The Electro-Mechanical Response of Electrostrictive Composite Laminates due to Actuation

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Summary

The classical laminated plate theory (CLPT) is extended to study the nonlinear electromechanical coupling in smart electrostrictive composite materials. Both numerical and analytic solutions are obtained to predict the in-plane strains, stresses, and electric displacements of electrostrictive laminated composites subjected to through-the-thickness electric fields. Both solutions give almost identical results, which shows that a higher-order nonlinear term in the constitutive relation of electrostrictive materials is negligible.

Introduction

Electrostrictive materials, like piezoelectric materials, can be regarded as smart materials because they exhibit mechanical and electrical response under mechanical or electrical excitation[1]. The differences between piezoelectric and electrostrictive materials are that electrostrictive materials, such as lead magnesium niobate (PMN), exhibit nonlinear electromechanical coupling without hysteresis[2].

In view of high precision positioning, quick response, low driving power, and miniaturization of devices, new transducers such as electrostrictive materials are candidates for advanced actuator materials. The present work conducts a feasibility study on combining PMN with ductile alloys or compliant polymers to make the transducer more compliant.

Lamina Analysis

The constitutive relations for three-dimensional electrostrictive materials can be expressed by[3]

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} + Q_{ijkl}D_k D_l, \tag{1}$$

$$E_i = \beta_{ij}P_j - 2Q_{jkli}\sigma_{jk}D_l, \tag{2}$$

where i, j, k and l = 1, 2, 3, ε is the strain tensor, S is the compliance tensor and is the inverse of the stiffness tensor C, σ is the stress tensor, Q are the electrostrictive

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constants, D is the electric displacement vector, E are the electric fields, and β is the permittivity tensor. The summation convention is used.

The plane stress condition can be assumed for a thin lamina lying in the x_1-x_2 plane:

$$\sigma_{33} = \sigma_{23} = \sigma_{31} = 0, \tag{3}$$

$$E_1 = E_2 = 0. \tag{4}$$

Eq. (1) and Eq. (2) can be reduced to

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C^*_{1111} & C^*_{1122} & 0 \\ C^*_{1122} & C^*_{2222} & 0 \\ 0 & 0 & C^*_{1212} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} - \begin{Bmatrix} H^*_{1133} \\ H^*_{2233} \\ 0 \end{Bmatrix} D_3^2, \tag{5}$$

$$D_3 = \frac{E_3}{\beta_{33} - 2(Q_{1133}\sigma_{11} + Q_{2233}\sigma_{22})}. \tag{6}$$

where

$$C^*_{1111} = C_{1111} - \frac{C^2_{1133}}{C_{3333}}, \tag{7}$$

$$C^*_{2222} = C_{2222} - \frac{C^2_{2233}}{C_{3333}}, \tag{8}$$

$$C^*_{1122} = C_{1122} - \frac{C_{1133}C_{2233}}{C_{3333}}, \tag{9}$$

$$C^*_{1212} = C_{1212}, \tag{10}$$

$$H^*_{1133} = H_{1133} - \frac{C_{1133}H_{3333}}{C_{3333}}, \tag{11}$$

$$H^*_{2233} = H_{2233} - \frac{C_{2233}H_{3333}}{C_{3333}}, \tag{12}$$

$$H_{ijkl} = C_{ijmn}Q_{mnkl}. \tag{13}$$

Laminate Analysis

Based on CLPT[4], one assumes that

$$\{\varepsilon\} = \{\varepsilon^{\circ}\} + z\{\rho\} \tag{14}$$

where z = x$_3$, $\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, 2\varepsilon_{12}\}^T$ are the laminate strains, $\{\varepsilon^{\circ}\} = \{\varepsilon_{11}^{\circ}, \varepsilon_{22}^{\circ}, 2\varepsilon_{12}^{\circ}\}^T$ are the laminate midplane strains, and $\{\rho\} = \{\rho_{11}, \rho_{22}, 2\rho_{12}\}^T$ are the laminate curvatures. The resultant forces and moments per unit length acting on a laminate are given by

$$\{N\} = \int_{-t/2}^{t/2} \{\sigma\} dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \{\sigma\}_k dz, \tag{15}$$

