Transient Solution of a Non-Linear Thermoelastic Instability Problem

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Summary

Sliding systems with frictional heating such as brakes and clutches often encounter variation in the contact area at the interface, making the thermoelastic contact problem non-linear. The transient evolution of the temperature field for a linear contact problem with frictional heating and constant sliding speed is obtained through superimposing the solution of the perturbation problem to the steady state solution. An approximate solution is sought in which the contact area variation in time is treated as piecewise constant, allowing the contact problem to be linear for a small time step. This method is presented here and tested in the context of a thermoelastic problem involving two contacting surfaces.

Introduction

During brake application or engagement of a transmission clutch, frictional heat is generated and the resulting non-uniform temperature cause thermoelastic distortion which in turn affects the contact pressure distribution [1]. The thermoelastic contact problem is linear as long as the contact area is constant, and the transient evolution of the temperature field can be written as a sum of eigenfunction series and steady state solution [2, 3]. The width of the contact area, however, may vary during the clutch or brake application, and hence the thermoelastic contact problem becomes non-linear and the solution of eigenfunction series expansion and steady state cease to apply. Evidence of contact area separation in clutch application has been observed [4] when the initial sliding speed is sufficiently high.

In this paper, we explore an approximate method for solving the non-linear thermoelastic contact problem. In this solution, the contact area is treated as a piecewise constant in time, which allows the problem to become linear during a small time step. This method will be tested on a single-sided clutch problem, in which the transient solution of the unperturbed axisymmetric problem is sought.

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Method of Solution

As for a linear non-homogenous differential equation, the general solution can consist of a homogenous and a particular solution

$$T(x,y,z,t) = T_h(x,y,z,t) + \theta_p(x,y,z,t), \tag{1}$$

where T_h and θ_p are respectively, the homogenous and particular solutions for the temperature field. Identifying solutions of the exponential form [5] for the homogenous problem

$$T_h(x,y,z,t) = e^{b_i t}\theta_i(x,y) \tag{2}$$

and solving the heat conduction, thermoelasticity and the boundary conditions leads to an eigenvalue problem for the exponential growth rate b_i and the associated eigenfunctions θ_i. The steady state solution can be used in place of the particular solution [6]. If the sliding speed is constant and the contact area is remained unchanged, the general solution for the transient evolution of the temperature field can be written as:

$$T(x,y,z,t) = \sum_{i=1}^{n} C_i\theta_i(x,y)e^{b_i t} + \theta_s(x,y) \tag{3}$$

where C_i is a set of arbitrary constants determined from the initial condition $T(\mathrm{x},\mathrm{y},0)$.

The eigenvalue b_i, eigenfunction θ_i, and the steady state solution θ_s of the expansion series are fixed as long as the sliding speed and the contact area remain unchanged. The contact area along the interface however may shift or reduce in size during brake application or clutch engagement, which in turn modifies the eigenvalues and eigenfunctions. Hence, the transient solution (3) ceases to apply.

The change in the contact area, Γ_c, can be treated as a piecewise constant in time. In other words, the system experiences a sequence of time periods at different constant contact areas. In mathematical terms, we can write

$$\Gamma_c(t) = \sum_{i=1}^{M} \Gamma_{cj} v_j(t), \tag{4}$$

where $v_j(t)$ are piecewise constant shape functions defined by

$$v_j(t) = 1 \;\; ; \;\; t_{j-1} < t < t_j \tag{5}$$

$$v_j(t) = 0 \quad ; \quad t < t_{j-1} \text{ and } t > t_j \tag{6}$$

and t_j are a set of nodal times with $t_0 = 0$.

During the *j*-th time period, the sliding speed is constant and equal to V_j and we can write the temperature field in the eigenfunction series

$$T^{(j)}(x, y, z, t) = \sum_{i=1}^{n} C_i^{(j)} e^{b_i^{(j)}(t - t_{j-1})} \theta_i^{(j)}(x, y, z) + \theta_s^{(j)}(x, y, z) \tag{7}$$

where $b_i^{(j)}$, $\theta_i^{(j)}$, $\theta_s^{(j)}$ are the eigenvalues, eigenfunctions, steady state temperatures respectively appropriate to contact area $\Gamma_{c\,j}$ and we have chosen to reset the zero for time in each time step.

