Analysis of Structural Dynamic Problems through the Node Activation Technique

Jin Yeon Cho[1], Sung Woon Woo[2], Jeong Ho Kim[3]

Summary

In this paper, the recently proposed node activation technique is extended to efficiently handle the structural dynamic problems only with reduced number of degree of freedoms. To investigate the performance of the proposed method, modal analysis and time domain analysis for the activated system are carried out, and the results are compared with the results for the finite element model with the same number of degree of freedoms. Through the modal analysis, it is confirmed that the natural frequencies obtained from the activated system generated by the proposed technique are more accurate compared with those obtained from the finite element model with the same number of degree of freedoms. Similarly, from the time domain responses it is observed that the results obtained from the activated system are more accurate than those obtained from the finite element model with the same number of degree of freedoms.

Introduction

Over the past decades, considerable research efforts have been given to numerical techniques by which one can obtain accurate solutions only with small number of degree of freedoms. And as a result, numerous algorithms have been proposed. Those are higher order accurate finite element methods, condensation techniques, and others. Recently, meshless methods[1-5] have been proposed to achieve higher order accuracy as well as to get rid of the dependency on mesh by using meshless interpolation techniques such as the moving least squares approximation technique[6], the reproducing kernel particle method[2], and others. And through the meshless interpolation character, higher order accuracy of numerical solution has been achieved while the problems of element distortion and mesh dependency are alleviated successfully.

This favorable nature of meshless interpolations plays a key role in the recently proposed node activation method[7,8] in which only the nodes of interest are dealt with as active degree of freedoms among the large set of the nodal points of the given original finite element system. Because the meshless approximation is utilized as "activating kernel" along with finite element shape function to arbitrarily activate the nodes of

[1] Assistant Professor, Department of Aerospace Engineering, INHA University, Incheon 402-751, Korea
[2] Research Assistant, Department of Aerospace Engineering, INHA University, Incheon 402-751, Korea
Currently, Researcher, Supercomputing Application Department, KISTI, Taejon 305-333, Korea
[3] Senior Researcher, Supercomputing Application Department, KISTI, Taejon 305-333, Korea

interest in the node activation technique, the favorable characteristics of meshless interpolation, such as higher order accuracy and alleviating mesh distortions, are inherited. Also there is no difficulty in integrating the weak form unlike the previously proposed meshless methods.

Because of this promising characters of the node activation techniques, we are aiming to extend the technique to the field of structural dynamics for which a large amount of computational cost should be paid compared with the static finite element analysis.

Node Activation Procedure

Although the node activation is conceptually related to condensation and sub-structuring technique, it does not incorporate an explicit matrix solving procedure. The node activation procedure consists of three stages. At first, nodes of interest are selected among the large set of nodal points in the given original finite element system. Secondly, the activating kernel $\psi_i(x)$ shown in Fig. 1 is obtained for each selected nodal point by using the moving least squares method, and subsequently the activating kernels are re-interpolated by using the original finite element shape functions $\phi_j(x)$ to construct the activating shape functions $\tilde{\phi}_i(x)$ for nodal points activation. The newly constructed activating shape function shown in Fig. 2 has the following form.

$$\tilde{\phi}_i(x) = \sum_{j=1}^{N} \psi_i(x_j)\phi_j(x) \tag{1}$$

where, N denotes the total number of nodes in the original finite element system. As mentioned in the previous work[7], the newly defined activating shape function satisfies the consistency condition to guarantee the convergence if the activating kernel satisfies the consistency condition.

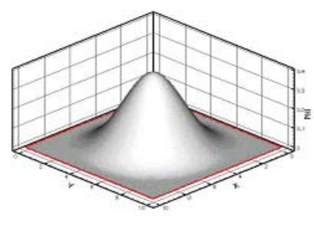


Fig. 1 Activating kernel obtained by the moving least squares approximation

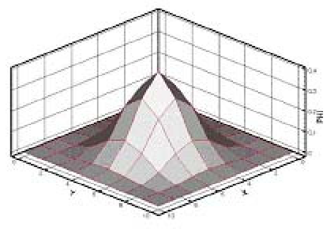


Fig. 2 Activating shape function

At the final stage, the displacement field is defined by the Eq. (2) through the activating shape function, and it is inserted to weak form to yield a system of algebraic equations for the activated system.

$$u(x) = \sum_{i=1}^{Na} \tilde{\phi}_i(x)\tilde{u}_i \quad and \quad u(x_k) = \sum_{i=1}^{Na} \tilde{\phi}_i(x_k)\tilde{u}_i = \sum_{i=1}^{Na}\left(\sum_{j=1}^{N} \psi_i(x_j)\phi_j(x_k)\right)\tilde{u}_i \tag{2}$$

where, Na denotes the number of activated nodes.

