Solving Maxwell's Equations by Multiquadrics Method

D. L. Young[1], T. K. Wong[1], C. W. Chen[1]

Summery

The multiquadrics (MQ) method (or Kansa's method) is used to simulate the electromagnetic field problems. The corresponding governing equations are the Maxwell's equations. The time-varying Maxwell's equations are simulated directly at the time-space domain (time-domain method) without transforming them into wave or Helmholtz equations, which are usually adapted in the literature. The multiquadrics method by the time-domain method is able to model the electromagnetic fields as long as the initial conditions and boundary conditions are given, and calculate the electromagnetic wave propagations until the requested time is terminated. MQ scheme is an excellent method not only for very accurate interpolation, but also for the estimates of partial derivatives. It is easy to deal with their appropriate partial derivatives, divergences, curls, gradient or integrals. The numerical model is chosen to calculate and analyze the distribution of the electric and magnetic fields in the homogeneous, isotropic and non-lossy two-dimensional rectangular waveguide and three-dimensional cubic cavity resonator. Good agreements are obtained as compared with analytical solutions.

Introduction

The early development of computational electromagnetism was prompted by intellectual curiosity concerning the implications of Maxwell's unified electric and magnetic field theory, especially after Hertz's verification of the predicted wireless propagation of electromagnetic energy. This early work more or less concentrated on obtaining exact analytical solutions of Maxwell's equations for a variety of diffraction problems. In the 1950s, Keller formulated the geometrical theory of diffraction (GTD). In the mid-1960s, Harrington set the agenda for the next 20 years by working out a systematic, functional-space description of electromagnetic interactions, which he called the method of moments [1].

---

[1]Department of Civil Engineering & Hydrotech Research Institute National Taiwan University, Taipei, Taiwan

In the mid-1960s, Yee introduced a computationally efficient algorithm to directly solve Maxwell's time-dependent curl equations using the finite difference method [2]. It is often called the finite-difference time-domain (FD-TD) method which has been used in many topics in the computational electromagnetism. Kolbehdari solved electromagnetic problems by explicit and implicit time-domain finite element methods using the Whiteny forms.

In this study, an innovative scheme – the multiquadrics method (MQ) or the so called Kansa's method [3 & 4] is used. The multiquadrics method (MQ) is first introduced by Hardy (1971), to approximate topographic surface from scattered data points and has been applied to surveying and mapping problems (1975). MQ scheme is a truly scattered, grid free scheme (or meshless) for representing surfaces and bodies in an arbitrary number of dimensions. The radial basis functions (RBF) depend only upon distances between pairs of points. So it is very powerful to deal with irregular domain problems. In comparison to other numerical methods, Maydych and Nelson (1990) and Wu and Schaback (1993) showed that MQ produces exponential convergence instead of linear or quadric convergence, with minimal semi-norm errors, meaning that MQ can provide a high accuracy of numerical solutions with relatively small amount of collocation points. And MQ scheme is an excellent method not only for very accurate interpolation, but also for partial derivative estimates. It is easy to deal with their appropriate partial derivatives, divergences, curls, gradient or integrals. So error analysis and huge scale computation of differential and integral equations will be much easy to deal with. The distribution of the electric and magnetic fields in the homogeneous, isotropic and non-lossy two-dimensional rectangular waveguide and three-dimensional cubic cavity resonator have been studied and employed to check the validity and efficiency of the MQ scheme in the present work.

Governing Equations

The Maxwell's equations were derived by James Clerk Maxwell (1831-1879), which constitute the theoretical study leading to the discovery of the electromagnetic waves. Maxwell summarized and modified some well-known laws of the electromagnetism, such as Gauss's law, Ampere's law and Faraday's law to form the so called Maxwell's equations. For any electromagnetic field, all the laws or principles are necessary to satisfy the Maxwell's equations. In order to simplify the problem of electromagnetic field to a first attempt, we assume：

1. The waveguide or resonator is filled with the electrically and magnetically linear, homogenous, isotropic and source free dielectric material

   Source free => ρ_v and $\vec{J}$ can both be regarded as zero.
2. The medium obeys the Ohm's law.
3. All materials that in vacuum are assumed to be non-lossy.

