Some Parallel Strategies for an Epidemic Genetic Algorithm Applied to an Inverse Heat Conduction Problem

Sabrina B. M. Sambatti, Haroldo F. de Campos Velho, Leonardo D. Chiwiacowsky, Stephan Stephany, Airam J. Preto[1]

Summary

Different strategies are investigated for the parallel implementation of a genetic algorithm (GA). The parallel GA (PGA) is employed to solve the inverse heat conduction problem of determining the initial temperature from the transient temperature noisy profile at a given time. The parallel code was generated using calls to the MPI (Message Passing Interface) library. Each processor executes the GA in its own population and migration of best-fitness individuals occurs periodically among processors. An epidemic operator purges each population whenever there is not fitness improvement. In this work, different migration topologies are tested, and the corresponding performance results, quality and convergence of the solution are discussed.

Introduction

Genetic Algorithms (GA) are methods of optimization based on human evolution mechanisms where the fittest ones survive and/or reproduce. The GA scheme has been successfully employed in many applications, although its efficiency in finding an optimal global solution depends on some parameters, such as the population size, selection and mutation policies, and the criterion for replacing individuals in the population.

One of the main aspects of GAs is its intrinsic suitability for parallel codification. A parallel genetic algorithms (PGA) may consist of independent populations being evolved in each processor. The parallel code was generated using calls to the message passing communication library MPI (Message Passing Interface) [Pacheco, 1996]. Each processor executes the GA in its own population and migration of best-fitness individuals occurs periodically among processors. An epidemic operator is proposed to purge each population whenever there is not fitness improvement. A new population is then generated and only the best-fitness individual of the former is preserved.

This work compares the performance of different migration topologies. The PGA is employed to solve the inverse heat conduction problem of determining the initial temperature from the transient temperature noisy profile at a given time. The inverse problem is formulated as an optimization problem, solved by the PGA. Each individual corresponds to a candidate solution and its fitness is evaluated by a square difference between model and given temperature profiles. The application of PGAs

[1]Laboratório Associado de Computação e Matemárica Aplicada & Instituto Nacional de Pesquisas Espaciais, São José dos Campos, Caixa Postal 515 12245-970 SP Brasil

to optimization problems is an open problem, since there are several parameters to be adjusted in order to achieve the best results.

Inverse Problem

The direct problem consists on a transient heat conduction problem in a slab with adiabatic boundary condition and $T(x,t=0)=f(x)$ (initial condition). The mathematical formulation of the problem, that is one-dimensional, is given by:

$$\frac{\partial^2(x,t)}{\partial x^2}=\frac{\partial T(x,t)}{\partial t} \;, \qquad x\in(0,\ 1)\;, \qquad t>0\;, \tag{1}$$

$$\frac{\partial T(x,t)}{\partial x}=0, \qquad x=0\;, \qquad x=1, \quad t>0\;, \tag{2}$$

$$T(x,t)=f(x)\;, \quad x\in[0,\ 1]\;, \quad t=0\;, \tag{3}$$

where $T(x,t)$ denotes temperature at a given point and time, $f(x)$, the initial condition, x, the spatial variable, and t, the time. These variables are treated as nondimensional quantities. The solution of the direct problem for $x\in(0,\ 1)$ and $t>0$ for a given initial condition $f(x)$ is given by

$$T(x,t)=\sum_{m=0}^{\infty} e^{-\beta_m^2 t}\frac{1}{N(\beta_m)}X(\beta_m,x)\int_0^1 X(\beta_m,x')f(x')dx' \tag{4}$$

where $X(\beta_m,x)$ are the eigenfunctions associated to the problem, and β_m and $N(\beta_m)$ are, respectively, the corresponding eigenvalues and *norms* [Özisik, 1980]. The function $f(x)$ is assumed to be bounded satisfying Dirichlet's conditions in the interval $[0,\ 1]$. In this transient conduction problem, the goal is to estimate the unknown initial temperature distribution $f(x)$, from the knowledge of the measured temperature T_i at the time $t=\tau>0$, for a finite number (N_{grid}) of different locations x_i in the domain. This inverse problem is formulated as an optimization problem, as follows.