$$\{M\} = \int_{-t/2}^{t/2} \{\sigma\} zdz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \{\sigma\}_k \, zdz. \tag{16}$$

where t is the laminate thickness, n is the number of lamine, z_k and z_{k-1} are the z-coordinates of the top and the bottom surface of the kth lamina, respectively. Substituting Eq. (14) and Eq. (5) into Eq. (15) and Eq. (16) gives

$$[A]\{\varepsilon^{\circ}\} + [B]\{\rho\} = \{N\} + \{N^e\}, \tag{17}$$

$$[B]\{\varepsilon^{\circ}\} + [D]\{\rho\} = \{M\} + \{M^e\}, \tag{18}$$

where the extensional stiffness [A], the coupling stiffness [B], the bending stiffness [D], and the loading $\{N^e\}$ and $\{M^e\}$ due to electric fields are given by

$$[A] = \sum_{k=1}^{n} [C^*]_k \,(z_k - z_{k-1}), \tag{19}$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} [C^*]_k \,(z_k^2 - z_{k-1}^2), \tag{20}$$

$$[D] = \frac{1}{3} \sum_{k=1}^{n} [C^*]_k \,(z_k^3 - z_{k-1}^3), \tag{21}$$

$$\left\{N^e\right\} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left\{H^*\right\}_k \left(\frac{E_3}{\beta_{33} - 2(Q_{1133}\sigma_{11} + Q_{2233}\sigma_{22})}\right)^2 dz, \tag{22}$$

$$\left\{M^e\right\} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left\{H^*\right\}_k \left(\frac{E_3}{\beta_{33} - 2(Q_{1133}\sigma_{11} + Q_{2233}\sigma_{22})}\right)^2 zdz, \tag{23}$$

respectively. One can solve the combination of Eq. (22), (23), (17), (18), (14), and (5) numerically to determine the mechanical and electric response of electrostrictive composite laminates due to actuation. If the term $2Q_{jkli}\sigma_{jk}P_l$ in Eq. (2) is ignored, $\{N^e\}$and $\{M^e\}$ given in Eq. (22) and Eq. (23) can be computed for a given E_3. Thus Eq. (17) and Eq. (18) can be solved analytically.

Results

Consider a [brass/PMN] laminate subjected to E_3 = 1 KV / mm. The ply thickness is 1 mm. The stresses $\sigma_1 (= \sigma_2)$ as a function of z are shown in Figure 1. Figure 2 shows the effect of the thickness of brass on strains. The relationship between the electric displacement D_3 and the electric field E_3 is shown in Figure 3. The effects of material properties on the electromechanical coupling in electrostrictive composites are shown in Figure 4, where [aluminum/PMN], [epoxy/PMN] and [brass/PMN] are considered.

