar and (b) concrete considering the aggregates as the microstructure

Despite the general agreement, the velocity rise predicted at low frequencies is more acute than in the experimental curves. This implies that the size of the aggregate is not necessarily the actual characteristic size of the microstructure. Indeed, concrete contains inhomogeneities from the order of size of nm (nanopores) up to even cm (large aggregates). This fact makes identification of the characteristic length of the material's microstructure a complicated task with other parameters as porosity or inter-particle distances possibly being important. Therefore, it was found proper to adjust the size of microstructure to a suitable value to fit experimental curves. The size of microstructure that, compared to the wavelength, yields g values reducing the discrepancy between theoretical and experimental results is approximately 600μm for mortar and 1.3mm for concrete. This is shown in Fig.2(a) and (b) for mortar and concrete respectively. Considering therefore, the size of effective microstructure around 1mm results in a curve less acute describing much more closely the experimental one. The values of h are not much different from the previous, resulting in g/1.029, g/1.0145 and g/1.17, g/1.145 for mortar and concrete respectively.

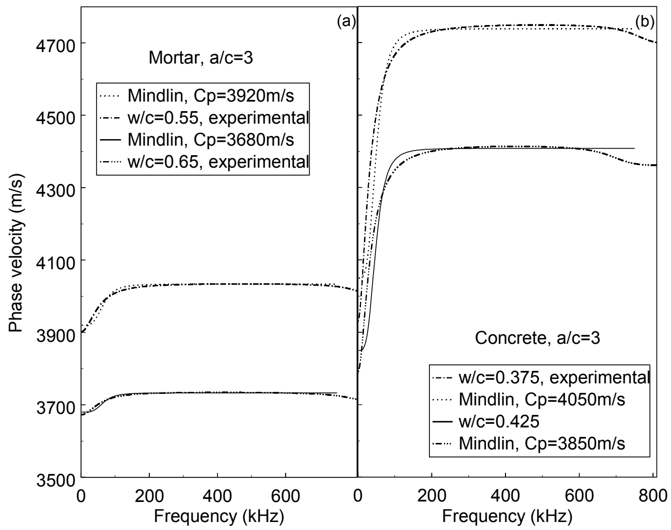

Fig.2 Comparison of experimental phase velocity and Mindlin's prediction for (a) mortar and (b) concrete with arbitrary microstructure

Generally, the dispersion trend differs slightly for each type of material. While experiments show that paste is non-dispersive, mortar undergoes phase velocity increase up to about 200kHz. Concrete undergoes even greater and more acute increase. This is why the acceleration parameter h, ranges between g/1.035 - g/1.015 for mortar, while for concrete that is more dispersive, it varies approximately between g/1.21 - g/1.14.

Conclusions

In the context of the present work the following conclusions can be drawn: at first, experimental observations show that low frequency longitudinal pulses propagating in concrete and mortar undergo dispersion. Also, considering concrete and mortar as solids characterized by microstructural effects, theoretical predictions made by the dipolar gradient elastic theory of Mindlin are very close to the experimental observations. Besides the classical Lame constants, Mindlin's theory introduces two new material constants, namely the volumetric strain gradient energy coefficient g^2 and the velocity gradient coefficient h^2, which correlate the microstructure with the dynamic macrostructural behavior of the considered gradient elastic continuum. Correlations between experimental observations and theoretical predictions have shown that h^2 is always smaller than g^2 while both constants depend on the aggregate size since they obtain quite different values for mortar and concrete.

References

1 Malhotra, V. M., Carino, N. J., (Eds.), (1991): *CRC Handbook on Nondestructive Testing of Concrete.* CRC Press, Florida.

2 Philippidis, T. P., Aggelis, D. G., (2003): “An acousto-ultrasonic approach for the determination of water to cement ratio in concrete”, *Cement Concrete Res 33, pp.525-538*

3 Aggelis, D. G., Polyzos D., “Wave propagation and microstructural effects in concrete”, submitted (2004).

4 Philippidis, T. P., Aggelis, D. G., “Wave dispersion and attenuation in concrete” submitted (2003).

5 Tsinopoulos, S. V., Verbis, J. T., Polyzos, D., (2000): “An iterative medium approximation for wave dispersion and attenuation predictions in particulate composites”, *Adv. Compos. Lett*. 9, pp.193-200

6 J.T. Verbis, S.E. Kattis, S.V. Tsinopoulos, D. Polyzos, (2001): “Wave dispersion and attenuation in fiber composites” *Comput. Mech.* 27, pp.244-252

7 H.F. Tiersten, J.L. Bleustein, (1974) “Generalizd elastic continua” in: R.D. Midlin and Applied Mechanics, (Hermann, G., ed.). Pergamon Press, New York, pp.67-103

8 A.C. Eringen,. (1992): “Vistas of Nonlocal Continuum Physics”. *International Journal of Engineering Science* 30, pp.1551-1565

9 G.E. Exadaktylos, I. Vardoulakis, (2001): “Microstructure in linear elasticity and scale effects: a reconsideration of basic rock mechanics and rock fracture mechanics”. *Tectonophysics* 335 (1-2) pp.81-109

10 K.G. Tsepoura, S. Papargyri-Beskou, D. Polyzos, (2002): “A boundary element method for solving 3D static gradient elastic problems with surface energy”, *Comput Mech* 29, pp.361-381

11 I. Vardoulakis, J. Sulem, (1995): *Bifurcation Analysis in Geomechanics*. Blackie/Chapman and Hall, London

12 B.S. Altan, E.C. Aifantis, (1992): “On the Structure of the Mode-III Crack-tip in Gradient Elasticity”, *Scripta Metallurgica et Materialia* 26, pp.319-324

13 B.S. Altan, H.A. Evensen, E.C. Aifantis, (1996): “Longitudinal vibrations of a beam: A gradient elasticity approach”, Mech. Res. Commun. 23, pp.35-40

14 D.Polyzos, K.G. Tsepoura, S. Tsinopoulos, D.E. Beskos, (2003): A boundary element method for solving 2-D and 3-D static gradient elastic problems – Part I: Integral formulation. *Comput. Method. Appl. M*. 192, pp.2845-2873

15 H.G. Georgiadis, I. Vardoulakis, (1998): Anti-plane shear Lamb’s problem treated by gradient elasticity with surface energy, *Wave Motion* 28, pp.353-366

16 R.D. Midlin, (1964): Micro-Structure in Linear Elasticity. *Arch. Ration. Mech. An.*, 16, pp.51-78

17 H.G. Georgiadis, (2003): “The mode III crack problem in microstructured solids governed by dipolar gradient elasticity: Static and dynamic analysis”, *J. Appl. Mech. T. ASME 70,* pp.517-530

18 Sachse, W., Pao, Y-H., 1978. “On the determination of phase and group velocities of dispersive waves in solids”, *Journal of Applied Physics 49 (8), pp.4320-4327.*