Application of Node Activation to Structural Dynamic System

In this section, the procedure for the application of node activation to the structural dynamic system is presented. From the principle of virtual work, equation of motion of structural dynamic system can be written as follows.

$$\int_V \delta\mathbf{u}^T \rho\ddot{\mathbf{u}} dV + \int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = \int_V \delta\mathbf{u}^T \mathbf{f} dV + \int_{\partial V} \delta\mathbf{u}^T \bar{\mathbf{t}} dA \tag{3}$$

where, ρ, $\mathbf{u}$, ε, σ, $\mathbf{f}$, and $\bar{\mathbf{t}}$ denote the density, displacement, strain, stress, body force per unit volume, and traction force, respectively. For semi-discretization, the displacement and strain fields over the space variable $\mathbf{x}$ are approximated by the activating shape function as follows.

$$\mathbf{u}(\mathbf{x},t) = \sum_{i=1}^{Na} \tilde{\boldsymbol{\Phi}}_i(\mathbf{x})\tilde{\mathbf{U}}_i(t), \quad \boldsymbol{\varepsilon}(\mathbf{x},t) = \sum_{i=1}^{Na} \tilde{\mathbf{B}}_i(\mathbf{x})\tilde{\mathbf{U}}_i(t) \tag{4}$$

where $\tilde{\mathbf{U}}_i(t)$ denotes the fictitious displacement vector at the activated nodal point i. And the interpolation matrices $\tilde{\boldsymbol{\Phi}}_i$ and $\tilde{\mathbf{B}}_i$ in two dimension can be written as follows.

$$\tilde{\boldsymbol{\Phi}}_i = \begin{bmatrix} \tilde{\phi}_i & 0 \\ 0 & \tilde{\phi}_i \end{bmatrix}, \quad \tilde{\mathbf{B}}_i = \begin{bmatrix} \tilde{\phi}_{i,x} & 0 \\ 0 & \tilde{\phi}_{i,y} \\ \tilde{\phi}_{i,y} & \tilde{\phi}_{i,x} \end{bmatrix} \tag{5}$$

Finally, substitution of Eq. (4) and Eq. (5) into Eq. (3) yields the following system of ordinary differential equations.

$$\tilde{\mathbf{M}}\ddot{\tilde{\mathbf{U}}} + \tilde{\mathbf{K}}\tilde{\mathbf{U}} = \tilde{\mathbf{F}} \tag{6}$$

where $\tilde{\mathbf{M}}, \tilde{\mathbf{K}}$, and $\tilde{\mathbf{F}}$ denote mass matrix, stiffness matrix, and force vector for the activated system, respectively.

Numerical Examples

In this section, modal analysis and time domain analysis are carried out for the activated system generated from the proposed node activation technique, and the results are compared with those obtained for the finite element system with the same number of degree of freedoms to investigate the performance of the proposed technique.

Test model for modal analysis is presented in Fig. 3. Among the total 4257 nodes, 85(2.00%), 297(6.97%), and 1105(25.96%) nodes are activated, and the relative errors of eigenvalues obtained from each activated system are presented in Fig. 4. To calculate relative error, eigenvalues obtained from the finite element system with 9457 nodal points are selected as reference values. As shown in Fig. 4, it is confirmed that the accuracy of eigenvalue from modal analysis is improved as the number of activated nodes is increased.

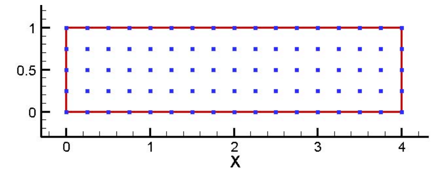


Fig. 3 Test model

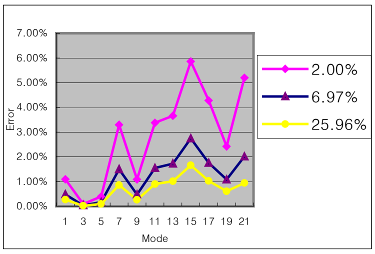


Fig. 4 Comparison of relative errors

In Fig. 5 and 6, value and relative error of eigenvalues, obtained from the system with 85(2%) activated nodes, are compared with those generated from the finite element systems with the same number of degree of freedoms to investigate the performance of the proposed node activation technique.