For determining the initial condition, a regularized solution is obtained by choosing the function $f(x)$ that minimizes the following functional:

$$J(f(x),\xi) \;=\; \|T^{\mathrm{Mod}} \;-\; T^{\mathrm{Exp}}\|_2^2 \;+\xi \sum_{k=1}^{N_{sol}} f_k^2 \;, \tag{5}$$

where $T^{\mathrm{Exp}}=T^{\mathrm{Exp}}(x_i,\tau)$ is the experimental data ($t=\tau>0$) and T^{Mod} is the temperature obtained using the candidate solution $f(x)$, and $\|.\|_2$ is the norm 2. The last term is the 0th-order Tikhonov regularization term [Tikhonov and Arsenin, 1977], weighted by ξ, the regularization parameter. Each candidate solution f_k is sampled by a set of N_{sol} discrete points f_k, with $k = 1, 2, \ldots, N_{sol}$. In the genetic algorithm approach, each individual is a particular instance f_k and its fitness is given by this functional.

Parallel Genetic Algorithm with Epidemic Operator

In a GA-based optimizer, an initial population is generated, composed by a group of random individuals, each one associated to a point of the domain. Every individual is evaluated, being assigned a score or fitness, according to the value of the function it maps. In every new generation, a new population of individuals is generated from the former one, by means of combining "parents" individuals using selection, crossing-over and mutation operators. Some schemes preserve best-fitness individuals from one generation to another. It is expected that, after many generations, the population will evolve and better-fitted individuals will appear. An epidemic genetic operator [Chiwiacowsky and de Campos Velho, 2002] is also used in this work. If the fitness does not improve after some number of generations, the population is renewed, preserving only best-fitness individuals.

An important feature of GA's is their suitability for parallelization. Nowadays GA's have been widely employed, but parallel implementations are recent. According to [Cantú-Paz, 1995], GA parallelization techniques are divided in: (a) Global Parallelization: in this approach all genetic operators and the evaluation of all individuals are explicitly parallelised; (b) Coarse Grained: such approach requires a division of population into some number of demes (subpopulations) that are separated one from another ('geographic isolation'). The individuals compete only within a deme. This approach introduces an additional operator called *migration* that is used to send some individuals from one deme to another; (c) Fine Grained: such approach requires a large number of processors because the population is divided into a large number of small demes, and each one evolves separately, but subject to migration.

Many GA researchers believe that a PGA, with its multiple distributed subpopulations and local rules and interactions, is a more realistic model for the evolution of species in nature than a single large population [Levine, 1994]. This work follows the coarse grained approach and implements some migration strategies.

In the island model, best-fitness individuals may migrate to all other processors. Island-1 denotes the sending of each processor best solution to a master processor that selects and broadcasts the "best-of-the-bests" to all others. Island-2 denotes multiple brodcasts in which each processor sends its particular best solution to all others. In the stepping-stone model, a logical ring of processors is defined and communication occurs in steps as each processor send its best individual to the left and right-side neighbours. After a finite number of steps, all processors have the best global solution. In the proposed implementation using a circular pipeline,

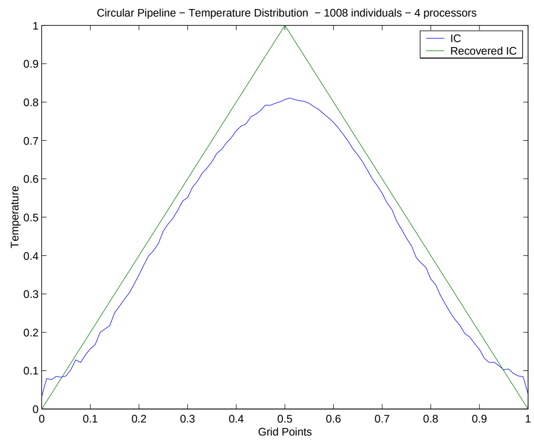

Figure 1: Reconstructed and exact initial temperature profile.

a logical ring is also defined and migration is also restricted to neighbours, but a communication sense is defined in a way that each processor always receive messages from the left neighbour and always send to the right one (assuming a clockwise sense). Considering a ring of p processors, after $p-1$ steps, all processors will have the global best individual.

Numerical Results

A PGA is used to solve the backward heat conduction problem for a one-dimensional slab. The chosen test case assumed the following initial temperature profile, given by the triangular test function

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 0.5 \\ 2(1-x), & 0.5 \leq x \leq 1; \end{cases}$$

The experimental data, corresponding to the measured temperatures at a time $\tau > 0$ is obtained from the the direct problem by adding a gaussian noise with 5% amplitude. For numerical purposes, it was adopted $\tau = 0.01$ and an experimental data grid of 101 points (N_{grid}). In order to accomplish the inversions, some parameters of the parallel genetic algorithm were defined: (a) fixed population size of 1008 individuals; (b) geometrical crossover operator $\mu = 1/2$; (c) non-uniform mutation operator $b = 5$; (d) mutation probability 5%; (e) epidemical operator (the best 5 individuals are kept); and (f) fixed maximal generation number of 10000.

