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Generating and Rendering Images Using the X-slits Projection

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Summary

This document describes the rendering of images based on a special mathematical projection model with properties that are different from those of the perspective projection. This mathematical model is known as crossed-slits projection, or x-slits. The procedure to obtain images based on the x-slits projection, starting from perspective images, is also described. Due to its characteristics this projection model could be very useful in wide-screen technologies and in other image rendering applications.

Introduction

The recovery of information about structure and other properties of a 3D dynamic scene from 2D images is an important problem in computer vision. To obtain that information it is essential the knowledge of the projection model that describes the image formation process.

The mathematical model that describes the formation of the most common type of images is the perspective projection model. Most of the optical devices that are commercialized generate images whose geometrical properties are described by the perspective model. However other models have been used which are simpler versions of the perspective model, as it is the case of the orthographic projection model. This model is applied in particular configurations namely when the 3D objects are located far from the camera (relative to the focal length) and are small (compared to the distance to the camera).

However certain special vision problems can benefit from the application of alternative projection models, as it is the case with image rendering applications. An example of such a model is the case of the so-called crossed-slits projection (x-slits). In this model the projection ray of a generic 3D point is defined by the 3D line that passes through the point and two lines, referred as slits. The image is obtained by the intersection of every projective ray with the image plane. This model was initially designed by one of the pioneers of the color photography, Ducos du Hauron, in 1888 [1]. However, it was a restricted model in terms of the slits positions, which were parallel and orthogonal between each other (this situation was later referred as POX). After one century, this model was revised and generalized by Kingslake, who concluded that the model was similar to the perspective projection in which the image is stretched or

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compressed in one direction more than the other [2]. This fact shows its adequacy to its use in wide-screen technologies (in television and in cinema). According to [3], one advantage for the use of this model in the referred technologies is the fact that x-slits images can be easily generated by perspective images. This means that the same optical devices can be used to obtain perspective or x-slits images. In short, this procedure is performed by pasting together vertical or horizontal samplings of a sequence of images captured from a perspective camera, which moves, respectively, along a horizontal or vertical line.

This paper is organized as follows. First the x-slits projection model is defined. Then the procedure to generate x-slits images from perspective images is presented. Finally, experimental results are shown.

X-slits projection model

Consider the x-slits projection configuration represented in figure 1.

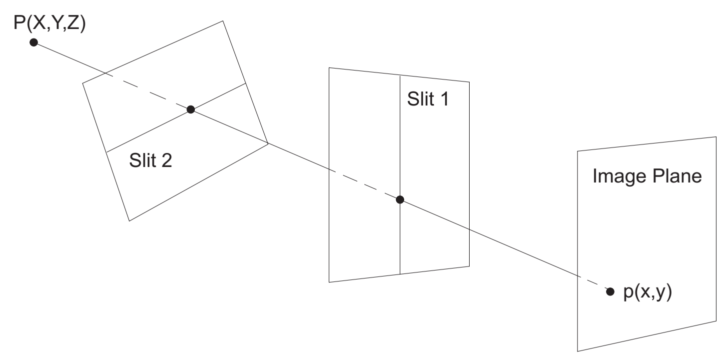

Figure 1 – X-slits projection model.

In this figure one can see that the projective ray of a generic 3D point, P, must intersect two slits l_1 and l_2. The point P together with each slit defines one plane. The intersection of those planes defines the projective ray. The projection of the 3D point in the image, p, is obtained by the intersection of the projective ray with the image plane. As it can be seen, this model defines a relation many to one, which means that every 3D points belonging to the same projective ray has the same projection point in the image.

To define the two slits, let $u_i = \begin{bmatrix} A_{u_i} & B_{u_i} & C_{u_i} & D_{u_i} \end{bmatrix}^T$ and $v_i = \begin{bmatrix} A_{v_i} & B_{v_i} & C_{v_i} & D_{v_i} \end{bmatrix}^T$, with $i = 1, 2$, be two generic planes defined in a space of 3 dimensions, given by their parametric coordinates. The slits, l_i, are defined through their intersection. Those slits can be represented by the dual Plüker matrix L_i^* [4], whose expression is

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$$L_i^* = u_i v_i^T - v_i u_i^T = \begin{bmatrix} 0 & A_{u_i}B_{v_i} - A_{v_i}B_{u_i} & -\left(C_{u_i}A_{v_i} - C_{v_i}A_{u_i}\right) & A_{u_i}D_{v_i} - A_{v_i}D_{u_i} \\ -\left(A_{u_i}B_{v_i} - A_{v_i}B_{u_i}\right) & 0 & B_{u_i}C_{v_i} - B_{v_i}C_{u_i} & -\left(D_{u_i}B_{v_i} - D_{v_i}B_{u_i}\right) \\ C_{u_i}A_{v_i} - C_{v_i}A_{u_i} & -\left(B_{u_i}C_{v_i} - B_{v_i}C_{u_i}\right) & 0 & C_{u_i}D_{v_i} - C_{v_i}D_{u_i} \\ -\left(A_{u_i}D_{v_i} - A_{v_i}D_{u_i}\right) & D_{u_i}B_{v_i} - D_{v_i}B_{u_i} & -\left(C_{u_i}D_{v_i} - C_{v_i}D_{u_i}\right) & 0 \end{bmatrix} \tag{1}$$