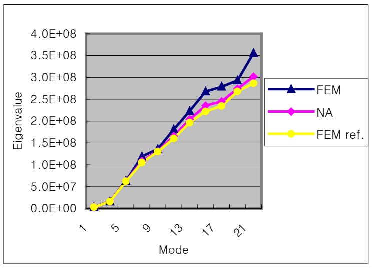


Fig. 5 Comparison of eigenvalues

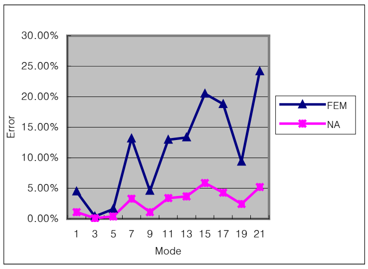


Fig. 6 Comparison of relative errors

In Fig 5, one can observe that eigenvalues for the activated system generated by node activation technique are much closer to reference eigenvalues than those for the finite element model with the same number of degree of freedoms. Also in Fig. 6 one can see that natural frequencies(eigenvalues) are greatly improved in high frequency regions.

Next, time domain analysis is carried out for the activated system by using the constant average acceleration method[9], which is the most famous form of Newmark-β direct time integration method. In Fig. 7, situation of impulse loading is sketched.

Among 1105 nodal points, only 85 nodal points are activated through the node activation technique, and the activated dynamic system is numerically integrated for the impulse loading problem of Fig. 7 by using the constant average acceleration method. The calculated response is compared with that from the finite element system with the same 85 nodal points. Also, both responses are compared with the reference response obtained from the finite element system composed of 4257 nodal points. In Fig. 8, the time domain history of y-directional displacement at the right-upper tip is presented. There is no big difference among each responses at the initial stage, however as time goes on, one can observe that the response from the activated system is more accurate compared with that from the finite element system with the same number of degree of freedoms.

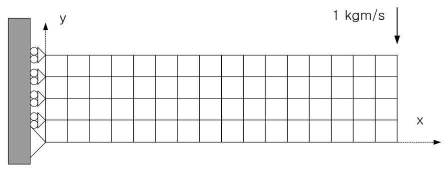
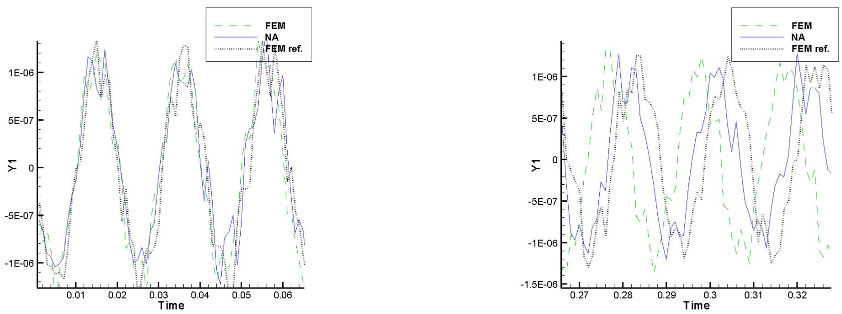


Fig. 7 Impulse loading



(a) At the initial time (b) After a long time

Fig. 8 Time domain history

Conclusions

In this paper, the node activation technique is extended to efficiently deal with the structural dynamic problems only with reduced number of degree of freedoms. To verify the efficiency of the proposed technique, both the modal analysis and time domain analysis are carried out. In modal analysis, natural frequencies are obtained for the activated system, and it is compared with the natural frequencies obtained for the finite element system composed of the same number of degree of freedoms. In time domain analysis, impulse response is analyzed for the activated system, and the result is compared with that for the finite element system having the same number of degree of freedoms. From the results, it is confirmed that one can carry out both the modal analysis and time domain analysis efficiently only with the small number of degree of freedoms by using the proposed node activation technique. It is also expected that 3 dimensional structural dynamic problems can be efficiently handled only with reduced number of degree of freedoms by using the proposed node activation technique.

Acknowledgements

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