The reconstructed initial temperature profile using 1008-individual population, 4 processors and the circular pipeline scheme, is compared to the exact profile in Fig. 1. The use of more processors split the population into smaller demes. Therefore, as each deme has less individuals, its variability is smaller and it can

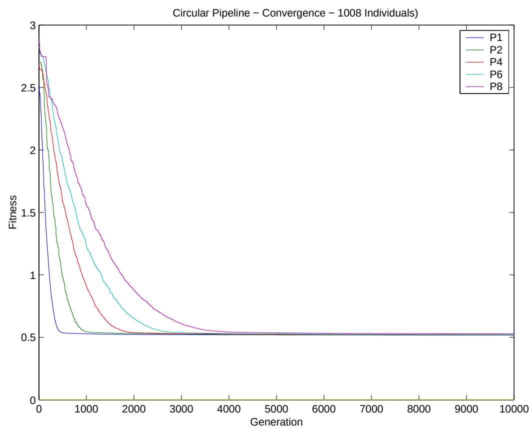

Figure 2: Convergence of the PGA solution.

Table 1: Performance values.

No. of processors	1	2	4	6	8
Communication time (sec)	9.882	9.976	9.703	4.536	0.001
Total Time (sec)	508.595	256.404	128.199	86.493	65.748
Speed-up	1.000	1.983	3.967	5.880	7.735
Efficiency	1.000	0.991	0.991	0.980	0.996

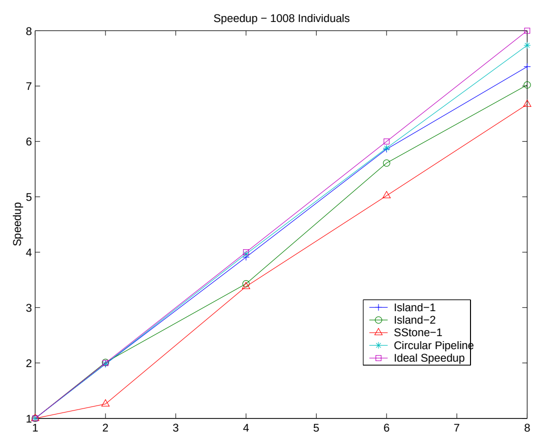

Figure 3: Speed-up for the 1008-individual population.

be expected a slower evolution – see Fig. 2. Usually, the performance of parallel implementation can be roughly evaluated by the *speed-up* $S_p = T_1/T_p$, being T_1 the sequential time and T_p the parallel time for p-processors. Another definition is the *efficiency* $E_p = S_p/p$. Table 1 shows speed-up and efficiency for different number of processors, using the 1008-individual population and the circular pipeline.

Conclusion

The inverse problem of estimating the unknown initial condition of heat conduction transfer in a one-dimensional slab was solved using a parallel genetic algorithm, that uses a new evolutionary operator called *epidemical*. The PGA was implemented using some migration topologies, two versions of island model, the stepping-stone model and the circular pipeline. The latter presented the best performance. The PGA performance is good, with speed-ups close to the linear one. However, the increase of the number of processors lead to a sligthly slower convergence of the solution due to the smaller demes.

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References

[Cantú-Paz, 1995] Cantú-Paz, E. (1995). A summary of research on parallel genetic algorithms. Illigal 95007, Illinois Genetic Algorithms Laboratory - University of Illinois at Urbana-Chapaign.

[Chiwiacowsky and de Campos Velho, 2002] Chiwiacowsky, L. and de Campos Velho, H. F. (2002). Different approaches for the solution of a backward heat conduction problem. *Inverse problem in engineering.*

[Levine, 1994] Levine, D. (1994). A parallel genetic algorithm for the set partitioning problem. Master's thesis, Mathematics and Computer Science Division - Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439. ANL-94/23.

[Pacheco, 1996] Pacheco, P. (1996). *Parallel Programming with MPI.* Morgan Kaufmann, USA.

[Tikhonov and Arsenin, 1977] Tikhonov, A. V. and Arsenin, V. Y. (1977). *Solutions of Ill-posed Problems.* Winston and Sons, New York.

[Özisik, 1980] Özisik, M. N. (1980). *Heat conduction.* Wiley Interscience, USA.