The definition of the projective ray is done as follows. As planes u_i and v_i, with $i = 1,2$, contain slits l_i, then also the plane $u_i + \alpha v_i$, with $i = 1,2$, contain them, whatever the value of α. The opposite, any plane which contains the slits l_i can be described by $u_i + \alpha v_i$, with $i = 1,2$, for some α, is also true. Thus, for any 3D generic point, P, there is an α_i such that

$$\left(u_i + \alpha_i v_i\right) P = 0 \text{, with } i = 1,2 \tag{2}$$

Solving equation (2) in order to α_i, we obtain

$$\alpha_i = -\frac{u_i^T P}{v_i^T P} \text{, with } i = 1,2 \tag{3}$$

Replacing equation (3) in the expression of the planes that pass through the point P and slits l_i, $u_i + \alpha_i v_i$, we obtain

$$u_i - \frac{u_i^T P}{v_i^T P} v_i = \left(v_i^T P\right) u_i - \left(u_i^T P\right) v_i = \left(u_i v_i^T - v_i u_i^T\right) P = L_i^* P \text{, with } i = 1,2$$

where L_i^* is the dual Plüker matrix that represents the slit i, defined in equation (1). In terms of the Plüker line coordinates, being the generic 3D point $P = k\begin{bmatrix} X & Y & Z & 1 \end{bmatrix}^T$, those planes are defined by

$$L_i^* P = k \begin{bmatrix} YL_{i34} + ZL_{i42} + L_{i23} \\ -XL_{i34} + ZL_{i14} - L_{i13} \\ -XL_{i42} - YL_{i14} + L_{i12} \\ -XL_{i23} + YL_{i13} - ZL_{i12} \end{bmatrix} \text{, with } i = 1,2 \text{, since } L_i^* = \begin{bmatrix} 0 & L_{i34} & L_{i42} & L_{i23} \\ -L_{i34} & 0 & L_{i14} & -L_{i13} \\ -L_{i42} & -L_{i14} & 0 & L_{i12} \\ -L_{i23} & L_{i13} & -L_{i12} & 0 \end{bmatrix} \text{ [4].}$$

Finally, projective ray, $l = \overline{Pp}$, is the intersection between those two planes, and can be defined by the dual Plüker matrix, through

$$L^* = \left(L_1^* P\right)\left(L_2^* P\right)^T - \left(L_2^* P\right)\left(L_1^* P\right)^T$$

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To define the image plane, let $P_0\left(X_0 \quad Y_0 \quad Z_0\right)$, $P_1\left(X_1 \quad Y_1 \quad Z_1\right)$ and $P_2\left(X_2 \quad Y_2 \quad Z_2\right)$, be three 3D known points and $P_i\left(X \quad Y \quad Z\right)$ a generic 3D point, all of them belonging to a plane, referred to as image plane I. The general equation of this plane is given by

$$\det\left(\begin{bmatrix} P_1 - P_0 \\ P_2 - P_0 \\ P_i - P_0 \end{bmatrix}\right) = 0 \Leftrightarrow \det\left(\begin{bmatrix} X_1 - X_0 & Y_1 - Y_0 & Z_1 - Z_0 \\ X_2 - X_0 & Y_2 - Y_0 & Z_2 - Z_0 \\ X_i - X_0 & Y_i - Y_0 & Z_i - Z_0 \end{bmatrix}\right) = 0 \Leftrightarrow AX_i + BY_i + CZ_i + D = 0$$

or, using its parametric co-ordinates, by $I = \left[A \quad B \quad C \quad D\right]$.

Besides, any point that belongs to this plane can be expressed by the linear combination of the points P_0, P_1 and P_2, given by $kxP_0 + kyP_1 + kP_2$. From that expression one can conclude that any point from a 2D space vector defined as the image plane, in homogeneous co-ordinates, is given by $p = \left[kx \quad ky \quad k\right]^T$ [5].

When choosing the points P_0, P_1 and P_2 to be used in the image plane definition, it must be considered that if two of them are in the infinity on the horizontal and vertical axis of that plane, the x-slits projection model keeps its generality, but reduces its complexity [3].

The projection of a 3D generic point P in the image plane I origins a 2D point p. This projection is given by the intersection of the projective ray l with the image plane I. This fact implies that p must belong to both planes $L_i^* P$. This idea can be mathematically formalized by the following linear equation,

$$\begin{bmatrix} \left(L_1^* P\right)^T p \\ \left(L_2^* P\right)^T p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} P^T L_1^* P_0 & P^T L_1^* P_1 & P^T L_1^* P_2 \\ P^T L_2^* P_0 & P^T L_2^* P_1 & P^T L_2^* P_2 \end{bmatrix} \begin{bmatrix} kx \\ ky \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow Mp = 0 \qquad (5)$$