The Maxwell's equations (1) to (4) then can be simplified to the following：

$$\nabla \cdot \vec{E} = 0 \tag{1}$$

$$\nabla \cdot \vec{H} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{3}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \tag{4}$$

where $\vec{E}$ is electric field intensity, $\vec{H}$ is magnetic field intensity, $\vec{J}$ is electric current density, ρ_v is electric charge density. These equations are the governing equations for the electromagnetic fields that we need. Therefore, the parameters of permittivity ε and permeability μ are assumed to be constants. In this study, we simply take ε and μ are equal to 1. The boundary conditions cannot be chosen freely. They should depend on the physical situation of the problem. In this study, we presume the boundary material is a perfectly electrical conductor. Consequently, the boundary conditions of the perfect conductor are shown as follow：

$$\vec{n} \cdot \vec{H} = 0 \tag{5}$$

$$\vec{n} \times \vec{E} = 0 \tag{6}$$

Numerical Analysis

In this study, the multiquadrics method (MQ) or so called Kansa's method is used to deal with our numerical simulation. Let,

$$E_{xi} = \sum_{j=1}^{n} a_{Exj} \sqrt{r_{ij}^2 + c^2} \tag{7}$$

$$E_{yi} = \sum_{j=1}^{n} a_{Eyj} \sqrt{r_{ij}^2 + c^2} \tag{8}$$

$$E_{zi} = \sum_{j=1}^{n} a_{Ezj} \sqrt{r_{ij}^2 + c^2} \tag{9}$$

$$H_{xi} = \sum_{j=1}^{n} a_{Hxj} \sqrt{r_{ij}^2 + c^2} \tag{10}$$

$$H_{yi} = \sum_{j=1}^{n} a_{Hyi} \sqrt{r_{ij}^2 + c^2} \tag{11}$$

$$H_{zi} = \sum_{j=1}^{n} a_{Hzj} \sqrt{r_{ij}^2 + c^2} \tag{12}$$

$$\left(\frac{\partial E_x}{\partial x}\right)_i = \sum_{j=1}^{n} a_{Exj} \left(\frac{x_i - x_j}{\sqrt{r_{ij}^2 + c^2}}\right) \tag{13}$$

where $r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2$ i = 1 ~ n

Equations (7) ~ (13) are substituted to equations (1) ~ (4) and the time derivative can be dealt with by the finite difference method of time marching scheme. For details it is referred to Wong's work [5].

Numerical Results and Discussions

For our numerical simulation, the dimension of the two-dimensional rectangular waveguide and three-dimensional cubic cavity resonator are with length a=1, width b=1and height h=1. We further take the value of Kansa's method parameter c = 0.246, with the mesh size 14*14 for the two-dimensional waveguide. Fig. 1 represents the time evolution history of E_x, E_y for TE_z mode respectively at fixed points (x, y) = (0.1428, 0.1428) for two-dimensional problem. And we further take Kansa's method parameter c=0.5, with the mesh size 8*8*8, for the three-dimensional cavity resonator. Fig. 2 represents the time evolution history of E_Z, and H_y respectively for TM mode at fixed point (x, y, z) = (0.75, 0.25, 0.25) for three-dimensional problem. The comparisons of both cases with the analytic solutions have demonstrated that the MQ algorithm is a very efficient and accurate tool for the simulation of the computational electromagnetism using the time-varying Maxwell's equations.

Conclusions

The time-varying Maxwell's equations are simulated directly at the time-space domain (time-domain method) without transforming them into wave or Helmholtz equations in the present work. The multiquadrics method by time-domain combination is capable to model the electromagnetic wave propagation and good agreements as comparing with the analytic solutions are obtained. The good performance of the MQ (Kansa's) scheme demonstrated by the two-dimensional waveguide and three-dimensional cavity resonator has shown that it is a powerful tool for the numerical solution of electromagnetic field problems.