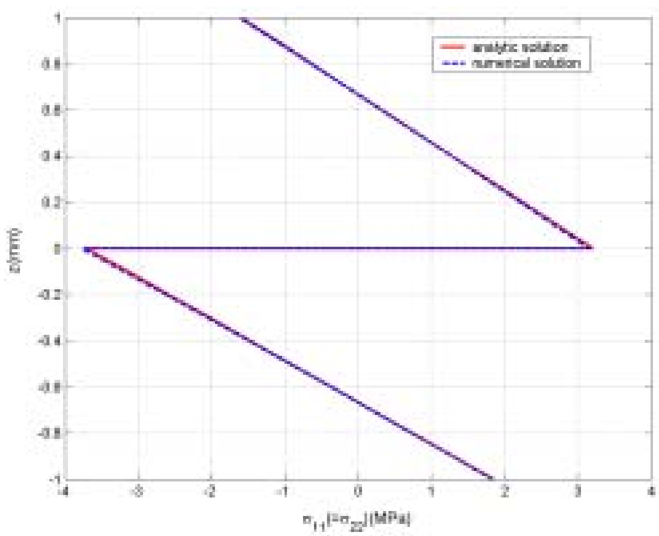

Figure 1: Stress $\sigma_{11} (= \sigma_{22})$ as a function of z

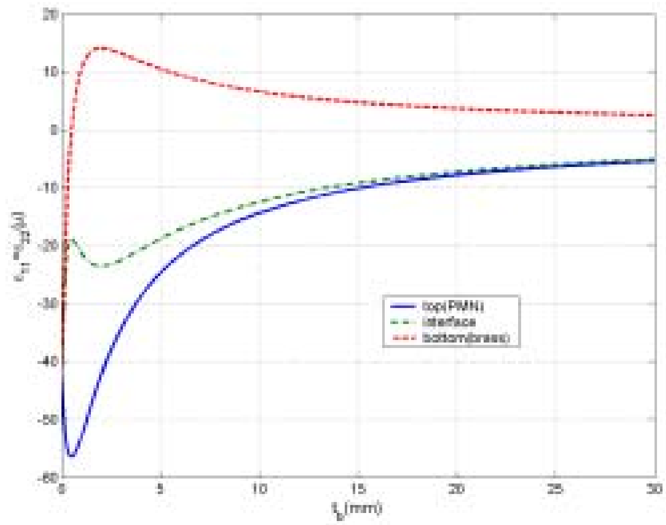

Figure 2: Strain $\varepsilon_{11}(=\varepsilon_{22})$ at the bottom, interface, and top of brass/PMN, subjected to $E_3 = 1\ kV/mm$, as a function of the thickness of brass

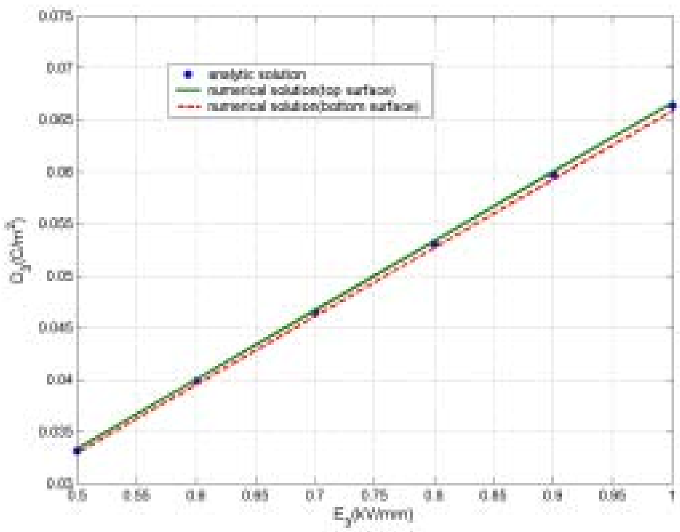

Figure 3: Electric displacement D_3 as a function of electric field E_3 in brass/PMN

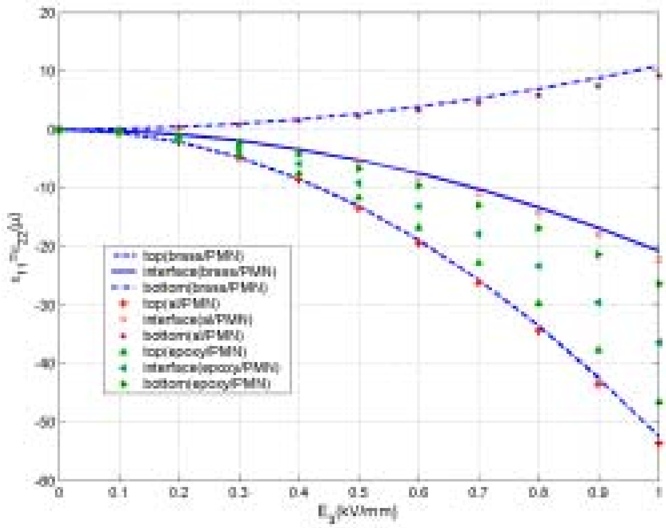

Figure 4: Strain $\varepsilon_{11}(=\varepsilon_{22})$ in brass/PMN, Al/PMN, and epoxy/PMN as a function of E_3

Conclusions

CLPT is extended to study the nonlinear electromechanical coupling in electrostrictive composite laminates. The effects of ply thickness, constituent properties on the elastic and electrical behavior of electrostrictive composites are studied. A higher-order nonlinear coupling term in the constitutive relation of electrostrictive materials is shown to be negligible.

Acknowledgement

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Reference

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