Being (5) a homogeneous equation system, its solution is $p = 0$. However, this solution is not the one we are looking for. Eliminating this solution, the solution for the equation system (5) is the right null space of the matrix M. This solution is obtained by the use of the cross product between the elements of the matrix M, as

$$\begin{cases} kx = P^T L_1^* P_1 P^T L_2^* P_2 - P^T L_1^* P_2 P^T L_2^* P_1 \\ ky = P^T L_1^* P_0 P^T L_2^* P_2 - P^T L_1^* P_2 P^T L_2^* P_0 \\ k = P^T L_1^* P_0 P^T L_2^* P_1 - P^T L_1^* P_1 P^T L_2^* P_0 \end{cases} \Leftrightarrow k \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} P^T L_1^* \left(P_1 P_2^T - P_2 P_1^T\right) L_2^* P \\ P^T L_1^* \left(P_0 P_2^T - P_2 P_0^T\right) L_2^* P \\ P^T L_1^* \left(P_0 P_1^T - P_1 P_0^T\right) L_2^* P \end{bmatrix} \Leftrightarrow p = \begin{bmatrix} P^T L_1^* I_0 L_2^* P \\ P^T L_1^* I_1 L_2^* P \\ P^T L_1^* I_2 L_2^* P \end{bmatrix}$$

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where I_0, I_1 and I_2 are the Plüker matrices that represent, in three dimensions, respectively, the X axis and the Y axis of the image and the image line at infinity. According to [3], this solution is unique unless p resides on the line joining the intersections of the two slits with the image plane.

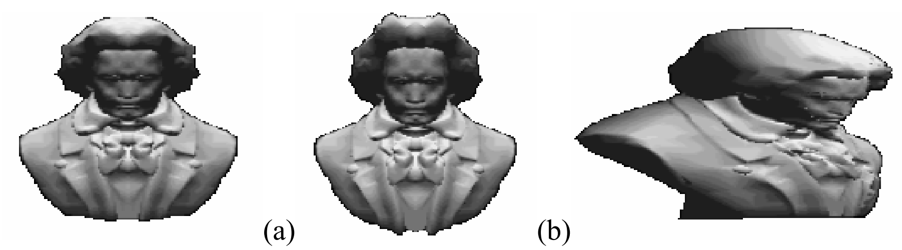
Generating x-slits images

The next proposed task is to generate x-slit images from perspective ones. According to [3], the perspective image sequence must be capture by a translating camera along a horizontal or vertical 3D line. The camera's speed must be approximately constant and its internal parameters must not change. This situation synthesizes x-slits images in a very specific case. If the horizontal slit lies on the path of the optical centre of the moving perspective camera, the vertical slit is parallel to the image's vertical axis. If it is the vertical slit that lies on the path of the optical centre of the moving perspective camera, it is the horizontal slit that is parallel to the image's horizontal axis.

The x-slit generation procedure is done as follows. From each perspective image, a vertical or horizontal strip is sampled, centered, respectively, on the horizontal or vertical co-ordinate. This means that, respectively, column or line i of the x-slit image should be a sample from a corresponding column or line i from the captured image. Then, those strips are past together into the x-slit image.

Experimental results

To enable a better understanding of the x-slits projection model, the generation of a 3D scene (Mozart bust) was simulated using both the x-slits projection and the perspective projection. The results of the simulation are presented in figure 2. Figure 2(a) resulted from the perspective projection model simulation, while figures 2(b) and 2(c) are the result from the x-slit projection model.



(a) (b) (c)

Figure 2 – Simulation of the virtual scenes: (a) in perspective projection; (b) and (c) in x-slits projection.

Comparing figure 2(a) with 2(b) one verifies that, as it was mentioned, image 2(b) is stretched in the vertical direction. In figure 2(b) the slits were in the POX situation, which means that they were parallel and orthogonal between each other. In figure 2(c), the slits were rotated around the axes X, Y and Z.

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Following the procedure described in above section, 246 images were acquired, at a rate of 10 per second, by a camera moving along a horizontal line. The corresponding result was the x-slit image presented in figure 3.

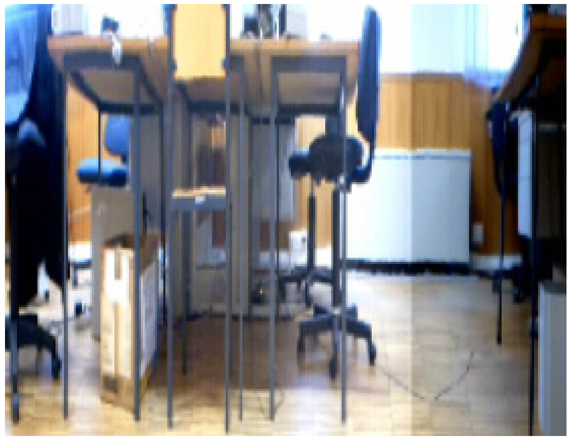

Figure 3 – Real x-slits images, generated with horizontal slices.

Observing the image from figure 3 one can notice that the field of vision was wider than the field of view corresponding to a perspective camera. The image shows also some degree of compression.

Conclusion

In this paper we show results from the application of an alternative image formation model to the generation of images. The images generated by using this model can be used in specific applications, namely in image rendering applications, were images present information obtained from multiple points of view.

Reference

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