Continuity of temperature at time t_j then requires that

$$T^{(j+1)}\left(x, y, z, t_j^+\right) = T^{(j)}\left(x, y, z, t_j^-\right) \tag{8}$$

and hence,

$$\sum_{i=1}^{n} C_i^{(j+1)} \theta_i^{(j+1)} + \theta_s^{(j+1)} = \sum_{i=1}^{n} C_i^{(j)} e^{b_i^{(j)}\left(t_j - t_{j-1}\right)} \theta_i^{(j)} + \theta_s^{(j)} \tag{9}$$

from equation (8). In other words, at the end of each time step, we need to re-expand the instantaneous temperature field as a series of the eigenfunctions appropriate to the next time step.

The viability of this idea was examined in the context of a single-sided clutch problem (Figure 1). The geometry is discretized by the finite element method; the instantaneous temperature field can be characterized by a vector $\boldsymbol{\Theta}$ whose components are the *n* nodal temperature. The representation (1) leads to n×n eigenvalue problem for θ [7] and it follows that there will be *n* terms in the eigenfunction series (3). Similarly the finite element solution of the steady state problem leads to a vector $\boldsymbol{\Theta}_s$ of *n* terms. A general solution for the evolution of the nodal temperature $T_i(t)$ at constant speed and contact area can then be written as:

$$T_i(t) = \sum_{k=1}^{n} C_k \Theta_k^i e^{b_k t} + \Theta_s^k \tag{10}$$

where the n constants C_k are to be determined from the n equations defining the initial nodal temperatures $T_i(0)$. This representation can be generalized to cases where the contact area is piecewise constant, as in (7). To assess the accuracy of this solution, a finite element solution was developed for the same transient thermoelastic contact problem using commercial FE package.

Results

The stability of the system is determined by the speed at which the first eigenfunction in equation (2) becomes unstable. This critical speed, ω_c, was evaluated for the single-sided clutch shown in Figure (1) and used to normalize the operating speeds in the figures presented in this paper.

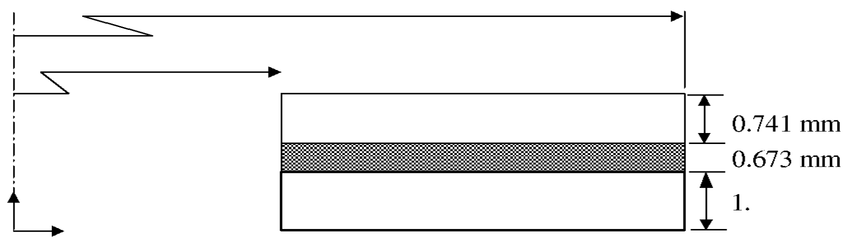

Figure 1: Single-sided clutch system

Figures 2 and 3 show the contact pressure distribution at different instances of time for $\omega = 1.5\omega_c$. Non-uniform distribution of the contact pressure is observed where the distribution initially forms a peak near the outer radius of the disk. This peak then shifts over time toward the main radius of the disk. Separation in the contact area starts at the outer radius, which is anticipated since the sliding speed and therefore the frictional heat generation is function of the disk radii. The contact area is reduced by almost 75% whereas the amplitude of the contact pressure is increased almost seven times the initial value. The transient solution of the contact pressure distribution was also evaluated for a sliding speed of $\omega = 0.75\omega_c$, as shown in Figure 4. To establish a comparison between the two speeds the same average heat flux of 2 Mw/mm^2 is used, which is achieved by increasing the applied pressure p_0. Although the operating speed is below the critical value, non-uniformity in the pressure distribution can still be observed. This non-uniformity, however, is much less severe than that observed in Figure 3. The system at this speed however takes longer time to converge to steady state.

The proposed method of solving the non-linear contact problem was proven to be more efficient compared to the conventional finite element solution. The number of time steps required to reach a certain level of accuracy was noted to be less than that needed by the conventional solution. Furthermore, when the system operates above the critical speed few unstable eigenfunctions dominate the output of the system. A reduced order model can be constructed by truncating the series in (10) at some values less than n. This in turn enhances the efficiency of the solution in term of reducing the computational time.