References

1. Harrington, R. F. (1968): *Field Computation by Moment Method*, Macmillanm New york.
2. Yee, K. S. (1966): "Numerical Solution of Initial Boundary Value Problems involving Maxwell's Equations in Isotropic Media", *IEEE Trans. Antennas Propaga.*, Vol. AP-14, No.3, pp.302-307.
3. Kansa, E. J. (1990): "Multiquadrics-A Scattered Data Approximation Scheme with Applications to Computational Fluid-dynamics-I. Surface Approximation and Partial Derivative Estimates", *Computers and Math. Applc.* , Vol.19, pp.127-145.
4. Kansa, E. J. (1990): "Multiquadrics-A Scattered Data Approximation Scheme with Applications to Computational Fluid-dynamics-II: Solutions to Parabolic, Hyperbolic and Elliptic Partial Differential Equations" *Computers and Math. Applc.* , Vol.19, pp.147-161.

5. Wong, T. K. (2002): “Analysis of Meshless Methods on Simple Electromagnetism and Flow Problems”, *MS Thesis*, National Taiwan University, Taipei, Taiwan.

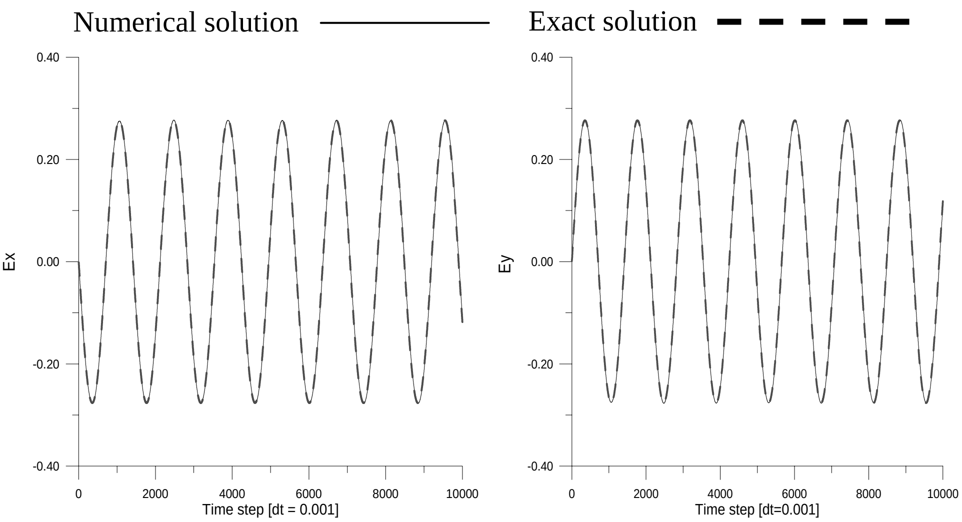

Fig. 1 The time history of (a) E_x (b) E_y, for TE_z mode of two-dimensional waveguide

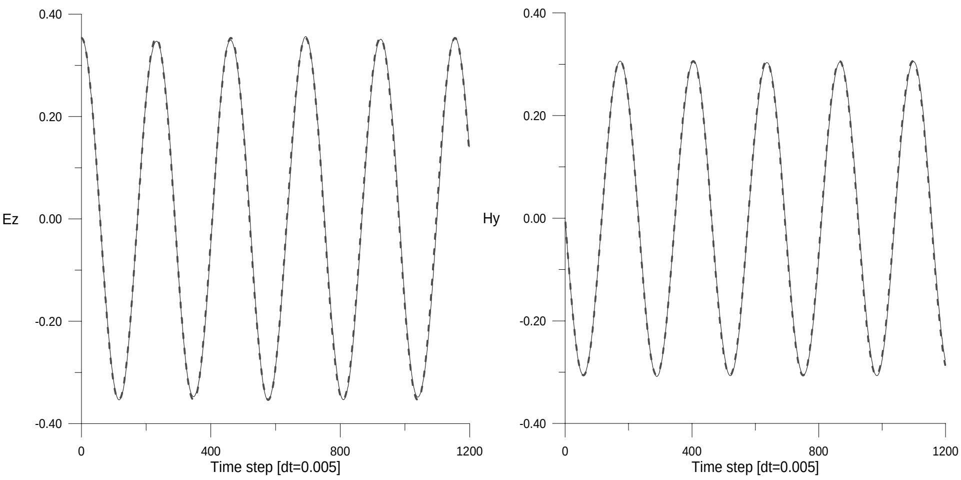

Fig. 2 The time history of (a) E_z (b) H_y, for TM mode of three-dimensional cavity resonator