Conclusion

In this paper, an approximate solution to the transient non-linear thermoelastic contact problem with frictional heating was explored, where the source of non-linearity is the variation of the contact area during the transient process. The contact area is approximated by a piecewise constant representation. This method was tested in the context of a single-sided clutch system for a constant sliding speed. When sliding speed is above the critical value, a significant reduction in contact area was observed contributing to high local contact pressure. For a sliding speed below the critical value, the reduction in the contact area is moderate and the system takes relatively a longer time to reach a steady state. The number of time steps required for a given level of accuracy was observed to be less for this solution. Moreover, a reduced model can be constructed by truncating the number of the eigenfunctions in the expansion series of the transient solution, resulted in an extra reduction in the computational time.

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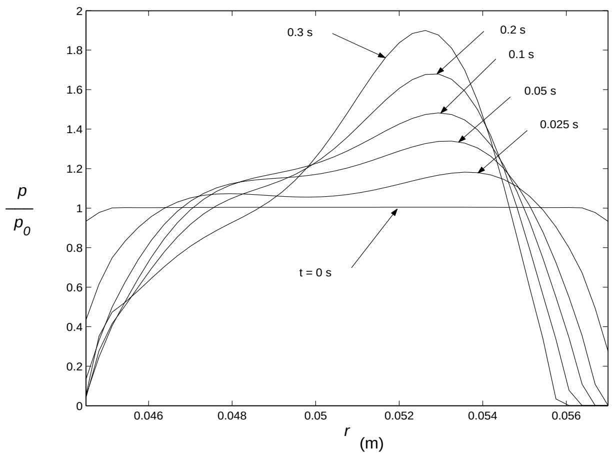
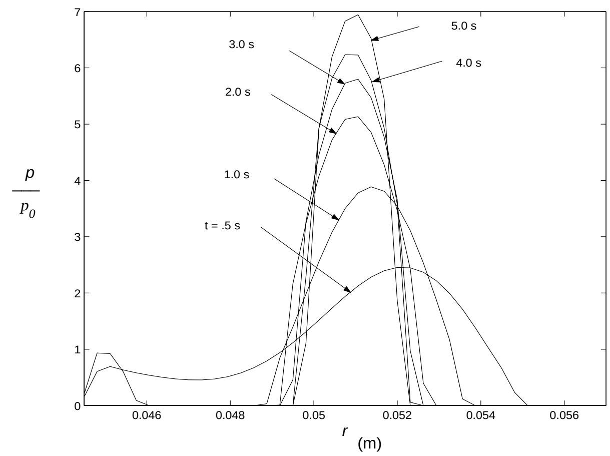
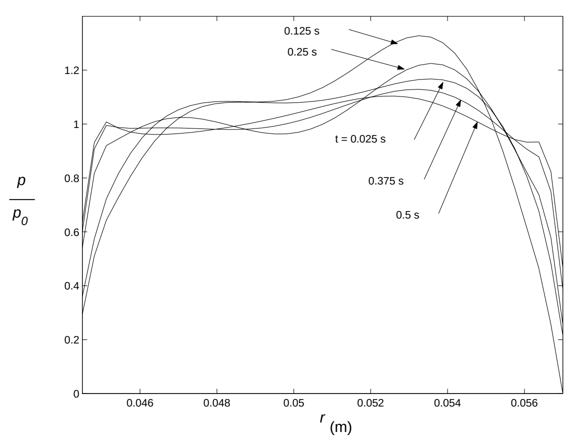
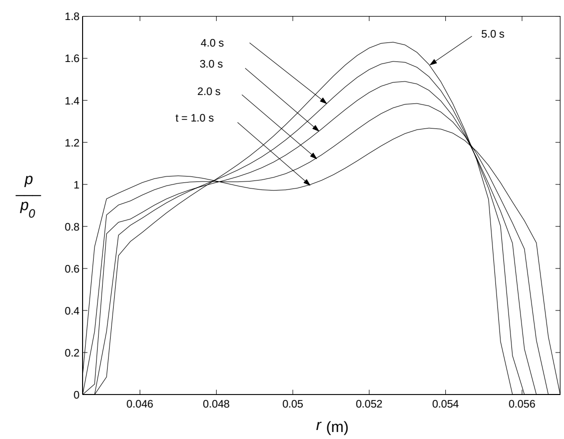




Figures (2, 3) Contact pressure distribution at different instants of time for $\omega(0) = 1.5\ \omega_c$





Figures (4, 5) Contact pressure distribution at different instants of time for $\omega(0) = 0.75\ \omega_c$